EECS 442 – Computer vision

Cameras

without cameras we wouldn’t have C.V.

• Pinhole cameras
• Cameras & lenses
• The geometry of pinhole cameras
• Other camera models

Reading: [FP] Chapters 1 – 3
[HZ] Chapter 6

Some slides in this lecture are courtesy to Profs. J. Ponce, S. Seitz, F-F Li
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How do we see the world?

• Let’s design a camera
  – Idea 1: put a piece of film in front of an object
  – Do we get a reasonable image?
Pinhole camera

- Add a barrier to block off most of the rays
  - This reduces blurring
  - The opening known as the aperture
Pinhole camera

f = focal length

c = center of the camera
Some history...

Milestones:
• Leonardo da Vinci (1452-1519): first record of camera *obscura*
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• Joseph Nicephore Niepce (1822): first photo - birth of photography

Photography (Niepce, “La Table Servie,” 1822)
Some history...

Milestones:
- Leonardo da Vinci (1452-1519): first record of camera *camera obscura*
- Johann Zahn (1685): first portable camera
- Joseph Nicephore Niepce (1822): first photo - birth of photography
- Daguerréotypes (1839)
- Photographic Film (Eastman, 1889)
- Cinema (Lumière Brothers, 1895)
- Color Photography (Lumière Brothers, 1908)
Let’s also not forget…

Motzu  
(468-376 BC)

Aristotle  
(384-322 BC)

Ibn al-Haitham  
(965-1040)
Pinhole camera

\[ P = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \rightarrow P' = \begin{bmatrix} x' \\ y' \end{bmatrix} \]

\[
\begin{aligned}
x' &= f \frac{x}{z} \\
y' &= f \frac{y}{z}
\end{aligned}
\]

Derived using similar triangles
Pinhole camera

\[ P = [x, z] \]

\[ P' = [x', f] \]

\[ \frac{x'}{f} = \frac{x}{z} \]
Common to draw image plane in front of the focal point. Moving the image plane merely scales the image.

\[ \begin{align*}
    x' &= f \frac{x}{z} \\
    y' &= f \frac{y}{z}
\end{align*} \]
Pinhole camera

Is the size of the aperture important?
Shrinking aperture size

- Rays are mixed up

- Why the aperture cannot be too small?
  - Less light passes through
  - Diffraction effect

Adding lenses!
Cameras & Lenses

- A lens focuses light onto the film
• A lens focuses light onto the film
  – Rays passing through the center are not deviated
  – All parallel rays converge to one point on a plane located at the *focal length* $f$
A lens focuses light onto the film
- There is a specific distance at which objects are “in focus”
  [other points project to a “circle of confusion” in the image]
Cameras & Lenses

Snell’s law

\[ n_1 \sin \alpha_1 = n_2 \sin \alpha_2 \]

- \( \alpha_1 \) = incident angle
- \( \alpha_2 \) = refraction angle
- \( n_i \) = index of refraction
Thin Lenses

Snell’s law:

\[ n_1 \sin \alpha_1 = n_2 \sin \alpha_2 \]

Small angles:

\[ n_1 \alpha_1 \approx n_2 \alpha_2 \]

\[ n_1 = n \, \text{(lens)} \]

\[ n_1 = 1 \, \text{(air)} \]

\[ \frac{1}{z'} - \frac{1}{z} = \frac{1}{f} \]

\[ f = \frac{R}{2(n-1)} \]
Thin Lenses

\[
\begin{align*}
\begin{cases}
  x' &= Z' \frac{X}{Z} \\
y' &= Z' \frac{Y}{Z} \\
z' &= f + z_o
\end{cases}
\end{align*}
\]

\[
f = \frac{R}{2(n-1)}
\]
Lenses are combined in various ways...
Issues with lenses: Chromatic Aberration

• Lens has different refractive indices for different wavelengths: causes color fringing

Near Lens Center

Near Lens Outer Edge
Issues with lenses: Spherical aberration

- Rays farther from the optical axis focus closer
Issues with lenses: Radial Distortion

- Caused by imperfect lenses
- Deviations are most noticeable for rays that pass through the edge of the lens

No distortion

Pin cushion

Barrel
Cameras

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- The geometry of pinhole cameras
- Other camera models
Pinhole perspective projection

Pinhole camera

\[ (x, y, z) \rightarrow (f \frac{x}{z}, f \frac{y}{z}) \]

\[ \mathbb{R}^3 \rightarrow \mathbb{R}^2 \]

\( f = \text{focal length} \)

\( c = \text{center of the camera} \)
\[
\frac{x'}{f} = \frac{x}{z}
\]

Is this a linear transformation?

No — division by z is nonlinear

How to make it linear?
Homogeneous coordinates

\[(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \]

homogeneous image coordinates

\[(x, y, z) \Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \]

homogeneous scene coordinates

• Converting *from* homogeneous coordinates

\[
\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w) \\
\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \Rightarrow (x/w, y/w, z/w)
\]
Perspective Projection Transformation

\[
X' = \begin{bmatrix}
    f & x \\
    f & y \\
    z & 1
\end{bmatrix}
= \begin{bmatrix}
f & 0 & 0 & 0 \\
0 & f & 0 & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z \\
1
\end{bmatrix}
\]

\[
X' = M X
\]

\[\mathbb{R}^4 \rightarrow \mathbb{R}^3\]
Normalized image plane

1. Off set

\[(x, y, z) \rightarrow (f \frac{x}{z} + c_x, f \frac{y}{z} + c_y)\]
Normalized image plane

1. Off set
2. From metric to pixels

\[(x, y, z) \to (f k \frac{x}{z} + c_x, f l \frac{y}{z} + c_y)\]

Units: \(k, l: \text{pixel/m}\)  Non-square pixels
\(f: m\)  \(\alpha, \beta: \text{pixel}\)
Normalized image plane

\[(x, y, z) \rightarrow (\alpha \frac{x}{z} + c_x, \beta \frac{y}{z} + c_y)\]

Matrix form?
Camera Matrix

\[
(x, y, z) \rightarrow (\alpha \frac{x}{z} + c_x, \beta \frac{y}{z} + c_y)
\]

\[
X' = \begin{bmatrix}
\alpha x + c_x z \\
\beta y + c_y z \\
z
\end{bmatrix} = \begin{bmatrix}
\alpha & 0 & c_x & 0 \\
0 & \beta & c_y & 0 \\
0 & 0 & 1 & 0 \\
z
\end{bmatrix} \begin{bmatrix}
x \\
y \\
z \\
1
\end{bmatrix}
\]
Camera Matrix

\[
X' = M X = K [ I \ 0 ] X
\]

\[
X' = \begin{bmatrix}
\alpha & 0 & c_x & 0 \\
0 & \beta & c_y & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
x \\ y \\ z \\ 1
\end{bmatrix}
\]
Finite projective cameras

\[
\begin{bmatrix}
X' = \begin{bmatrix}
\alpha & s & c_x & 0 \\
0 & \beta & c_y & 0 \\
0 & 0 & 1 & 0 \\
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z \\
1 \\
\end{bmatrix}
\end{bmatrix}
\]

K has 5 degrees of freedom!
World reference system
3D Rotation of Points

Rotation around the coordinate axes, counter-clockwise:

\[
R_x(\alpha) = \begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \alpha & -\sin \alpha \\
0 & \sin \alpha & \cos \alpha
\end{bmatrix}
\]

\[
R_y(\beta) = \begin{bmatrix}
\cos \beta & 0 & \sin \beta \\
0 & 1 & 0 \\
-\sin \beta & 0 & \cos \beta
\end{bmatrix}
\]

\[
R_z(\gamma) = \begin{bmatrix}
\cos \gamma & -\sin \gamma & 0 \\
\sin \gamma & \cos \gamma & 0 \\
0 & 0 & 1
\end{bmatrix}
\]
World reference system

In 4D homogeneous coordinates:

\[ X = \begin{bmatrix} R & T \end{bmatrix} X_w \]

\[ X' = M X_w = K \begin{bmatrix} R & T \end{bmatrix} X_w \]

Internal parameters

External parameters
Projective cameras

\[ X'_3 = M X_w = K_{3 \times 3} [R \quad T]_{3 \times 4} X_w_{4 \times 1} \]

How many degrees of freedom?

5 + 3 + 3 = 11!
Projective cameras

\[ X'_3 \times 1 = M X_w = K_{3 \times 3} [R \ T]_{3 \times 4} X_{w 4 \times 1} \]

\[ (x, y, z)_w \rightarrow \left( \frac{m_1 X_w}{m_3 X_w}, \frac{m_2 X_w}{m_3 X_w} \right) \]

M is defined up to scale!

Multiplying M by a scalar won’t change the image.
Theorem (Faugeras, 1993)

Let \( \mathcal{M} = (\mathcal{A} \ b) \) be a \( 3 \times 4 \) matrix and let \( a_i^T \) \( (i = 1, 2, 3) \) denote the rows of the matrix \( \mathcal{A} \) formed by the three leftmost columns of \( \mathcal{M} \).

- A necessary and sufficient condition for \( \mathcal{M} \) to be a perspective projection matrix is that \( \text{Det}(\mathcal{A}) \neq 0 \).

- A necessary and sufficient condition for \( \mathcal{M} \) to be a zero-skew perspective projection matrix is that \( \text{Det}(\mathcal{A}) \neq 0 \) and

\[
(a_1 \times a_3) \cdot (a_2 \times a_3) = 0.
\]

- A necessary and sufficient condition for \( \mathcal{M} \) to be a perspective projection matrix with zero skew and unit aspect-ratio is that \( \text{Det}(\mathcal{A}) \neq 0 \) and

\[
\begin{cases} 
(a_1 \times a_3) \cdot (a_2 \times a_3) = 0, \\
(a_1 \times a_3) \cdot (a_1 \times a_3) = (a_2 \times a_3) \cdot (a_2 \times a_3).
\end{cases}
\]
Properties of Projection

• Points project to points
• Lines project to lines
Properties of Projection

- Distant objects look smaller

\[ x' = f' \frac{x}{z} \]
\[ y' = f' \frac{y}{z} \]
Properties of Projection

• Angles are not preserved
• Parallel lines meet!

Vanishing point
Vanishing points

• Each set of parallel lines meets at a different point
  [The *vanishing point* for this direction]

• Sets of parallel lines on the same plane lead to *collinear*
  vanishing points [The line is called the *horizon* for that plane]
One-point perspective

• Masaccio, *Trinity*, Santa Maria Novella, Florence, 1425-28
Properties of Projection
Objects on the periphery are expanded

• The exterior columns appear bigger
• The distortion is not due to lens flaws
• Problem pointed out by DaVinci
Properties of Projection

• Degenerate cases
  – Line through focal point projects to a point.
  – Plane through focal point projects to line
  – Plane perpendicular to image plane projects to part of the image (with horizon).
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Weak perspective projection

\[
\begin{aligned}
&x' = -mx \\
y' = -my
\end{aligned}
\]

where \( m = -\frac{f'}{z_0} \) is the magnification.

When the scene depth is small compared its distance from the Camera, \( m \) can be taken constant: weak perspective projection.
Orthographic (affine) projection

- Distance from center of projection to image plane is infinite

\[
\begin{align*}
    x' &= x \\
    y' &= y
\end{align*}
\]

When the camera is at a (roughly constant) distance from the scene, take \( m = 1 \).
Orthographic (affine) projection

\[ K = \begin{bmatrix} \alpha & s & c_x & 0 \\ 0 & \beta & c_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \]

\[
\begin{cases}
    x' = x \\
    y' = y
\end{cases}
\]

\[
K = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}
\]
Weak perspective projection
Pros and Cons of These Models

• **Weak perspective much simpler math.**
  – Accurate when object is small and distant.
  – Most useful for recognition.

• **Pinhole perspective much more accurate for scenes.**
  – Used in structure from motion.
Next lecture

• How to calibrate a camera?

• No conversation hour tomorrow!
• Further questions during office hour