EECS 442 – Computer vision

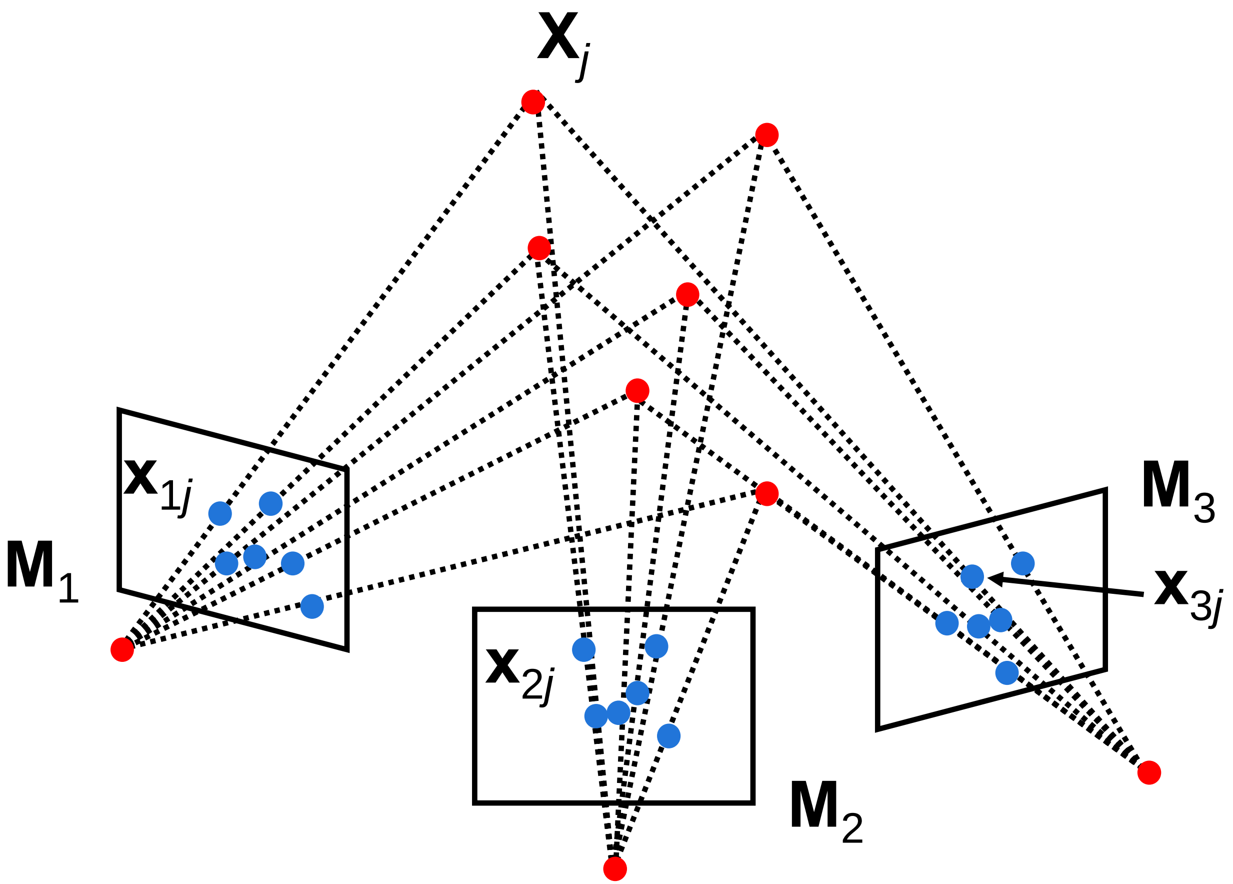
Multiple view geometry
Affine structure from Motion

- Affine structure from motion problem
- Algebraic methods
- Factorization methods

Reading: [HZ] Chapters: 6,14,18
[FP] Chapter: 12

Some slides of this lectures are courtesy of prof. J. Ponce, prof FF Li, prof S. Lazebnik & prof. M. Hebert

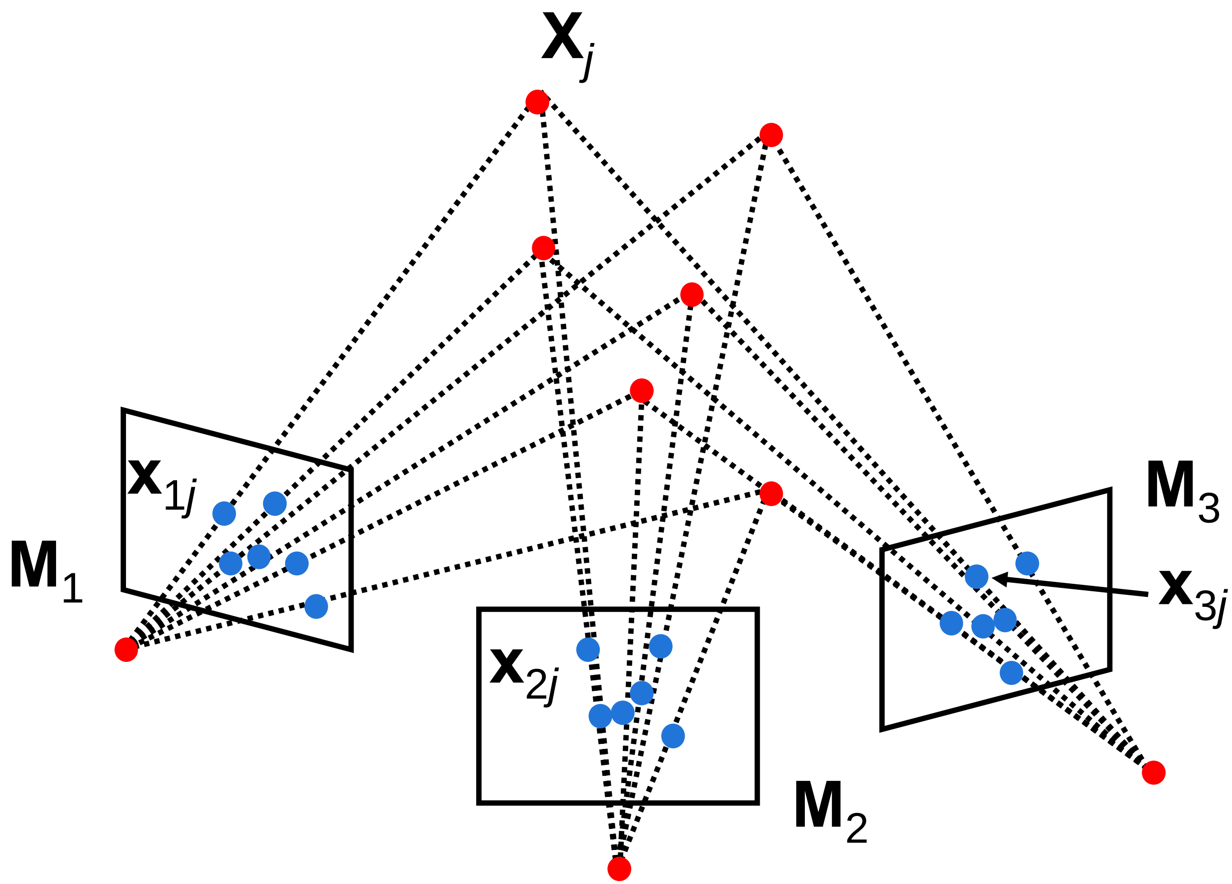
Structure from motion problem



Given *m* images of *n* fixed 3D points

- $\mathbf{x}_{ij} = \mathbf{M}_i \mathbf{X}_j, \quad i = 1, \ldots, m, \quad j = 1, \ldots, n$

Structure from motion problem

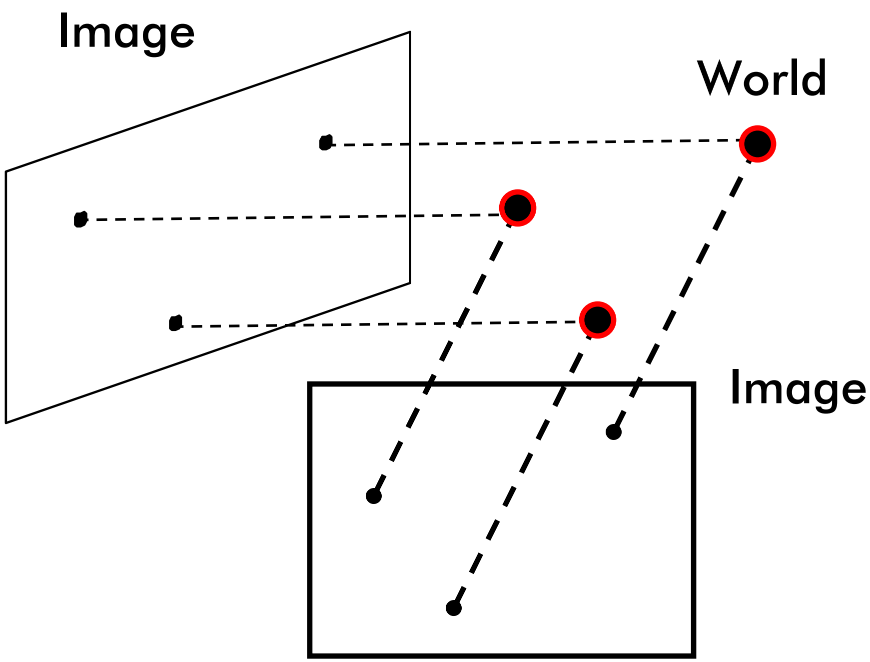


From the mxn correspondences $\mathbf{x}_{ij}$, estimate:

- m projection matrices $\mathbf{M}_i$ — motion
- n 3D points $\mathbf{X}_j$ — structure

Affine structure from motion

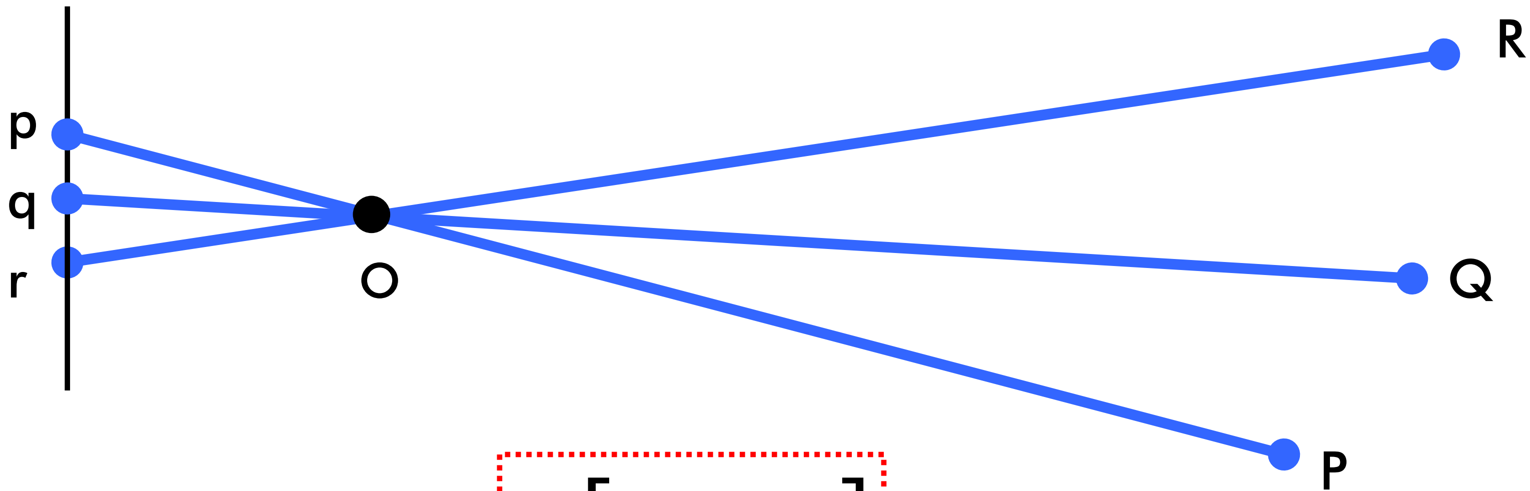
(simpler problem)



From the mxn correspondences $\mathbf{x}_{ij}$, estimate:

- m projection matrices $\mathbf{M}_i$ (affine cameras)
- n 3D points $\mathbf{X}_j$

Finite cameras



$$x = \underbrace{\boxed{K\begin{bmatrix} R & T \end{bmatrix}}}_{M} X$$

Perspective projection matrix

$$K = \begin{bmatrix} \alpha_x & s & x_o \\ 0 & \alpha_y & y_o \\ 0 & 0 & 1 \end{bmatrix} \qquad M = \boxed{K_{3x3}} \boxed{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}} \boxed{\begin{bmatrix} R & T \\ 0 & 1 \end{bmatrix}}$$

Homography (in 2D)

Homography (in 3D)

Affine cameras

$$x = K\begin{bmatrix} R & T \end{bmatrix} X$$

Projective case

$$K = \begin{bmatrix} \alpha_x & s & x_o \\ 0 & \alpha_y & y_o \\ 0 & 0 & 1 \end{bmatrix} \qquad M = K\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} R & T \\ 0 & 1 \end{bmatrix}$$

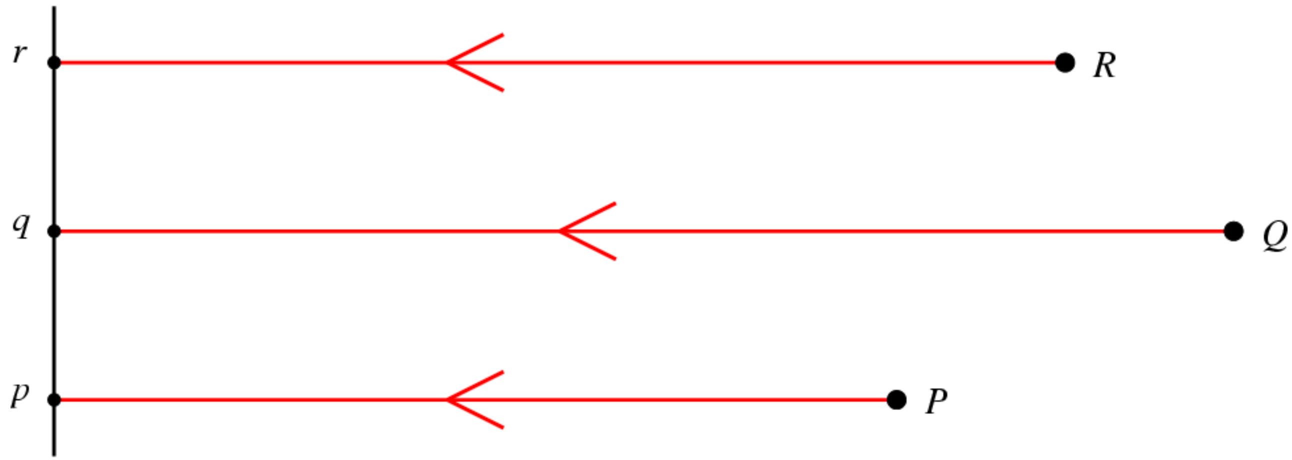
Affine case

$$K = \begin{bmatrix} \alpha_x & s & \boxed{0} \\ 0 & \alpha_y & \boxed{0} \\ 0 & 0 & 1 \end{bmatrix} \qquad M = K\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \boxed{0} & 1 \end{bmatrix}\begin{bmatrix} R & T \\ 0 & 1 \end{bmatrix}$$

Parallel projection matrix
(points at infinity are mapped as points at infinity)

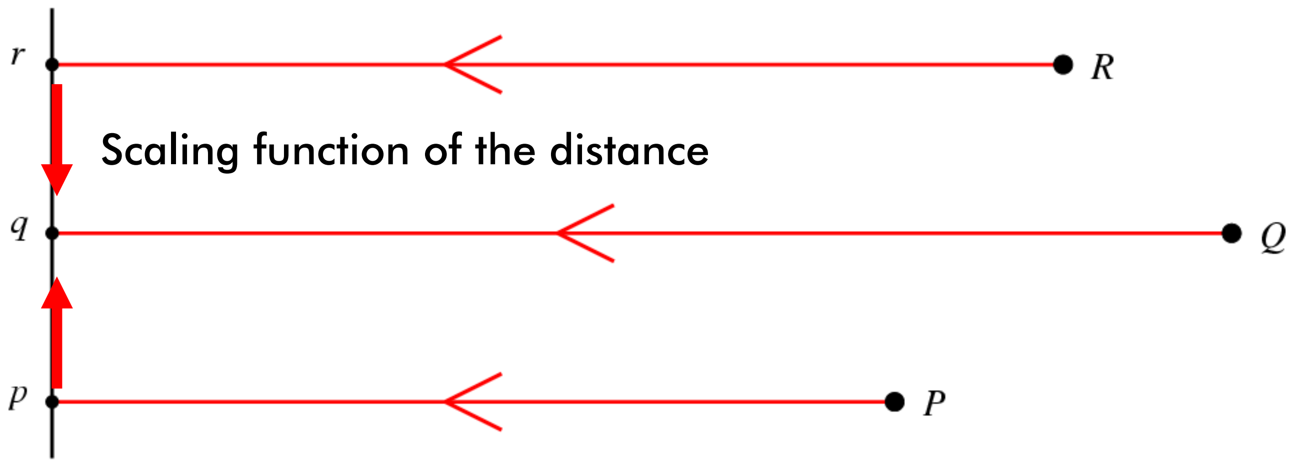
Orthographic Projection

$$K = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



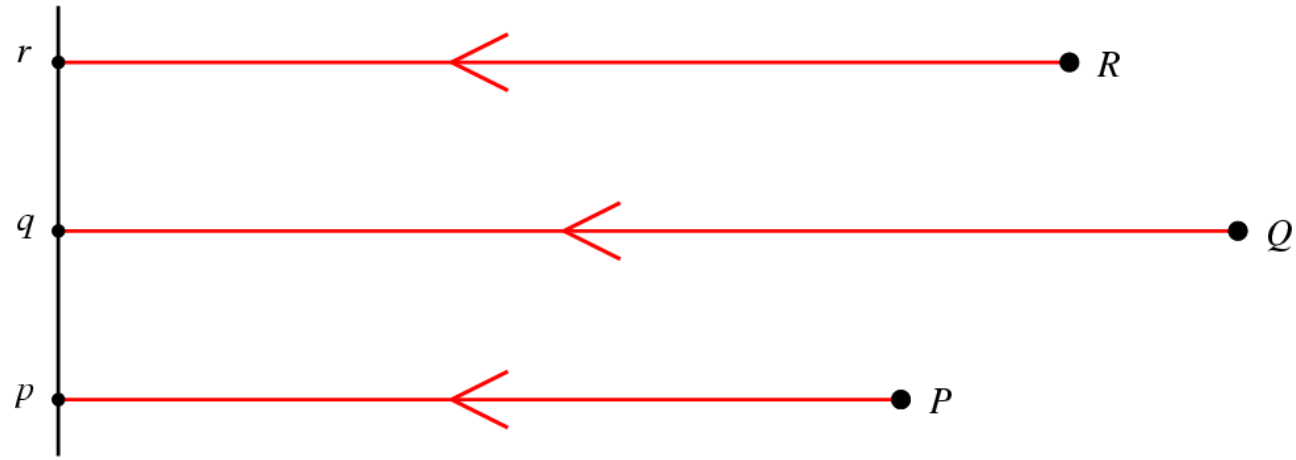
Weak-Perspective Projection

$$K = \begin{bmatrix} \alpha_x & 0 & 0 \\ 0 & \alpha_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



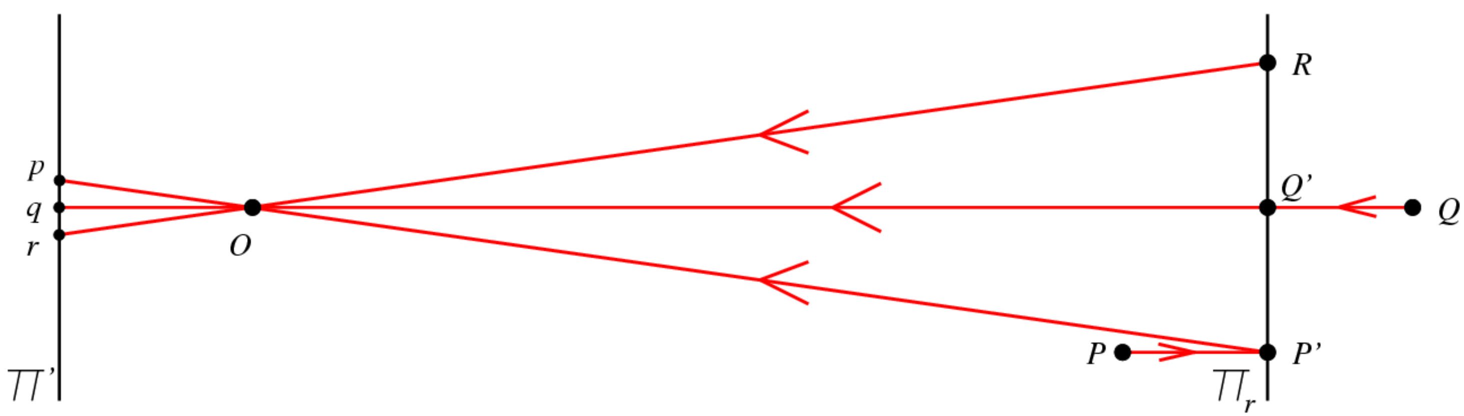
Orthographic Projection

$$K = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



Weak-Perspective Projection

$$K = \begin{bmatrix} \alpha_x & 0 & 0 \\ 0 & \alpha_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



Affine cameras

$$x = K\begin{bmatrix} R & T \end{bmatrix} X$$ [Homogeneous]

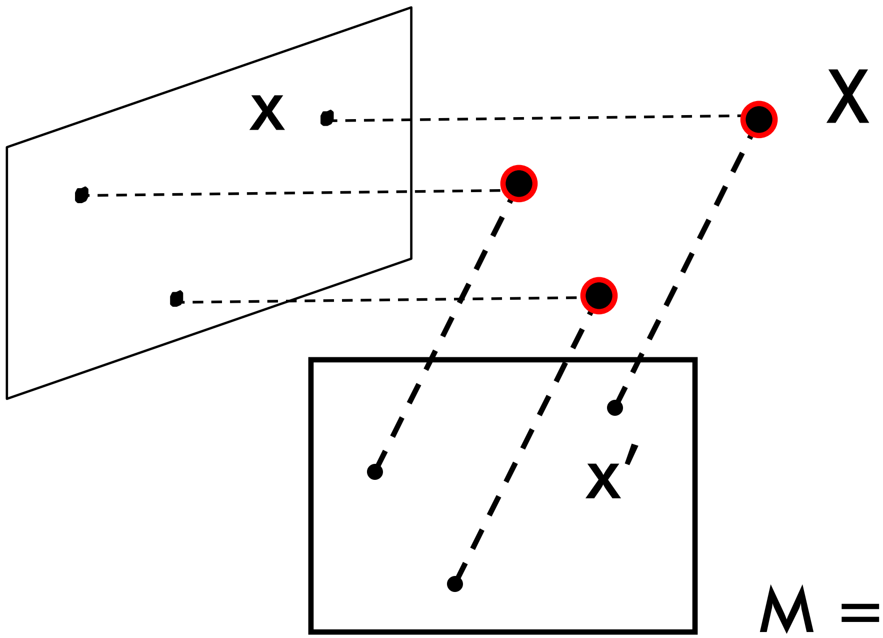
$$K = \begin{bmatrix} \alpha_x & s & 0 \\ 0 & \alpha_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad M = K \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} R & T \\ 0 & 1 \end{bmatrix}$$

$$M = [3\times 3\,\text{affine}] \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} [4\times 4\,\text{affine}] = \begin{bmatrix} a_{11} & a_{12} & a_{13} & b_1 \\ a_{21} & a_{22} & a_{23} & b_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{b} \\ \mathbf{0} & \mathbf{1} \end{bmatrix}$$

$$\mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} + \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \mathbf{AX} + \mathbf{b} = M_{Euc} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix};$$ [non-homogeneous image coordinates]

$$M_{Euc} = M = \begin{bmatrix} A & b \end{bmatrix}$$

Affine cameras



M = camera matrix

To recap:

from now on we define M as the camera matrix for the affine case

$$\mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix} = \mathbf{A}\mathbf{X} + \mathbf{b} = \mathrm{M}\begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}; \qquad \mathrm{M} = \begin{bmatrix} \mathrm{A} & \mathrm{b} \end{bmatrix}$$

The Affine Structure-from-Motion Problem

Given m images of n fixed points P_j (=X_i) we can write

$$\boldsymbol{p}_{ij} = \mathcal{M}_i \begin{pmatrix} \boldsymbol{P}_j \\ 1 \end{pmatrix} = \mathcal{A}_i \boldsymbol{P}_j + \boldsymbol{b}_i \quad \text{for} \quad i = 1, \ldots, m \quad \text{and} \quad j = 1, \ldots, n.$$

Problem: estimate the m 2×4 matrices M_i and the n positions P_i from the $m\times n$ correspondences p_{ij} .

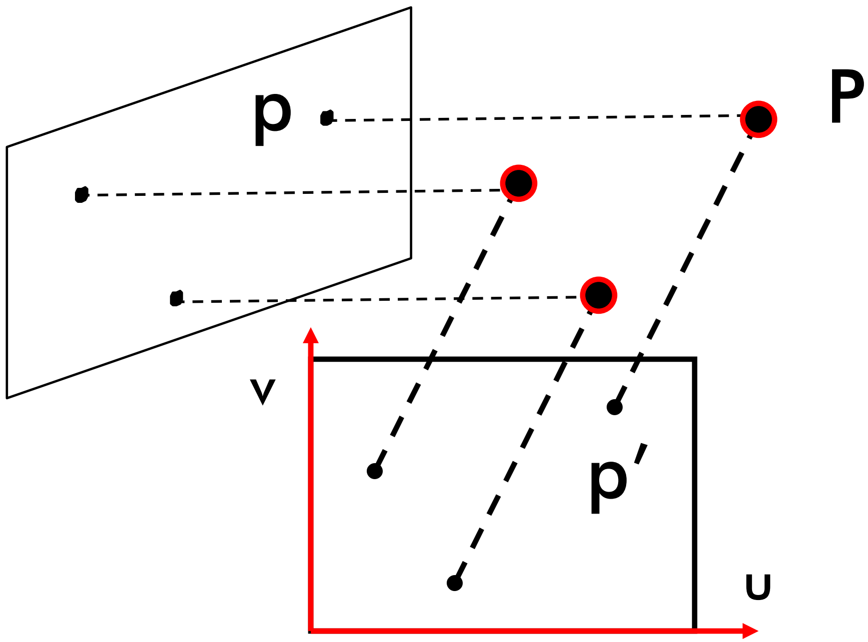
How many equations and how many unknown?

$2m \times n$ equations in $8m+3n$ unknowns

Two approaches:

- Algebraic approach (affine epipolar geometry)
- Factorization method

Algebraic analysis (2-view case)



Homogeneous system

$$\begin{cases} \boldsymbol{p} = \mathcal{A}\boldsymbol{P} + \boldsymbol{b} \\ \boldsymbol{p}' = \mathcal{A}'\boldsymbol{P} + \boldsymbol{b}' \end{cases} \Rightarrow \begin{pmatrix} \mathcal{A} & \boldsymbol{p} - \boldsymbol{b} \\ \mathcal{A}' & \boldsymbol{p}' - \boldsymbol{b}' \end{pmatrix} \begin{pmatrix} \boldsymbol{P} \\ -1 \end{pmatrix} = \boldsymbol{0}$$

$$\Rightarrow \operatorname{Det} \begin{pmatrix} \mathcal{A} & \boldsymbol{p} - \boldsymbol{b} \\ \mathcal{A}' & \boldsymbol{p}' - \boldsymbol{b}' \end{pmatrix} = 0 \Rightarrow \boxed{\alpha u + \beta v + \alpha' u' + \beta' v' + \delta = 0}$$

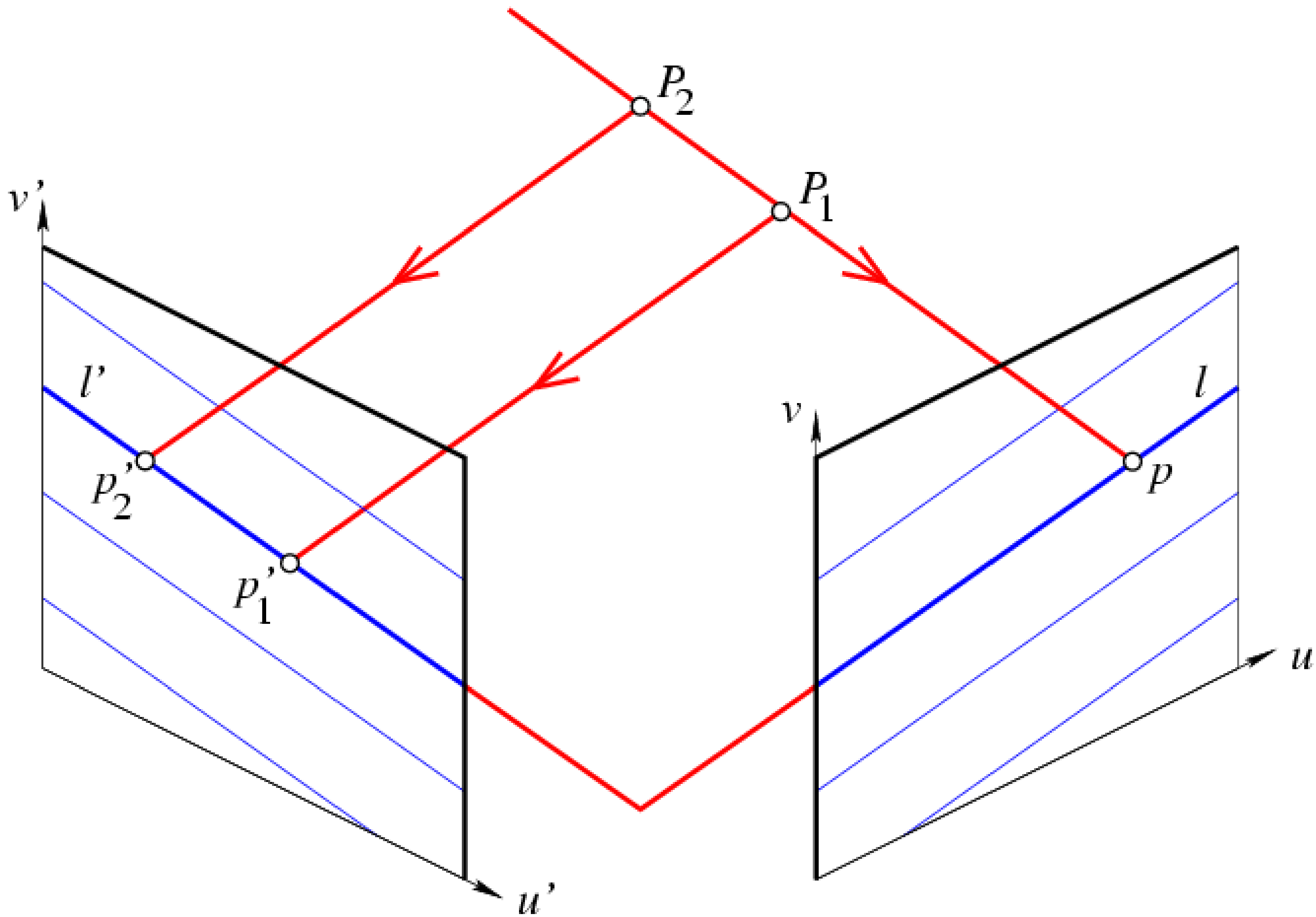
Algebraic analysis (2-view case)

$$\alpha u + \beta v + \alpha' u' + \beta' v' + \delta = 0$$

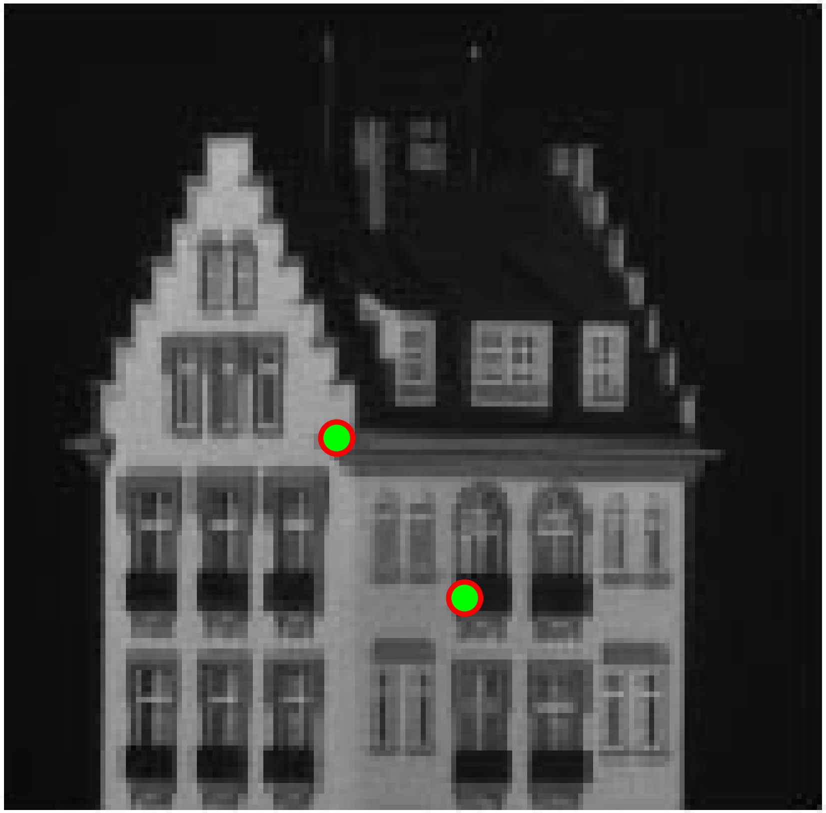
$$(u, v, 1)\mathcal{F}\begin{pmatrix} u' \\ v' \\ 1 \end{pmatrix} = 0 \quad \text{where} \quad \mathcal{F} \stackrel{\text{def}}{=} \begin{pmatrix} 0 & 0 & \alpha \\ 0 & 0 & \beta \\ \alpha' & \beta' & \delta \end{pmatrix}$$

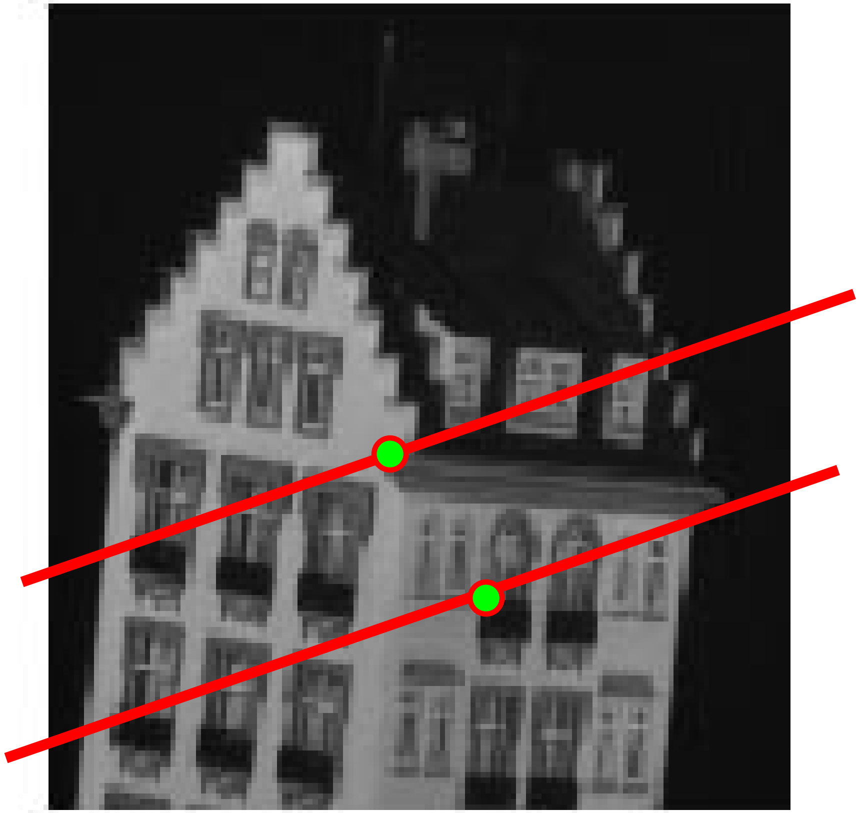
The Affine Fundamental Matrix!

Affine Epipolar Geometry



Note: the epipolar lines are parallel.





Estimating F

$$\alpha u + \beta v + \alpha' u' + \beta' v' + \delta = 0$$

- Measurements: u, u′, v, v′
- From at least 4 correspondences, we obtain a linear system on the unknown alpha, beta, etc…

$$\begin{bmatrix} u'_1 & v'_1 & u_1 & v_1 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ u'_n & v'_n & u_n & v_n & 1 \end{bmatrix} \mathbf{f} = 0$$

- Computed by least square and by enforcing |f|=1

Estimating projection matrices from epipolar constraints

If M_i and P_i are solutions,
then M_i' and P_i' are also solutions,

where

$$\mathcal{M}_i' = \mathcal{M}_i \mathcal{Q} \quad \text{and} \quad \begin{pmatrix} \boldsymbol{P}_j' \\ 1 \end{pmatrix} = \mathcal{Q}^{-1} \begin{pmatrix} \boldsymbol{P}_j \\ 1 \end{pmatrix}$$

and

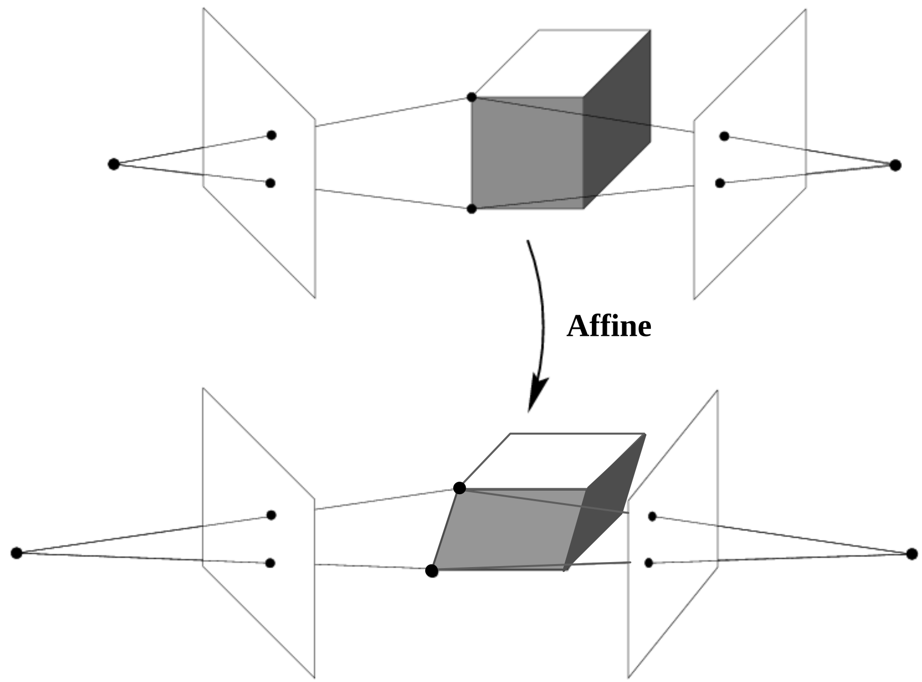
$$\mathcal{Q} = \begin{pmatrix} \mathcal{C} & \boldsymbol{d} \\ \boldsymbol{0}^T & 1 \end{pmatrix} \quad \text{with} \quad \mathcal{Q}^{-1} = \begin{pmatrix} \mathcal{C}^{-1} & -\mathcal{C}^{-1}\boldsymbol{d} \\ \boldsymbol{0}^T & 1 \end{pmatrix}$$

Q is an affine transformation.

Proof:

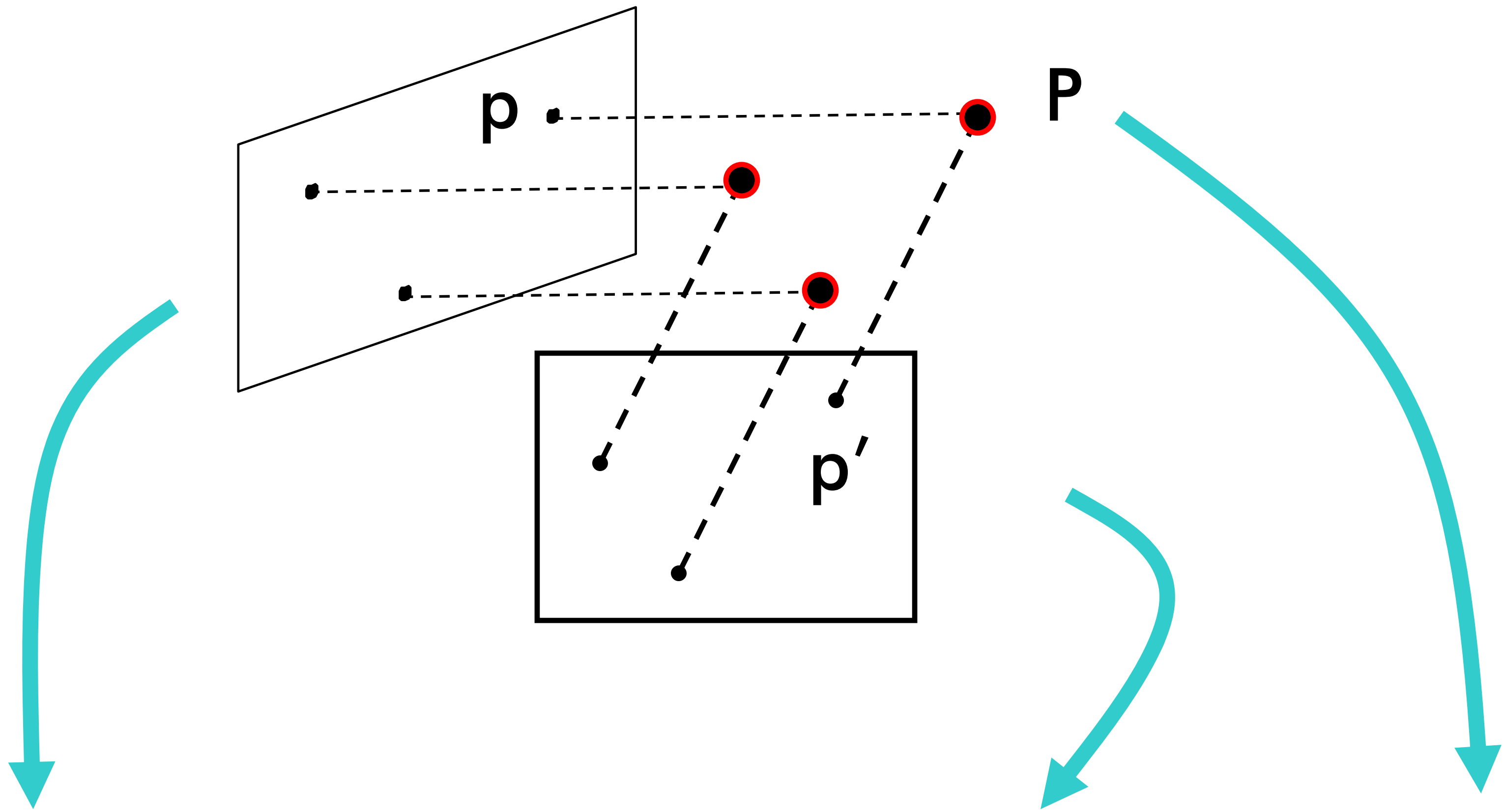
$$\boldsymbol{p}_{ij} = \mathcal{M}_i \begin{pmatrix} \boldsymbol{P}_j \\ 1 \end{pmatrix} = (\mathcal{M}_i \mathcal{Q}) \left(\mathcal{Q}^{-1} \begin{pmatrix} \boldsymbol{P}_j \\ 1 \end{pmatrix} \right) = \mathcal{M}_i' \begin{pmatrix} \boldsymbol{P}_j' \\ 1 \end{pmatrix} \quad \blacksquare$$

Affine ambiguity



$$\mathbf{x} = \mathbf{P}\mathbf{X} = \left(\mathbf{P}\mathbf{Q}_{\mathbf{A}}^{\mathbf{-1}}\right)\left(\mathbf{Q}_{\mathbf{A}}\,\mathbf{X}\right)$$

Estimating projection matrices from epipolar constraints



$$\mathcal{M} = (\,\mathcal{A} \quad \boldsymbol{b}\,) \qquad \mathcal{M}' = (\,\mathcal{A}' \quad \boldsymbol{b}'\,) \qquad \boldsymbol{P}$$

$$\tilde{\mathcal{M}} = \mathcal{M}\mathcal{Q} \qquad \tilde{\mathcal{M}}' = \mathcal{M}'\mathcal{Q} \qquad \tilde{\boldsymbol{P}} = \mathcal{Q}^{-1}\boldsymbol{P}$$

Estimating projection matrices from epipolar constraints

$$\mathcal{M} = (\mathcal{A} \quad \boldsymbol{b}) \qquad \mathcal{M}' = (\mathcal{A}' \quad \boldsymbol{b}') \qquad \boldsymbol{P}$$

$$\downarrow \qquad\qquad \downarrow \qquad\qquad \downarrow$$

$$\tilde{\mathcal{M}} = \mathcal{M}\mathcal{Q} \qquad \tilde{\mathcal{M}}' = \mathcal{M}'\mathcal{Q} \qquad \tilde{\boldsymbol{P}} = \mathcal{Q}^{-1}\boldsymbol{P}$$

Choose Q such that…

$$\tilde{\mathcal{M}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \qquad \tilde{\mathcal{M}}' = \begin{pmatrix} 0 & 0 & 1 & 0 \\ a & b & c & d \end{pmatrix} \qquad \tilde{\boldsymbol{P}}$$

Canonical affine cameras

$$\tilde{A} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \qquad \tilde{A}' = \begin{bmatrix} 0 & 0 & 1 \\ \boxed{a \quad b \quad c} \end{bmatrix}$$

$$\tilde{b} = \begin{bmatrix} 0 & 0 \end{bmatrix}^T \qquad \tilde{b}' = \begin{bmatrix} 0 & \boxed{d} \end{bmatrix}^T$$

Function of the parameters of F

Estimating projection matrices from epipolar constraints

$$\mathcal{M} = (\mathcal{A} \quad \boldsymbol{b}) \qquad \mathcal{M}' = (\mathcal{A}' \quad \boldsymbol{b}') \qquad \boldsymbol{P}$$

$$\downarrow \qquad\qquad \downarrow \qquad\qquad \downarrow$$

$$\tilde{\mathcal{M}} = \mathcal{M}\mathcal{Q} \qquad \tilde{\mathcal{M}}' = \mathcal{M}'\mathcal{Q} \qquad \tilde{\boldsymbol{P}} = \mathcal{Q}^{-1}\boldsymbol{P}$$

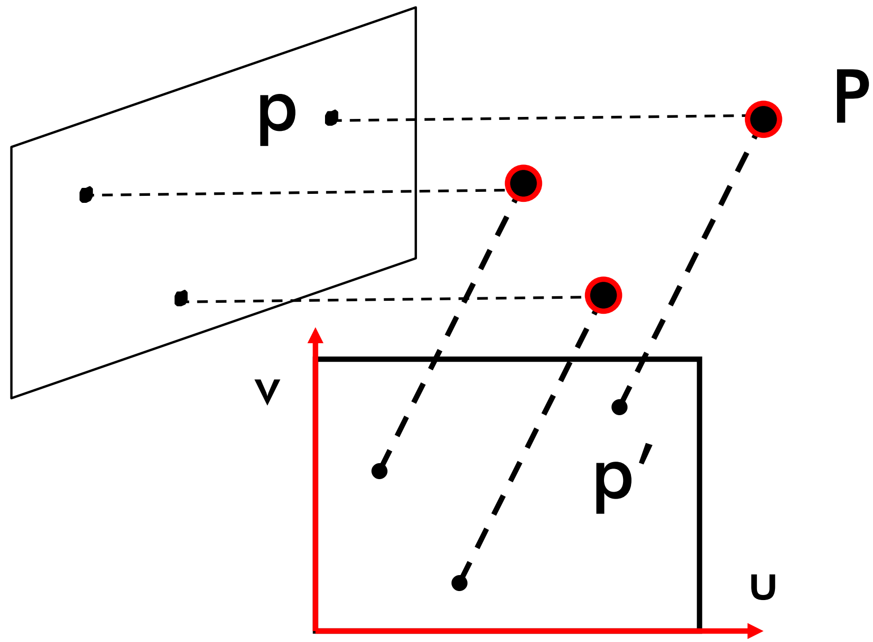
Choose Q such that…

$$\downarrow \qquad\qquad \downarrow \qquad\qquad \downarrow$$

$$\tilde{\mathcal{M}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \qquad \tilde{\mathcal{M}}' = \begin{pmatrix} 0 & 0 & 1 & 0 \\ a & b & c & d \end{pmatrix} \qquad \tilde{\boldsymbol{P}}$$

By re-enforcing the epipolar constraint, we can compute a, b, c, d directly from the measurements

Reminder: epipolar constraint



Homogeneous system

$$\begin{cases} \boldsymbol{p} = \mathcal{A}\boldsymbol{P} + \boldsymbol{b} \\ \boldsymbol{p}' = \mathcal{A}'\boldsymbol{P} + \boldsymbol{b}' \end{cases} \Rightarrow \begin{pmatrix} \mathcal{A} & \boldsymbol{p} - \boldsymbol{b} \\ \mathcal{A}' & \boldsymbol{p}' - \boldsymbol{b}' \end{pmatrix} \begin{pmatrix} \boldsymbol{P} \\ -1 \end{pmatrix} = \boldsymbol{0}$$

$$\Rightarrow \boxed{\mathrm{Det} \begin{pmatrix} \mathcal{A} & \boldsymbol{p} - \boldsymbol{b} \\ \mathcal{A}' & \boldsymbol{p}' - \boldsymbol{b}' \end{pmatrix} = 0} \Rightarrow \alpha u + \beta v + \alpha' u' + \beta' v' + \delta = 0$$

Estimating projection matrices from epipolar constraints

$$\mathcal{M} = (\,\mathcal{A} \quad \boldsymbol{b}\,) \qquad \mathcal{M}' = (\,\mathcal{A}' \quad \boldsymbol{b}'\,) \qquad \boldsymbol{P}$$

$$\tilde{\mathcal{M}} = \mathcal{M}\mathcal{Q} \qquad \tilde{\mathcal{M}}' = \mathcal{M}'\mathcal{Q} \qquad \tilde{\boldsymbol{P}} = \mathcal{Q}^{-1}\boldsymbol{P}$$

Choose Q such that…

$$\tilde{\mathcal{M}} = \left(\underbrace{\begin{matrix}1 & 0 & 0\\ 0 & 1 & 0\end{matrix}}_{A} \;\; \underbrace{\begin{matrix}0\\0\end{matrix}}_{b}\right) \qquad \tilde{\mathcal{M}}' = \begin{pmatrix}0 & 0 & 1 & 0\\ a & b & c & d\end{pmatrix} \qquad \tilde{\boldsymbol{P}}$$

Re-enforce the Epipolar constraint

$$\mathrm{Det}\begin{pmatrix}\mathcal{A} & \boldsymbol{p}-\boldsymbol{b}\\ \mathcal{A}' & \boldsymbol{p}'-\boldsymbol{b}'\end{pmatrix} = 0 \;\longrightarrow\; \mathrm{Det}\begin{pmatrix}1 & 0 & 0 & u\\ 0 & 1 & 0 & v\\ 0 & 0 & 1 & u'\\ a & b & c & v'-d\end{pmatrix} = 0$$

Estimating projection matrices from epipolar constraints

$$\mathcal{M} = \begin{pmatrix} \mathcal{A} & \boldsymbol{b} \end{pmatrix} \qquad \mathcal{M}' = \begin{pmatrix} \mathcal{A}' & \boldsymbol{b}' \end{pmatrix} \qquad \boldsymbol{P}$$

$$\tilde{\mathcal{M}} = \mathcal{M}\mathcal{Q} \qquad \tilde{\mathcal{M}}' = \mathcal{M}'\mathcal{Q} \qquad \tilde{\boldsymbol{P}} = \mathcal{Q}^{-1}\boldsymbol{P}$$

Choose Q such that…

$$\tilde{\mathcal{M}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \qquad \tilde{\mathcal{M}}' = \begin{pmatrix} 0 & 0 & 1 & 0 \\ a & b & c & d \end{pmatrix} \qquad \tilde{\boldsymbol{P}}$$

A b

$$\boxed{\mathrm{Det}\begin{pmatrix} 1 & 0 & 0 & u \\ 0 & 1 & 0 & v \\ 0 & 0 & 1 & u' \\ a & b & c & v'-d \end{pmatrix} = au - bv + cu' + v' - d = 0}$$

Estimating projection matrices from epipolar constraints

$$\mathrm{Det}\begin{pmatrix} 1 & 0 & 0 & u \\ 0 & 1 & 0 & v \\ 0 & 0 & 1 & u' \\ a & b & c & v'-d \end{pmatrix} = au - bv + cu' + v' - d = 0$$

- Linear relationship between measurements and unknown

  Unknown: a, b, c, d
  Measurements: u, u', v, v'

- From at least 4 correspondences, we can solve this linear system and compute a, b, c, d (via least square)
- The cameras can be computed
- How about the structure?

Estimating the structure from epipolar constraints

$$\tilde{\mathcal{M}} = \left(\underbrace{\begin{matrix}1 & 0 & 0\\ 0 & 1 & 0\end{matrix}}_{\text{A}}\;\underbrace{\begin{matrix}0\\ 0\end{matrix}}_{\text{b}}\right) \qquad \tilde{\mathcal{M}}' = \begin{pmatrix}0 & 0 & 1 & 0\\ a & b & c & d\end{pmatrix} \qquad \tilde{\boldsymbol{P}}$$

$$\begin{pmatrix}\mathcal{A} & \boldsymbol{p}-\boldsymbol{b}\\ \mathcal{A}' & \boldsymbol{p}'-\boldsymbol{b}'\end{pmatrix}\begin{pmatrix}\boldsymbol{P}\\ -1\end{pmatrix} = \boldsymbol{0} \quad \Rightarrow$$

$$\begin{pmatrix}1 & 0 & 0 & u\\ 0 & 1 & 0 & v\\ 0 & 0 & 1 & u'\\ a & b & c & v'-d\end{pmatrix}\begin{pmatrix}\tilde{\boldsymbol{P}}\\ -1\end{pmatrix} = 0 \quad \Rightarrow \quad \boxed{\tilde{\boldsymbol{P}} = \begin{pmatrix}u\\ v\\ u'\end{pmatrix}}$$

Can be solved by least square again

A factorization method – Tomasi & Kanade algorithm

C. Tomasi and T. Kanade. Shape and motion from image streams under orthography: A factorization method. *IJCV*, 9(2):137-154, November 1992.

- Centering the data
- Factorization

A factorization method - Centering the data

Centering: subtract the centroid of the image points

$$\hat{\mathbf{x}}_{ij} = \mathbf{x}_{ij} - \frac{1}{n}\sum_{k=1}^{n}\mathbf{x}_{ik}$$

A factorization method - Centering the data

Centering: subtract the centroid of the image points

$$\hat{\mathbf{x}}_{ij} = \mathbf{x}_{ij} - \frac{1}{n}\sum_{k=1}^{n}\mathbf{x}_{ik} = \mathbf{A}_i\mathbf{X}_j + \mathbf{b}_i - \frac{1}{n}\sum_{k=1}^{n}\left(\mathbf{A}_i\mathbf{X}_k + \mathbf{b}_i\right)$$

A factorization method - Centering the data

Centering: subtract the centroid of the image points

$$\hat{\mathbf{x}}_{ij} = \mathbf{x}_{ij} - \frac{1}{n}\sum_{k=1}^{n}\mathbf{x}_{ik} = \mathbf{A}_i\mathbf{X}_j + \mathbf{b}_i - \frac{1}{n}\sum_{k=1}^{n}\left(\mathbf{A}_i\mathbf{X}_k + \mathbf{b}_i\right)$$

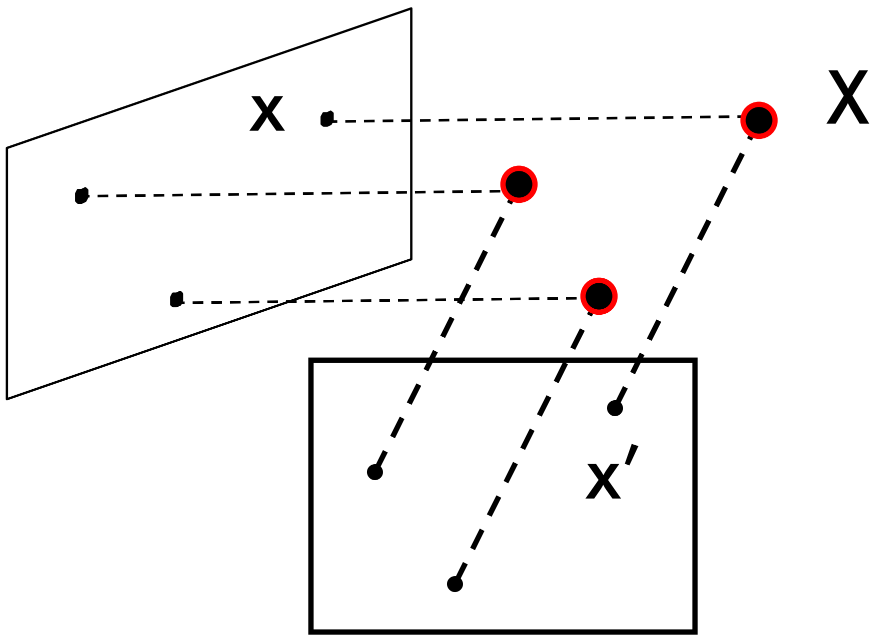
$$= \mathbf{A}_i\left(\mathbf{X}_j - \boxed{\frac{1}{n}\sum_{k=1}^{n}\mathbf{X}_k}\right) = \mathbf{A}_i\hat{\mathbf{X}}_j$$

Assume that the origin of the world coordinate system is at the $\boxed{\text{centroid}}$ of the 3D points

After centering, each normalized point x_{ij} is related to the 3D point X_i by

$$\hat{\mathbf{x}}_{ij} = \mathbf{A}_i\mathbf{X}_j$$

A factorization method - Centering the data



$$\hat{\mathbf{x}}_{ij} = \mathbf{A}_i \mathbf{X}_j$$

A factorization method - factorization

Let's create a $2m \times n$ data (measurement) matrix:

$$\mathbf{D} = \begin{bmatrix} \hat{\mathbf{x}}_{11} & \hat{\mathbf{x}}_{12} & \cdots & \hat{\mathbf{x}}_{1n} \\ \hat{\mathbf{x}}_{21} & \hat{\mathbf{x}}_{22} & \cdots & \hat{\mathbf{x}}_{2n} \\ & & \ddots & \\ \hat{\mathbf{x}}_{m1} & \hat{\mathbf{x}}_{m2} & \cdots & \hat{\mathbf{x}}_{mn} \end{bmatrix}$$

cameras ($2m$)

points (n)

A factorization method - factorization

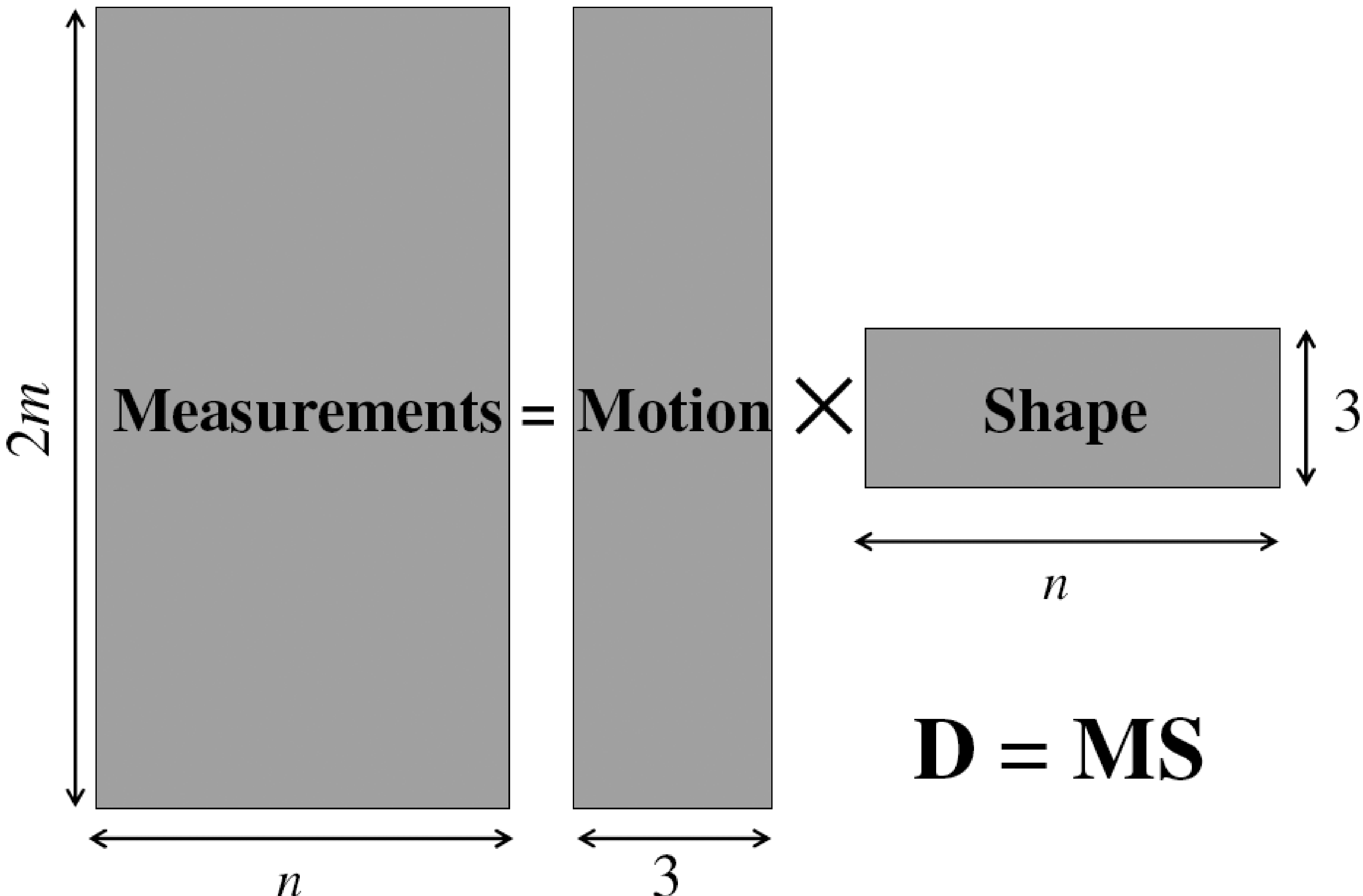
Let's create a 2m × n data (measurement) matrix:

$$\mathbf{D} = \underbrace{\begin{bmatrix} \hat{\mathbf{x}}_{11} & \hat{\mathbf{x}}_{12} & \cdots & \hat{\mathbf{x}}_{1n} \\ \hat{\mathbf{x}}_{21} & \hat{\mathbf{x}}_{22} & \cdots & \hat{\mathbf{x}}_{2n} \\ & & \ddots & \\ \hat{\mathbf{x}}_{m1} & \hat{\mathbf{x}}_{m2} & \cdots & \hat{\mathbf{x}}_{mn} \end{bmatrix}}_{(2\,m \times \mathrm{n})} = \underbrace{\boxed{\begin{bmatrix} \mathbf{A}_1 \\ \mathbf{A}_2 \\ \vdots \\ \mathbf{A}_m \end{bmatrix}}}_{\substack{\text{cameras} \\ (2\,m \times 3)}}{}_{\mathrm{M}} \; \overset{\mathrm{S}}{\underbrace{\boxed{\begin{bmatrix} \mathbf{X}_1 & \mathbf{X}_2 & \cdots & \mathbf{X}_n \end{bmatrix}}}_{\text{points } (3 \times n)}}$$

The measurement matrix D = M S must have rank 3

(it's a product of a 2mx3 matrix and 3xn matrix)

Factorizing the measurement matrix

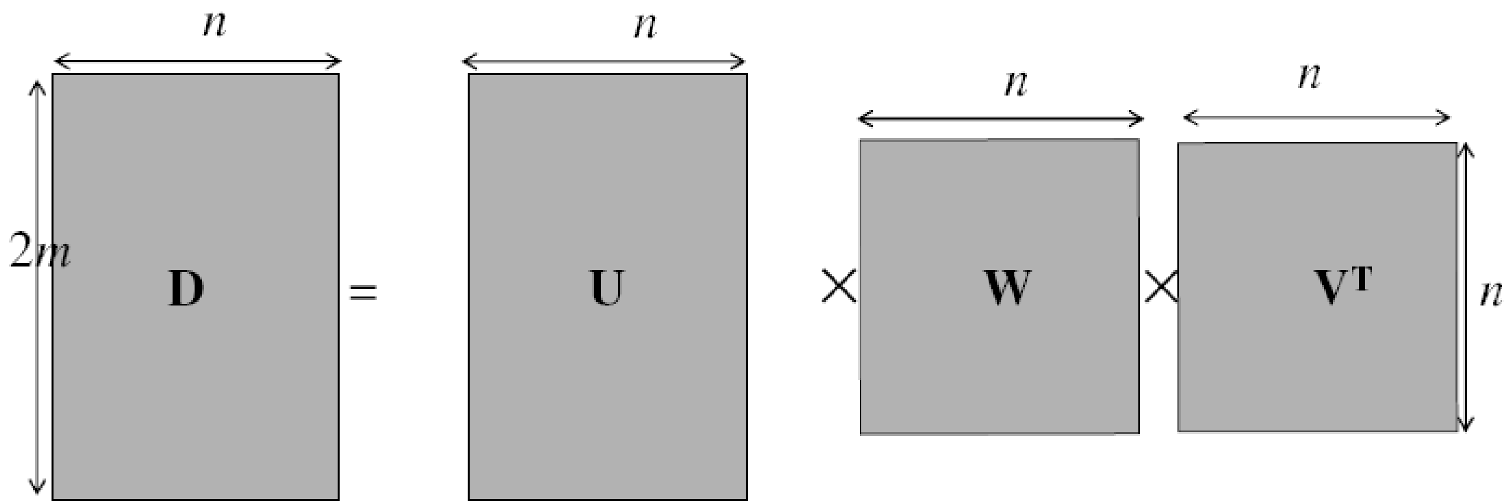


$$\mathbf{D} = \mathbf{MS}$$

Source: M. Hebert

Factorizing the measurement matrix

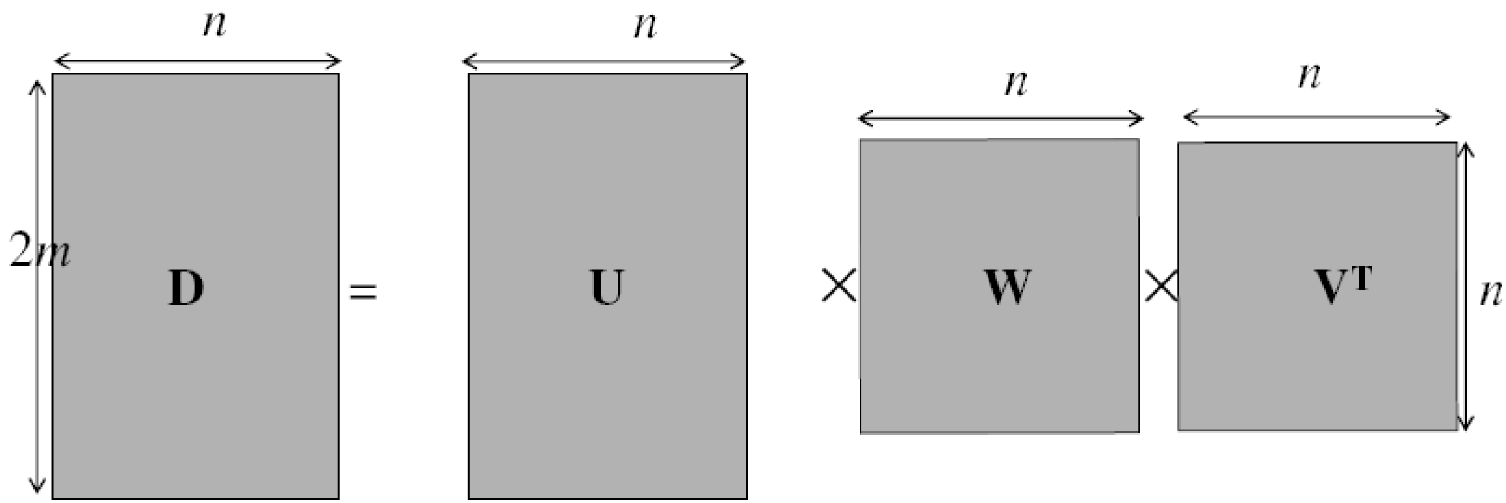
Singular value decomposition of D:



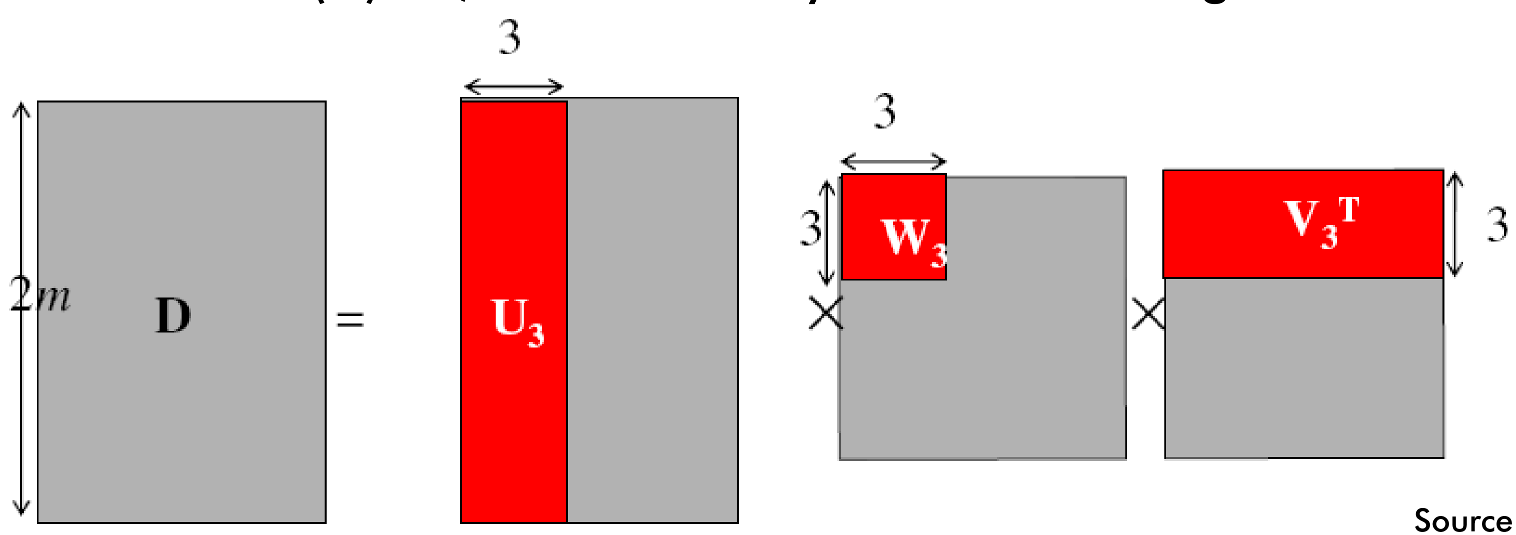
Source: M. Hebert

Factorizing the measurement matrix

Singular value decomposition of D:



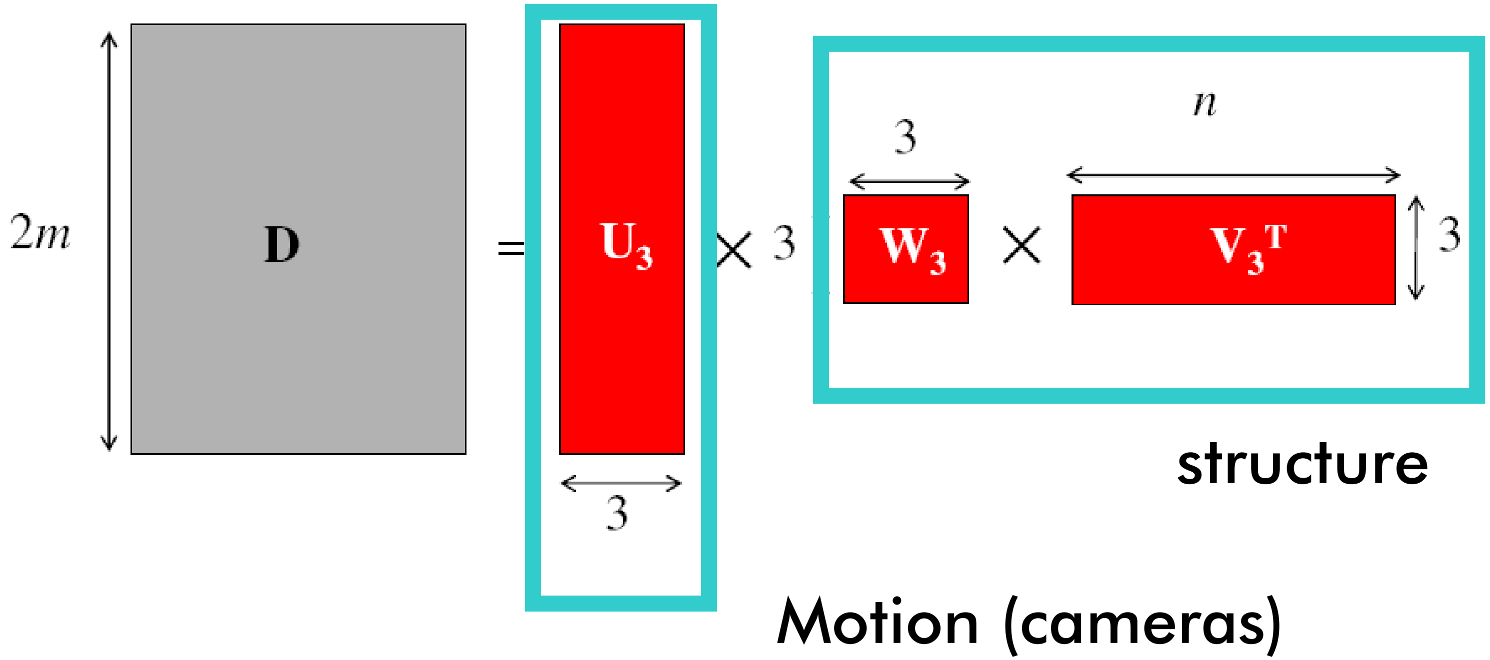
Since rank (D)=3, there are only 3 non-zero singular values



Source: M. Hebert

Factorizing the measurement matrix

Obtaining a factorization from SVD:

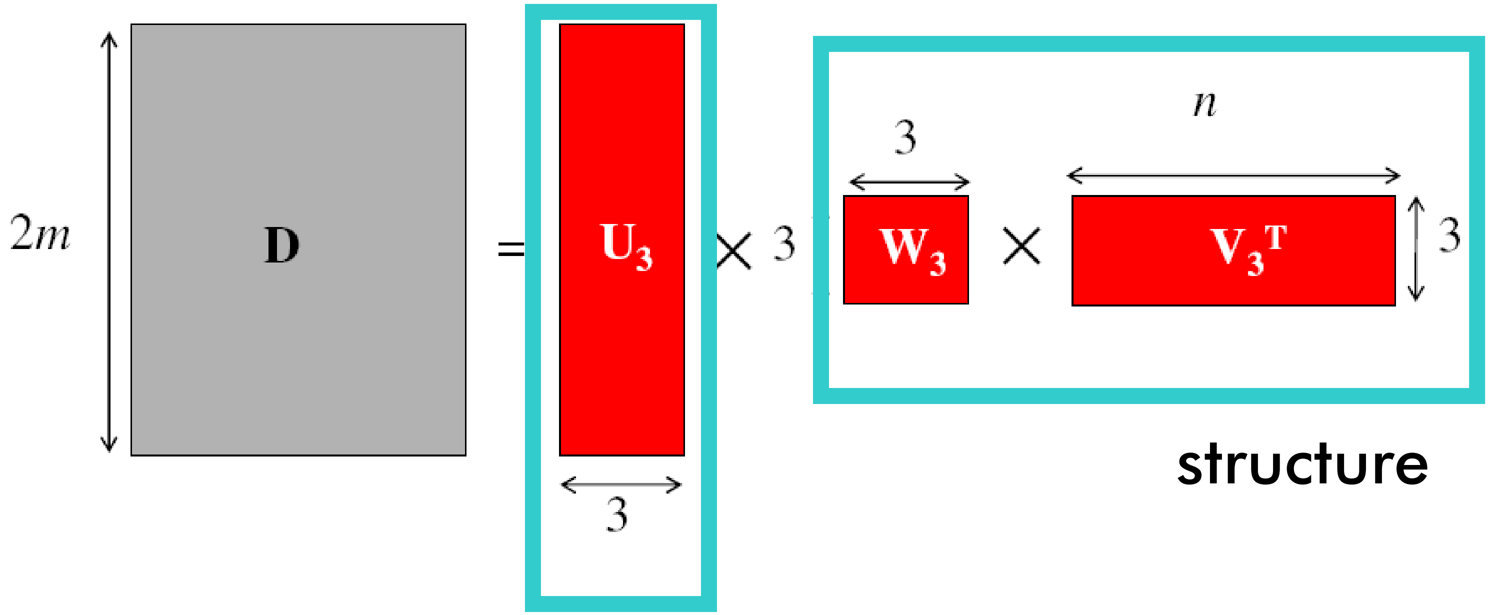


What is the issue here?

D has rank>3 because of - measurement noise
- affine approximation

Factorizing the measurement matrix

Obtaining a factorization from SVD:

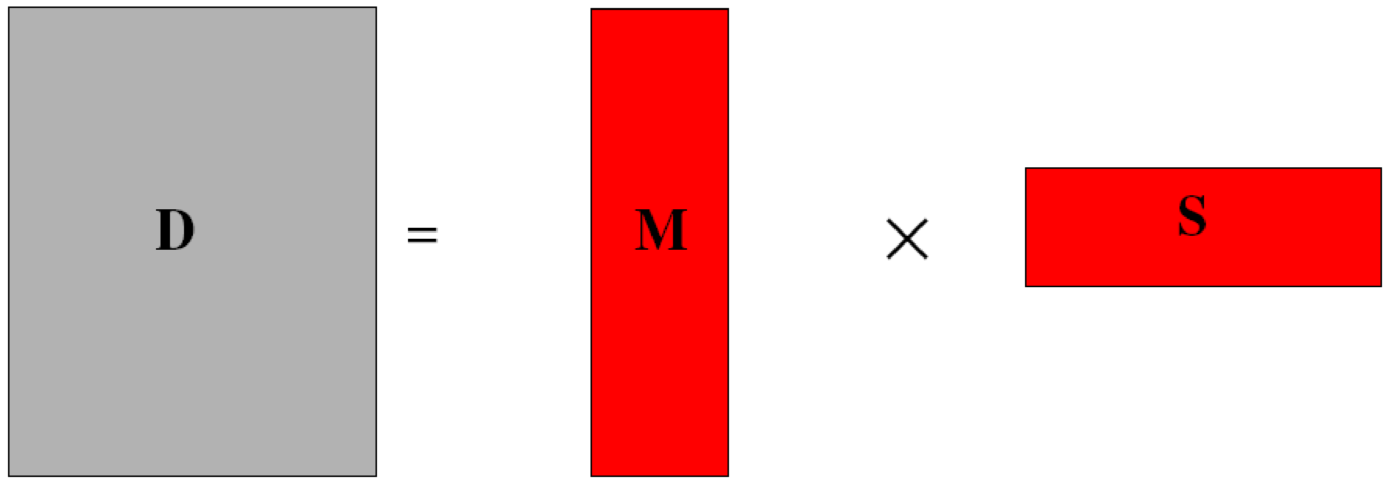


Theorem: When D has a rank greater than p, $\mathcal{U}_p \mathcal{W}_p \mathcal{V}_p^T$ is the best possible rank-p approximation of D in the sense of the Frobenius norm.

$$\mathcal{D} = \mathcal{U}_3 \mathcal{W}_3 \mathcal{V}_3^T$$

$$\boxed{\begin{cases} \mathcal{A}_0 = \mathcal{U}_3 \\ \mathcal{P}_0 = \mathcal{W}_3 \mathcal{V}_3^T \end{cases}}$$

Affine ambiguity



The decomposition is not unique. We get the same D by using any 3×3 matrix C and applying the transformations $M \rightarrow MC$, $S \rightarrow C^{-1}S$

We can enforce some Euclidean constraints to resolve The ambiguity (more on this next lecture!)

Algorithm summary

Given: m images and n features x_{ij}

For each image i, center the feature coordinates

Construct a $2m \times n$ measurement matrix D:

- Column j contains the projection of point j in all views
- Row i contains one coordinate of the projections of all the n points in image i

Factorize D:

- Compute SVD: $D = U\ W\ V^T$
- Create U_3 by taking the first 3 columns of U
- Create V_3 by taking the first 3 columns of V
- Create W_3 by taking the upper left 3×3 block of W

Create the motion and shape matrices:

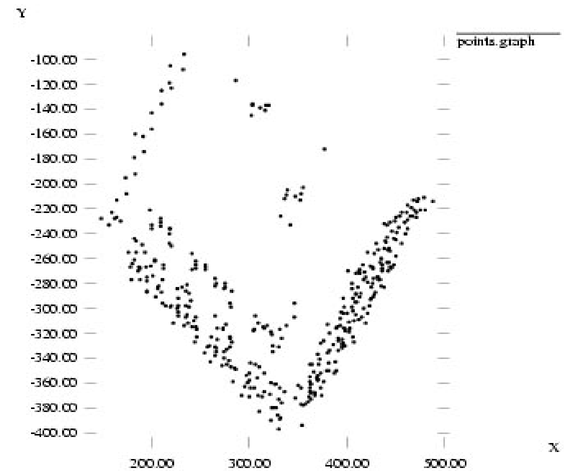
- $M = U_3 W_3^{½}$ and $S = W_3^{½}\ V_3^T$ (or $M = U_3$ and $S = W_3 V_3^T$)

Eliminate affine ambiguity

Reconstruction results

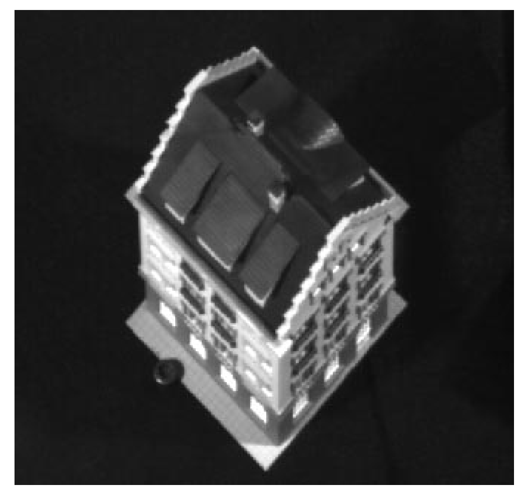
1

60



120

150



C. Tomasi and T. Kanade. Shape and motion from image streams under orthography: A factorization method. *IJCV*, 9(2):137-154, November 1992.

Next lecture

Multiple view geometry

Perspective structure from Motion