



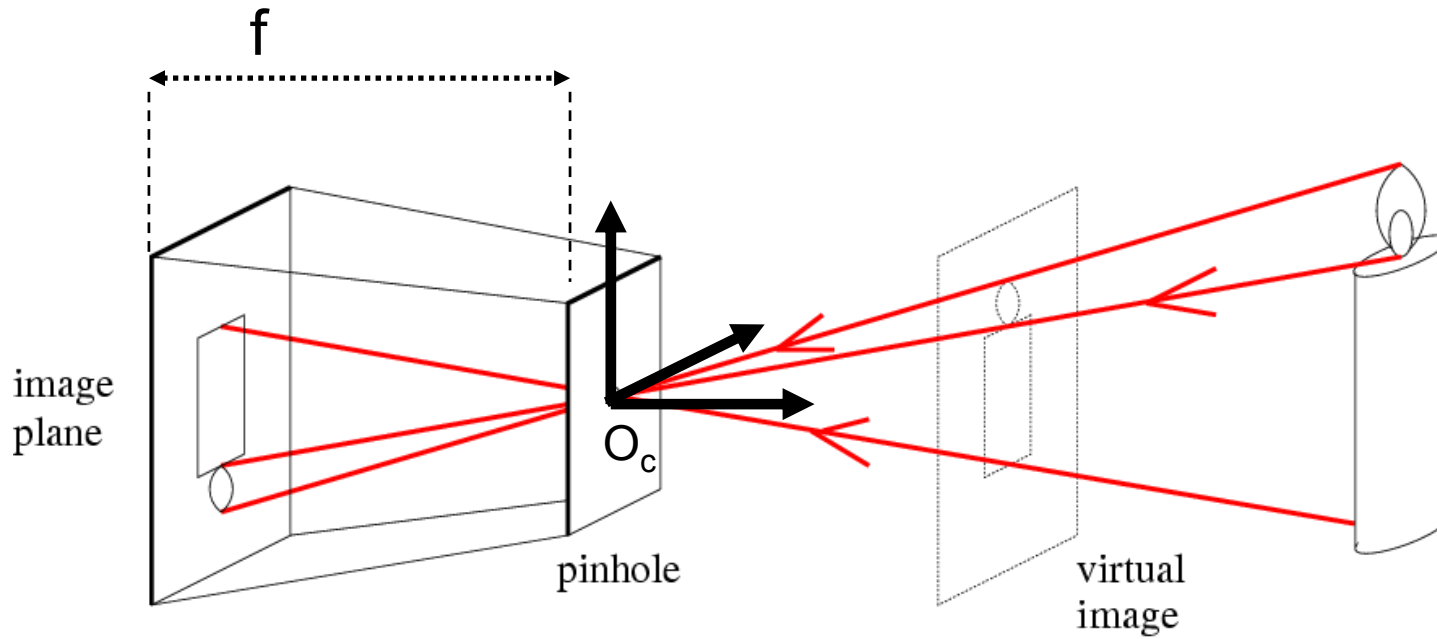
EECS 442 – Computer vision

Camera Calibration

- Review camera parameters
- Camera calibration problem
- Example

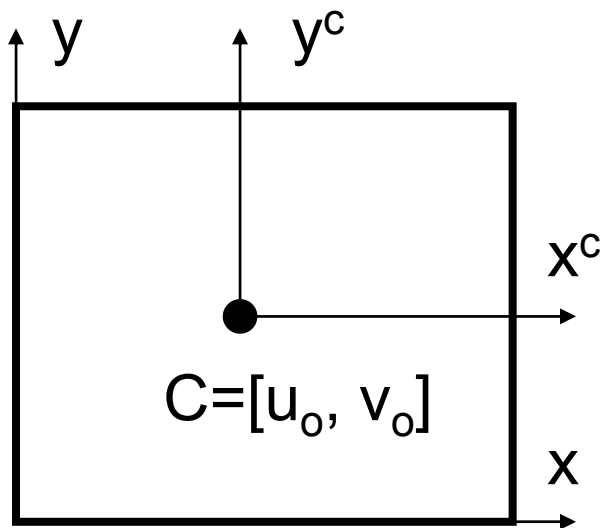
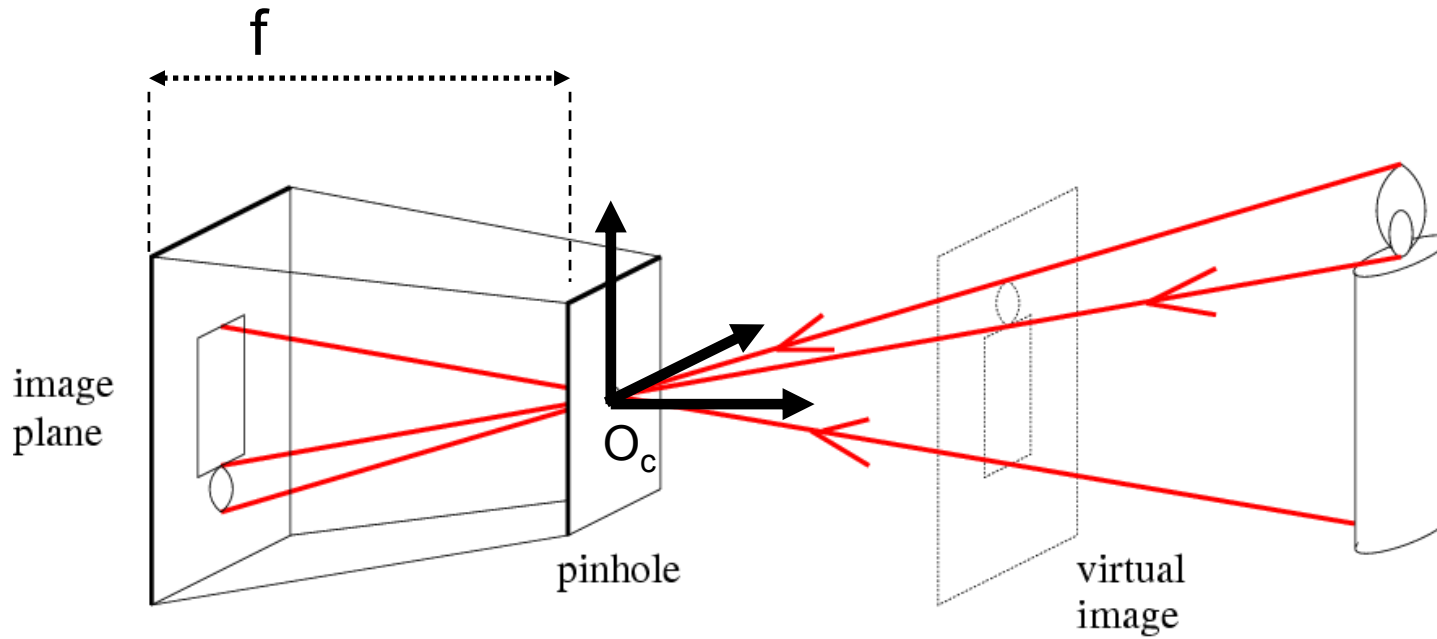
Reading: [FP] Chapter 3
[HZ] Chapter 7

Projective camera



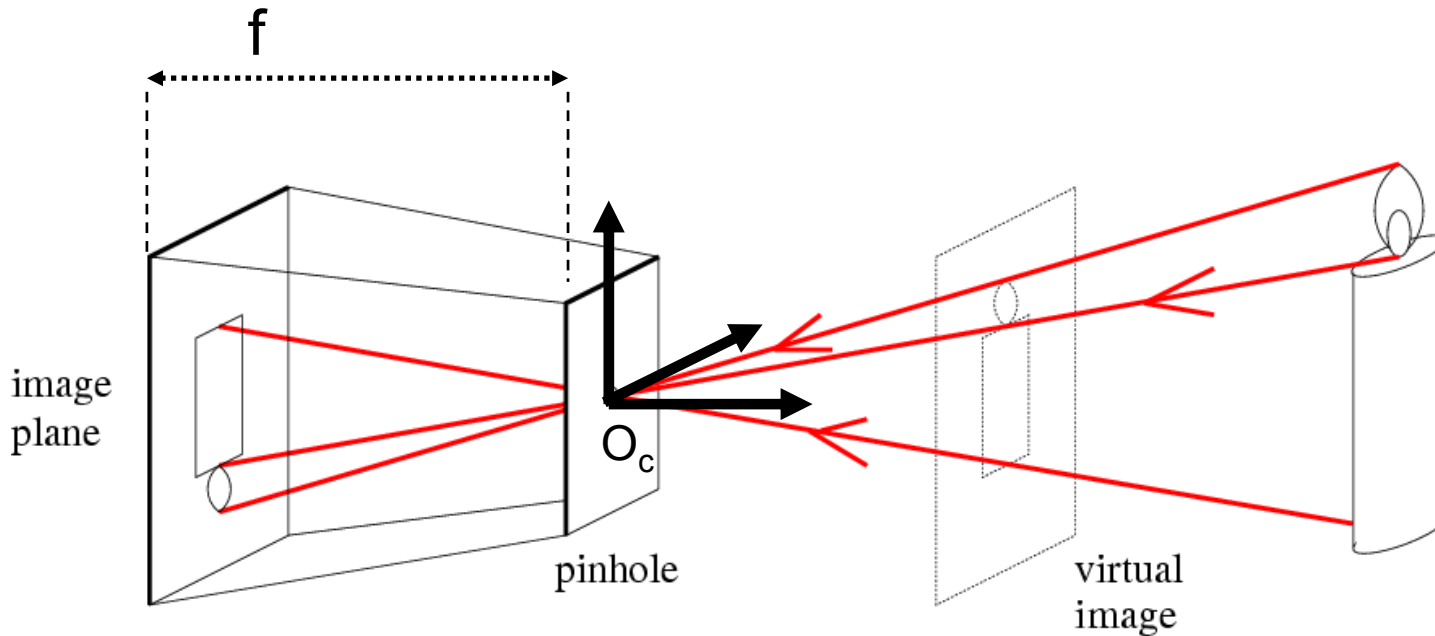
f = focal length

Projective camera



f = focal length
 u_o, v_o = offset

Projective camera



Units: k, l [pixel/m]

f [m]

Non-square pixels

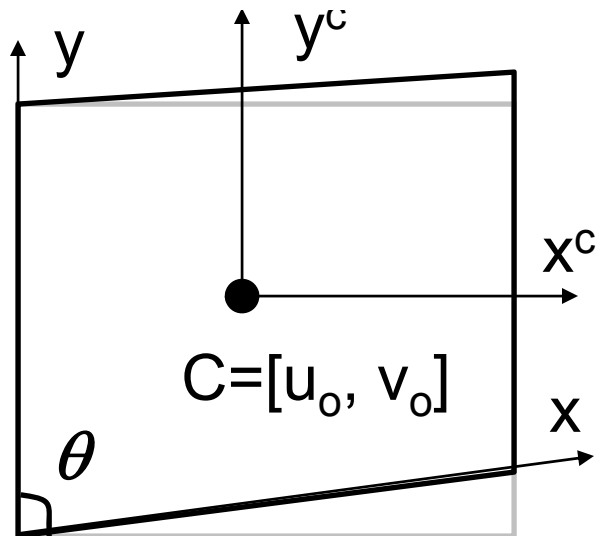
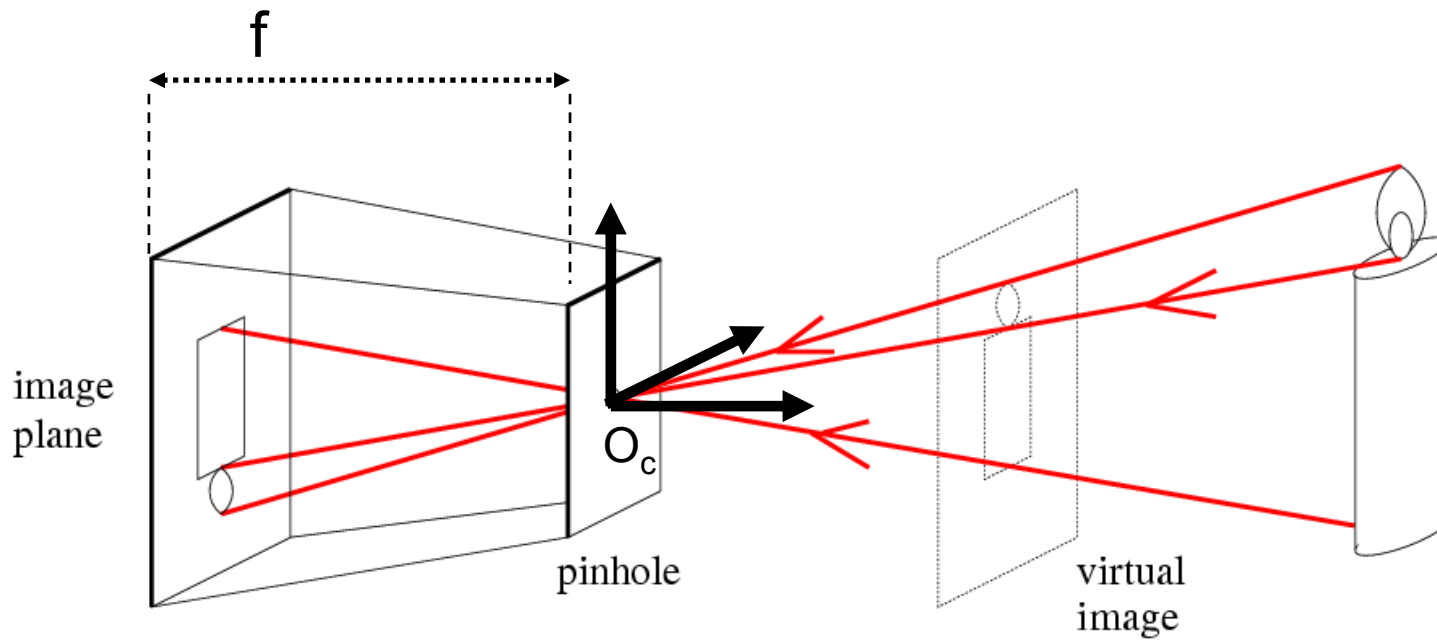
α, β [pixel]

f = focal length

u_o, v_o = offset

α, β \rightarrow non-square pixels

Projective camera



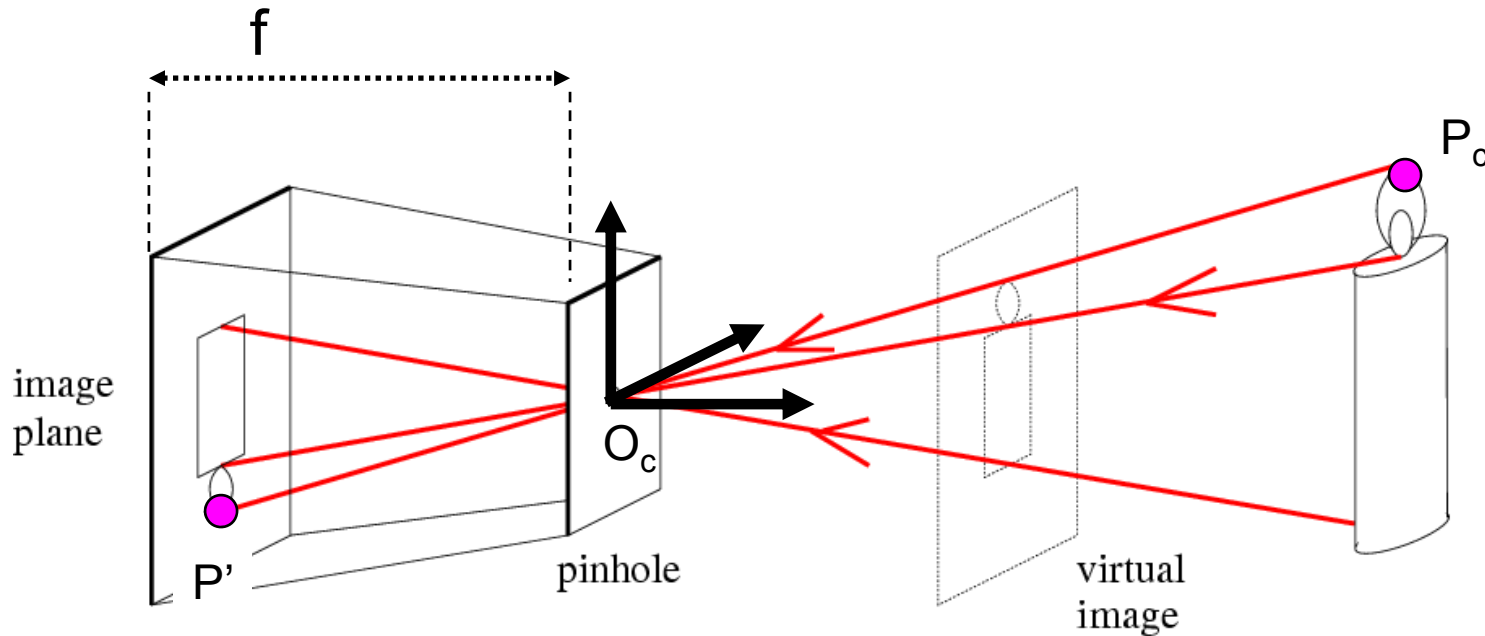
f = focal length

u_o, v_o = offset

α, β \rightarrow non-square pixels

θ = skewness

Projective camera



$$P' = \begin{bmatrix} \alpha & -\alpha \cot \theta & u_o & 0 \\ 0 & \frac{\beta}{\sin \theta} & v_o & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

f = focal length

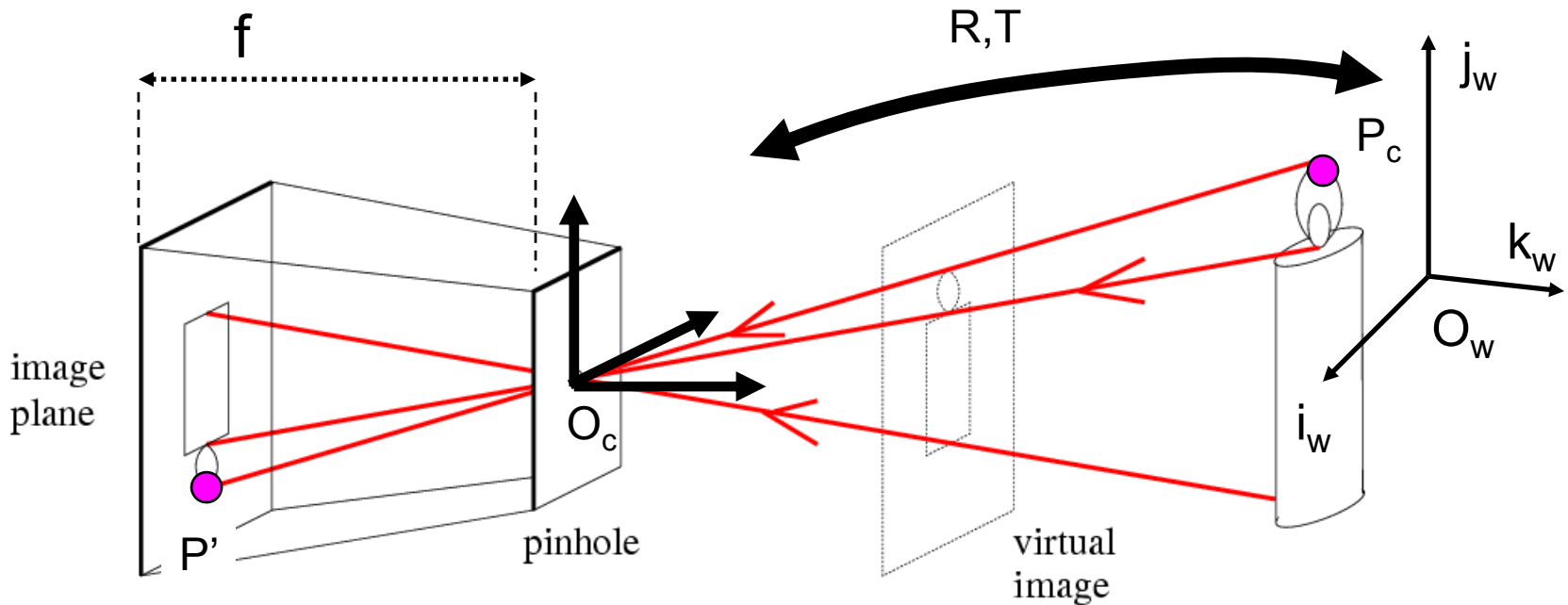
u_o, v_o = offset

$\alpha, \beta \rightarrow$ non-square pixels

θ = skewness

K has 5 degrees of freedom!

Projective camera



$$P_c = [R \quad T] P_w$$

f = focal length

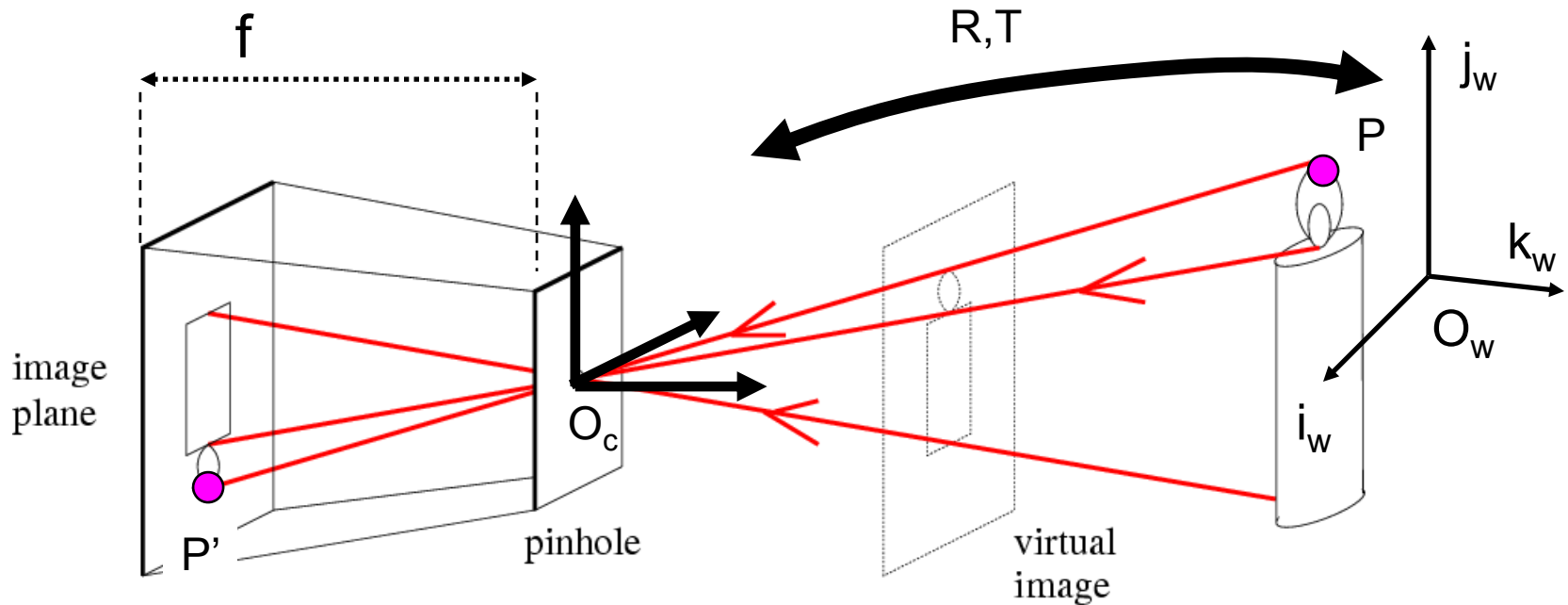
u_o, v_o = offset

α, β \rightarrow non-square pixels

θ = skewness

R, T = rotation, translation

Projective camera



$$P' = M P_w$$

$$= \mathbf{K} \begin{bmatrix} \mathbf{R} & \mathbf{T} \end{bmatrix} P_w$$

Internal parameters

External parameters

f = focal length

u_o, v_o = offset

α, β → non-square pixels

θ = skewness

R, T = rotation, translation

Goal of calibration

Estimate intrinsic and extrinsic parameters from 1 or multiple images

$$P' = M P_w = K [R \quad T] P_w$$

Internal parameters

External parameters

Goal of calibration

Estimate intrinsic and extrinsic parameters from 1 or multiple images

$$P' = M P_w = K [R \quad T] P_w$$

$$M = \begin{pmatrix} \alpha \mathbf{r}_1^T - \alpha \cot \theta \mathbf{r}_2^T + u_0 \mathbf{r}_3^T & \alpha t_x - \alpha \cot \theta t_y + u_0 t_z \\ \frac{\beta}{\sin \theta} \mathbf{r}_2^T + v_0 \mathbf{r}_3^T & \frac{\beta}{\sin \theta} t_y + v_0 t_z \\ \mathbf{r}_3^T & t_z \end{pmatrix}$$

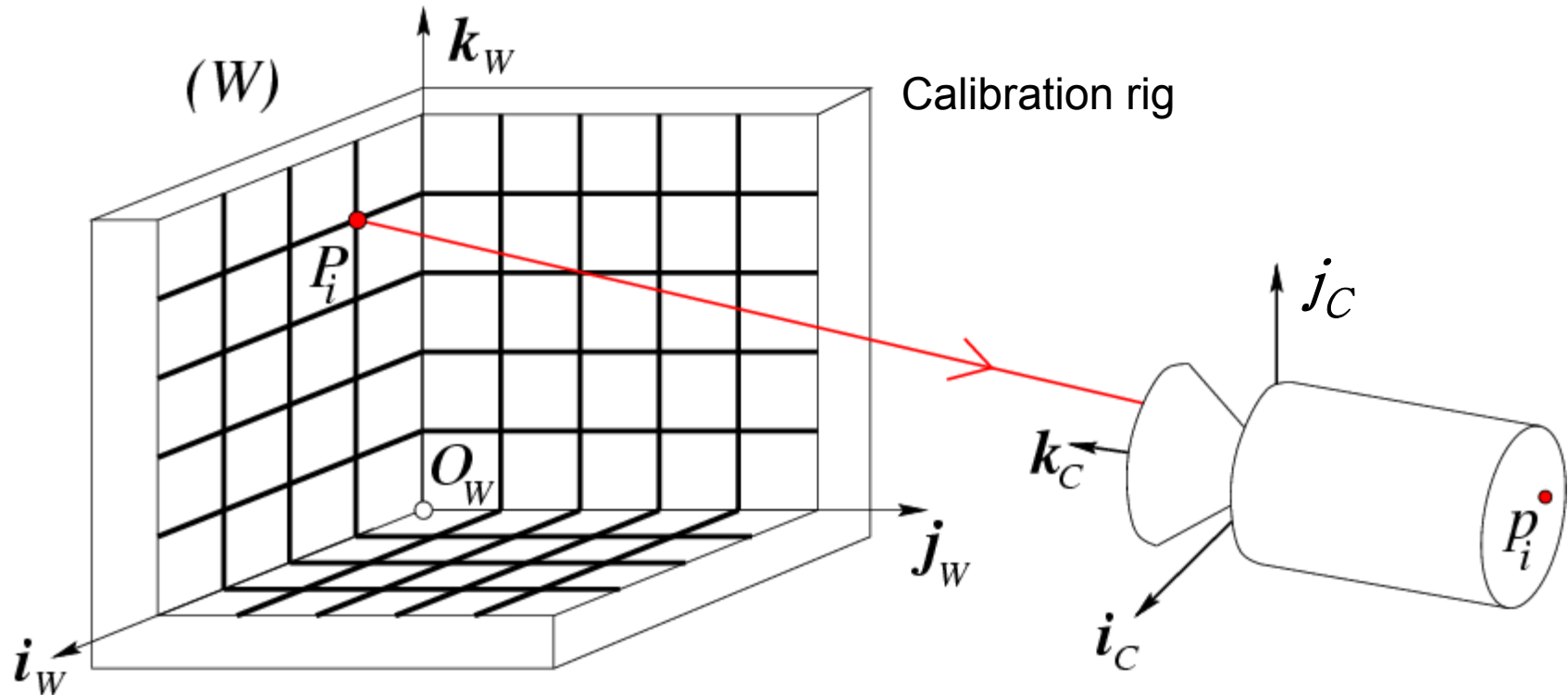
$$K = \begin{bmatrix} \alpha & -\alpha \cot \theta & u_0 \\ 0 & \frac{\beta}{\sin \theta} & v_0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R = \begin{bmatrix} \mathbf{r}_1^T \\ \mathbf{r}_2^T \\ \mathbf{r}_3^T \end{bmatrix}$$

$$T = \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}$$

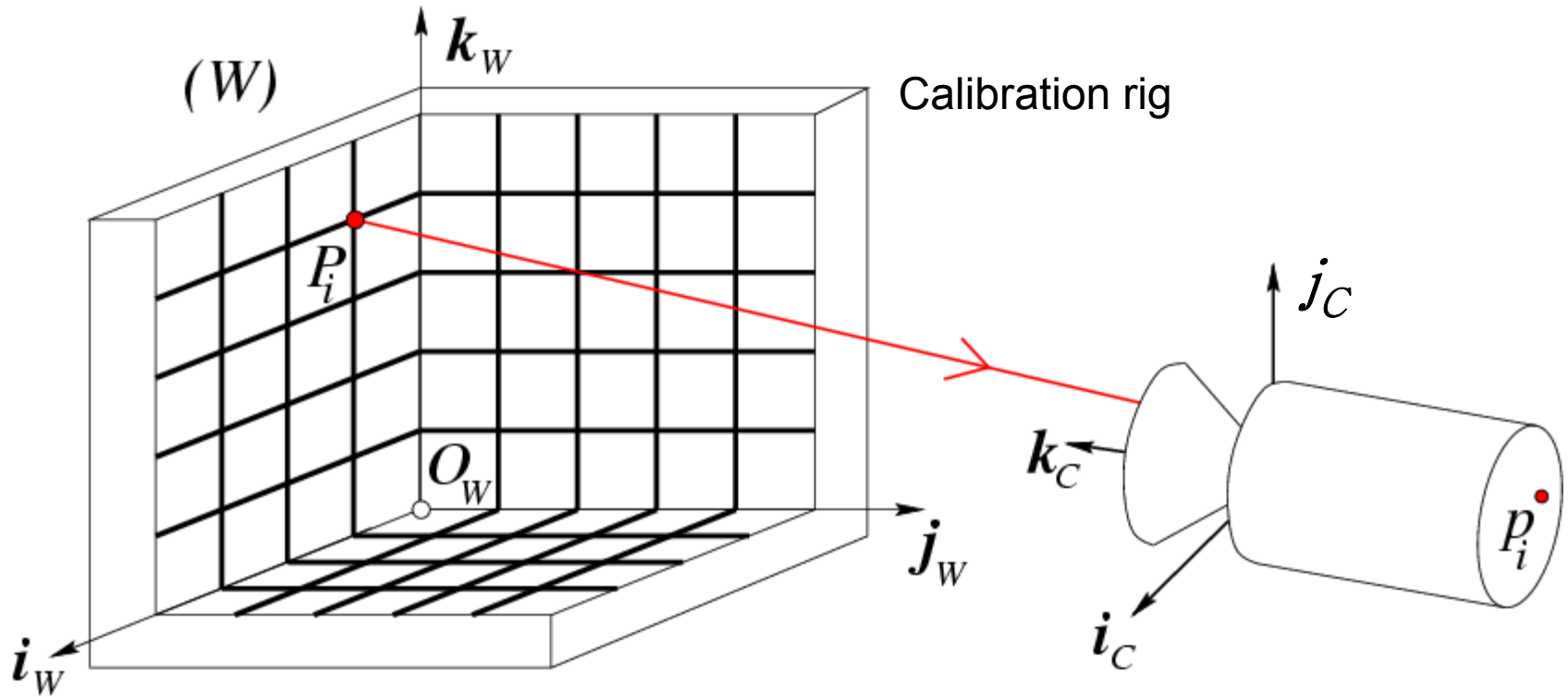
Note: To simplify notation let $P = P_w$

Calibration Problem



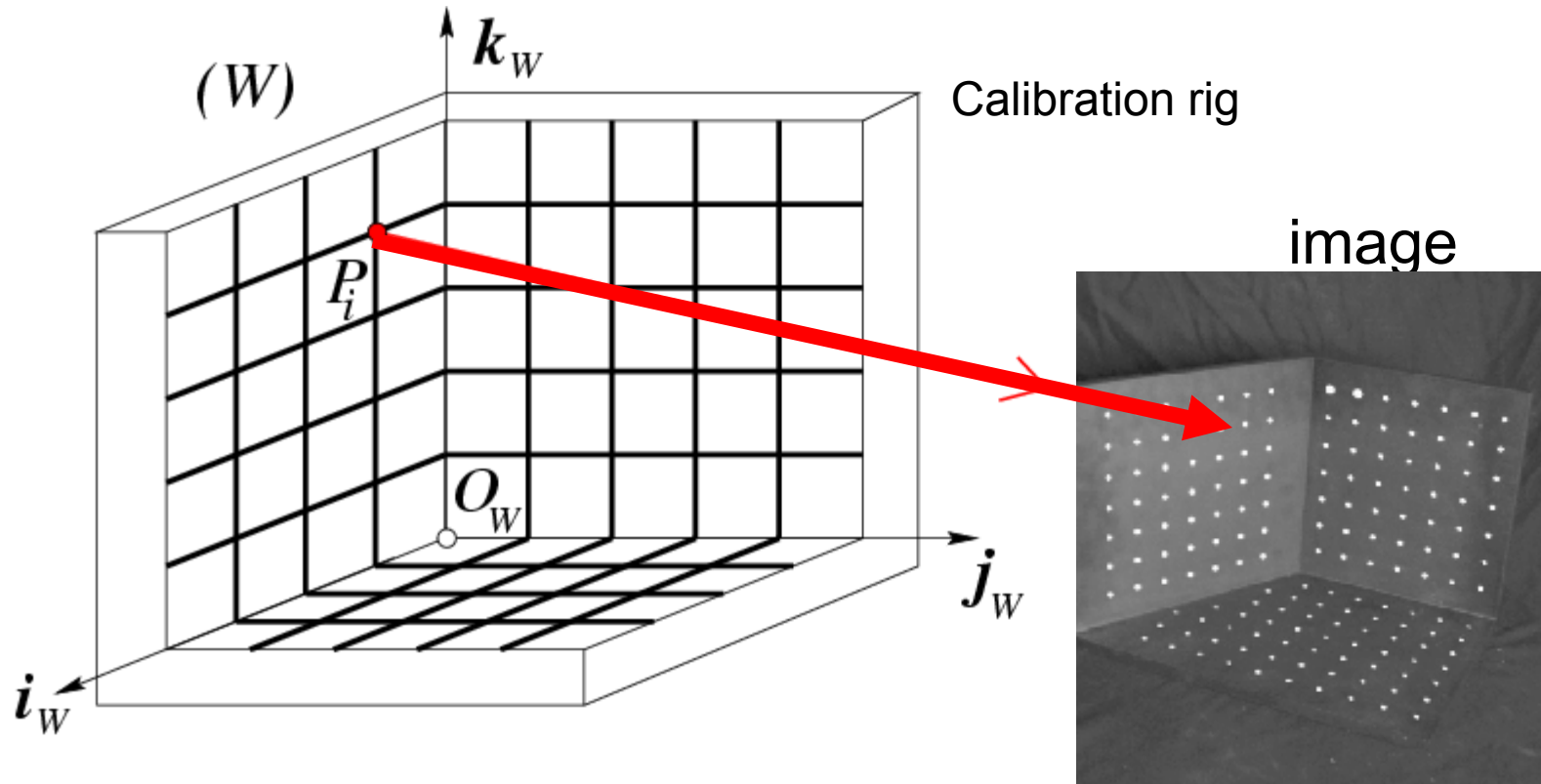
- $P_1 \dots P_n$ with **known** positions in $[O_w, i_w, j_w, k_w]$
 - p_1, \dots, p_n **known** positions in the image
- Goal:** compute intrinsic and extrinsic parameters

Calibration Problem



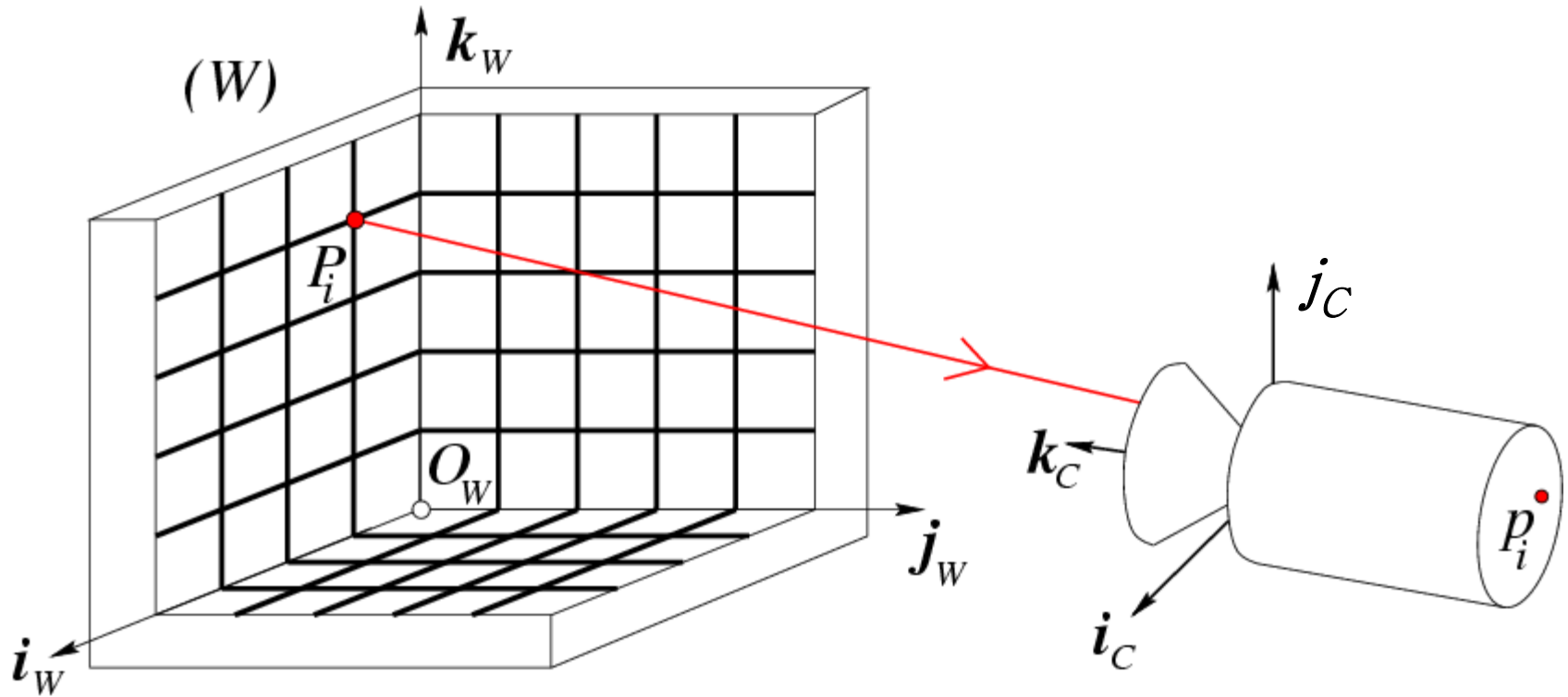
How many correspondences do we need?

Calibration Problem



In practice: user may need to look at the image and select the $n \geq 6$ correspondences

Calibration Problem



$$P_i \rightarrow M P_i \rightarrow p_i = \begin{bmatrix} u_i \\ v_i \end{bmatrix} = \begin{bmatrix} \frac{\mathbf{m}_1 P_i}{\mathbf{m}_3 P_i} \\ \frac{\mathbf{m}_2 P_i}{\mathbf{m}_3 P_i} \end{bmatrix} \quad M = \begin{bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3 \end{bmatrix}$$

in pixels

Calibration Problem

$$\begin{bmatrix} u_i \\ v_i \end{bmatrix} = \begin{bmatrix} \frac{\mathbf{m}_1 P_i}{\mathbf{m}_3 P_i} \\ \frac{\mathbf{m}_2 P_i}{\mathbf{m}_3 P_i} \end{bmatrix}$$

$$\begin{aligned} u_i &= \frac{\mathbf{m}_1 P_i}{\mathbf{m}_3 P_i} \rightarrow u_i (\mathbf{m}_3 P_i) = \mathbf{m}_1 P_i \\ v_i &= \frac{\mathbf{m}_2 P_i}{\mathbf{m}_3 P_i} \rightarrow v_i (\mathbf{m}_3 P_i) = \mathbf{m}_2 P_i \end{aligned} \rightarrow \begin{pmatrix} \mathbf{m}_1 - u_i \mathbf{m}_3 \\ \mathbf{m}_2 - v_i \mathbf{m}_3 \end{pmatrix} P_i = 0$$

Calibration Problem

$$\left\{ \begin{array}{l} \begin{array}{l} \left(\begin{array}{l} \mathbf{m}_1 - u_1 \mathbf{m}_3 \\ \mathbf{m}_2 - v_1 \mathbf{m}_3 \end{array} \right) \\ \vdots \\ \left(\begin{array}{l} \mathbf{m}_1 - u_i \mathbf{m}_3 \\ \mathbf{m}_2 - v_i \mathbf{m}_3 \end{array} \right) \\ \vdots \\ \left(\begin{array}{l} \mathbf{m}_1 - u_n \mathbf{m}_3 \\ \mathbf{m}_2 - v_n \mathbf{m}_3 \end{array} \right) \end{array} \right. P_i = 0$$

Block Matrix Multiplication

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{21} & \mathbf{A}_{22} \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} \mathbf{B}_{11} & \mathbf{B}_{12} \\ \mathbf{B}_{21} & \mathbf{B}_{22} \end{bmatrix}$$

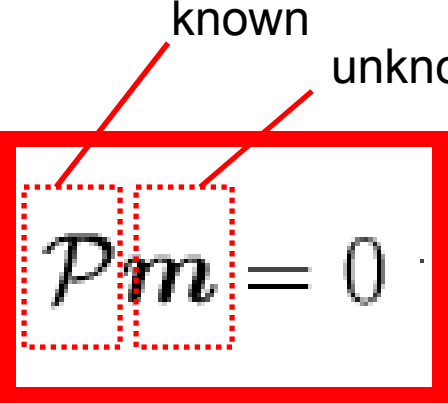
What is \mathbf{AB} ?

$$\mathbf{AB} = \begin{bmatrix} \mathbf{A}_{11}\mathbf{B}_{11} + \mathbf{A}_{12}\mathbf{B}_{21} & \mathbf{A}_{11}\mathbf{B}_{12} + \mathbf{A}_{12}\mathbf{B}_{22} \\ \mathbf{A}_{21}\mathbf{B}_{11} + \mathbf{A}_{22}\mathbf{B}_{21} & \mathbf{A}_{21}\mathbf{B}_{12} + \mathbf{A}_{22}\mathbf{B}_{22} \end{bmatrix}$$

Calibration Problem

$$\left\{ \begin{array}{l} \vdots \\ \left(\begin{array}{l} \mathbf{m}_1 - u_i \mathbf{m}_3 \\ \mathbf{m}_2 - v_i \mathbf{m}_3 \end{array} \right) P_i = 0 \\ \vdots \\ \left(\begin{array}{l} \mathbf{m}_1 - u_n \mathbf{m}_3 \\ \mathbf{m}_2 - v_n \mathbf{m}_3 \end{array} \right) P_n = 0 \end{array} \right.$$

known unknown



$$\mathcal{P} \mathbf{m} = 0$$

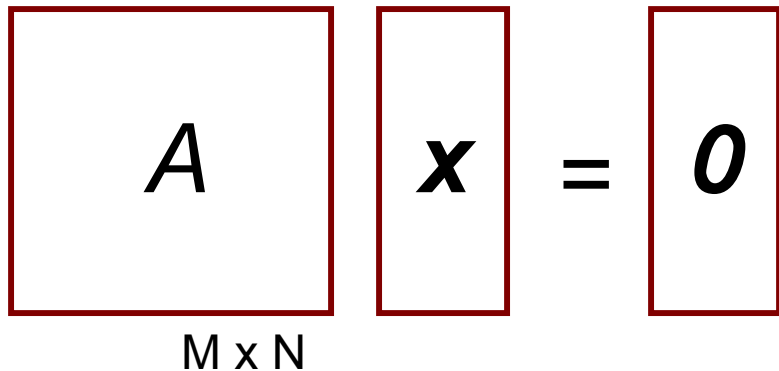
Homogenous linear system

$$\mathcal{P} \stackrel{\text{def}}{=} \begin{pmatrix} P_1^T & \mathbf{0}^T & -u_1 P_1^T \\ \mathbf{0}^T & P_1^T & -v_1 P_1^T \\ \dots & \dots & \dots \\ P_n^T & \mathbf{0}^T & -u_n P_n^T \\ \mathbf{0}^T & P_n^T & -v_n P_n^T \end{pmatrix} \begin{matrix} 1 \times 4 \\ \\ \\ 2n \times 12 \end{matrix}$$

$$\mathbf{m} \stackrel{\text{def}}{=} \begin{pmatrix} \mathbf{m}_1^T \\ \mathbf{m}_2^T \\ \mathbf{m}_3^T \end{pmatrix} \begin{matrix} 4 \times 1 \\ \\ 12 \times 1 \end{matrix}$$

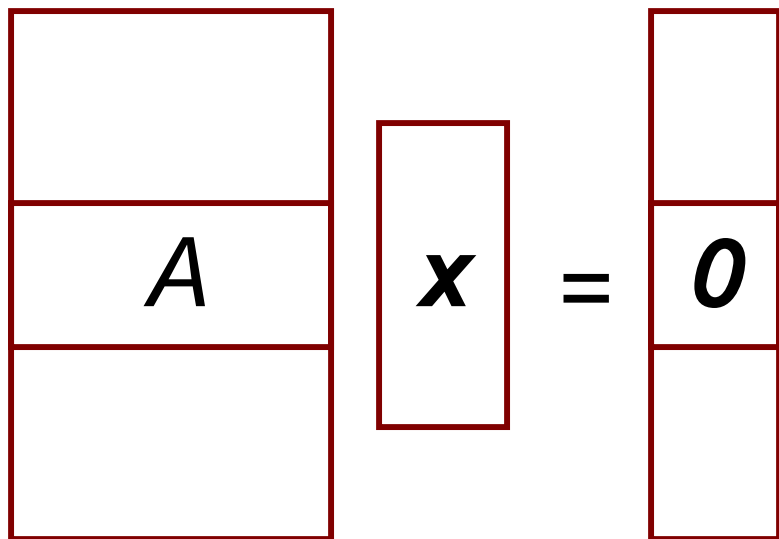
Homogeneous M x N Linear Systems

M=number of equations
N=number of unknown


$$\begin{matrix} \boxed{A} & \boxed{x} & = & \boxed{0} \\ M \times N & & & \end{matrix}$$

Square system (M=N):

- if $\text{Det}(A) \neq 0$,
unique solution: 0


$$\begin{matrix} \boxed{A} & \boxed{x} & = & \boxed{0} \end{matrix}$$

Rectangular system

- 0 is always a solution
- non-zero solution if $\det(A) \neq 0$
- noisy measurements

Minimize $|Ax|^2$

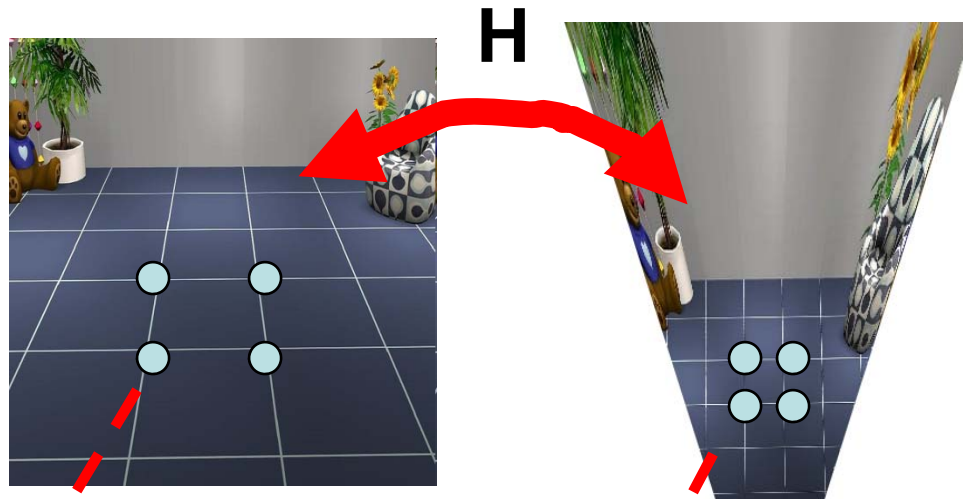
under the constraint $|x|^2 = 1$

Calibration Problem

$$\mathcal{P}m = 0$$

How do we solve this homogenous linear system?

DLT algorithm (Direct Linear Transformation)



\mathbf{x}_i

\mathbf{x}'_i

$$\mathbf{x}'_i = \mathbf{H} \mathbf{x}_i$$

$$\mathbf{x}'_i \times \mathbf{H} \mathbf{x}_i = \mathbf{0}$$



$$\underbrace{\mathbf{A}_i}_{\text{Function of measurements}} \overbrace{\mathbf{h}}^{\text{unknown}} = \mathbf{0}$$

Function of measurements

General Calibration Problem

$$\mathcal{P}m = 0$$

$$\mathbf{U}_{2n \times 12} \mathbf{D}_{12 \times 12} \mathbf{V}^T_{12 \times 12}$$

Last column of \mathbf{V} gives m

$$\mathbf{M} \mathbf{P}_i \rightarrow \mathbf{p}_i$$

Extracting camera parameters

$$\mathcal{M} = \left(\begin{array}{c|c} \alpha \mathbf{r}_1^T - \alpha \cot \theta \mathbf{r}_2^T + u_0 \mathbf{r}_3^T & \alpha t_x - \alpha \cot \theta t_y + u_0 t_z \\ \frac{\beta}{\sin \theta} \mathbf{r}_2^T + v_0 \mathbf{r}_3^T & \frac{\beta}{\sin \theta} t_y + v_0 t_z \\ \mathbf{r}_3^T & t_z \end{array} \right) = \mathbf{K} \begin{bmatrix} \mathbf{R} \\ \mathbf{T} \end{bmatrix}$$

$\rho \mathbf{A}$
 $\rho \mathbf{b}$

$$\mathbf{A} = \begin{bmatrix} \mathbf{a}_1^T \\ \mathbf{a}_2^T \\ \mathbf{a}_3^T \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

Estimated values

Intrinsic

$$\rho = \frac{\pm 1}{|\mathbf{a}_3|} \quad \begin{array}{l} u_0 = \rho^2 (\mathbf{a}_1 \cdot \mathbf{a}_2) \\ v_0 = \rho^2 (\mathbf{a}_2 \cdot \mathbf{a}_3) \end{array}$$

$$\cos \theta = \frac{(\mathbf{a}_1 \times \mathbf{a}_3) \cdot (\mathbf{a}_2 \times \mathbf{a}_3)}{|\mathbf{a}_1 \times \mathbf{a}_3| \cdot |\mathbf{a}_2 \times \mathbf{a}_3|}$$

Theorem (Faugeras, 1993)

Let $\mathcal{M} = (\mathcal{A} \ \mathbf{b})$ be a 3×4 matrix and let \mathbf{a}_i^T ($i = 1, 2, 3$) denote the rows of the matrix \mathcal{A} formed by the three leftmost columns of \mathcal{M} .

- A necessary and sufficient condition for \mathcal{M} to be a perspective projection matrix is that $\text{Det}(\mathcal{A}) \neq 0$.

- A necessary and sufficient condition for \mathcal{M} to be a zero-skew perspective projection matrix is that $\text{Det}(\mathcal{A}) \neq 0$ and

$$(\mathbf{a}_1 \times \mathbf{a}_3) \cdot (\mathbf{a}_2 \times \mathbf{a}_3) = 0.$$

- A necessary and sufficient condition for \mathcal{M} to be a perspective projection matrix with zero skew and unit aspect-ratio is that $\text{Det}(\mathcal{A}) \neq 0$ and

$$\begin{cases} (\mathbf{a}_1 \times \mathbf{a}_3) \cdot (\mathbf{a}_2 \times \mathbf{a}_3) = 0, \\ (\mathbf{a}_1 \times \mathbf{a}_3) \cdot (\mathbf{a}_1 \times \mathbf{a}_3) = (\mathbf{a}_2 \times \mathbf{a}_3) \cdot (\mathbf{a}_2 \times \mathbf{a}_3). \end{cases}$$

Extracting camera parameters

$$\mathcal{M} = \left(\begin{array}{c|c} \alpha \mathbf{r}_1^T - \alpha \cot \theta \mathbf{r}_2^T + u_0 \mathbf{r}_3^T & \alpha t_x - \alpha \cot \theta t_y + u_0 t_z \\ \hline \frac{\beta}{\sin \theta} \mathbf{r}_2^T + v_0 \mathbf{r}_3^T & \frac{\beta}{\sin \theta} t_y + v_0 t_z \\ \hline \mathbf{r}_3^T & t_z \end{array} \right) = \mathbf{K} [\mathbf{R} \quad \mathbf{T}]$$

$\rho \mathbf{A}$
 $\rho \mathbf{b}$

$$\mathbf{A} = \begin{bmatrix} \mathbf{a}_1^T \\ \mathbf{a}_2^T \\ \mathbf{a}_3^T \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

Estimated values

Intrinsic

$$\alpha = \rho^2 |\mathbf{a}_1 \times \mathbf{a}_3| \sin \theta \quad \rightarrow \quad \mathbf{f}$$

$$\beta = \rho^2 |\mathbf{a}_2 \times \mathbf{a}_3| \sin \theta$$

Extracting camera parameters

$$\mathcal{M} = \left(\begin{array}{c|c} \alpha \mathbf{r}_1^T - \alpha \cot \theta \mathbf{r}_2^T + u_0 \mathbf{r}_3^T & \alpha t_x - \alpha \cot \theta t_y + u_0 t_z \\ \hline \frac{\beta}{\sin \theta} \mathbf{r}_2^T + v_0 \mathbf{r}_3^T & \frac{\beta}{\sin \theta} t_y + v_0 t_z \\ \hline \mathbf{r}_3^T & t_z \end{array} \right) = \mathbf{K} [\mathbf{R} \quad \mathbf{T}]$$

$\rho \mathbf{A}$ $\rho \mathbf{b}$

$$\mathbf{A} = \begin{bmatrix} \mathbf{a}_1^T \\ \mathbf{a}_2^T \\ \mathbf{a}_3^T \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

Estimated values

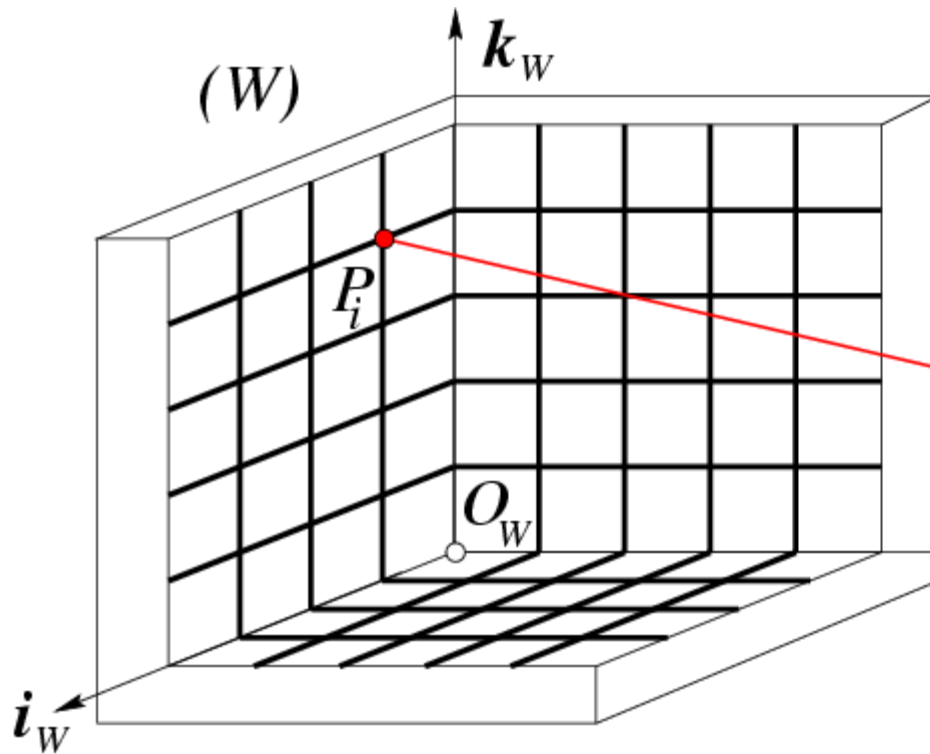
Extrinsic

$$\mathbf{r}_1 = \frac{(\mathbf{a}_2 \times \mathbf{a}_3)}{|\mathbf{a}_2 \times \mathbf{a}_3|} \quad \mathbf{r}_3 = \frac{\pm 1}{|\mathbf{a}_3|}$$

$$\mathbf{r}_2 = \mathbf{r}_3 \times \mathbf{r}_1$$

$$\mathbf{T} = \rho \mathbf{K}^{-1} \mathbf{b}$$

Degenerate cases

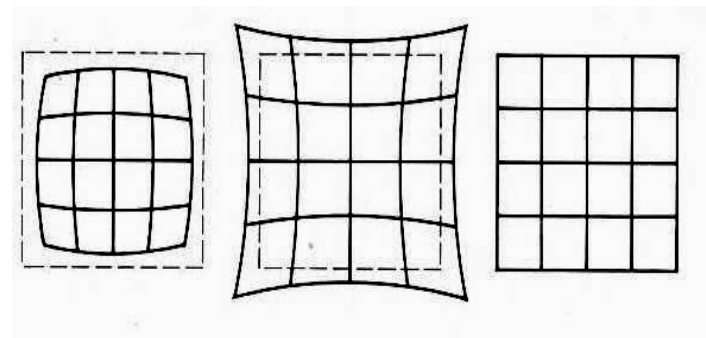
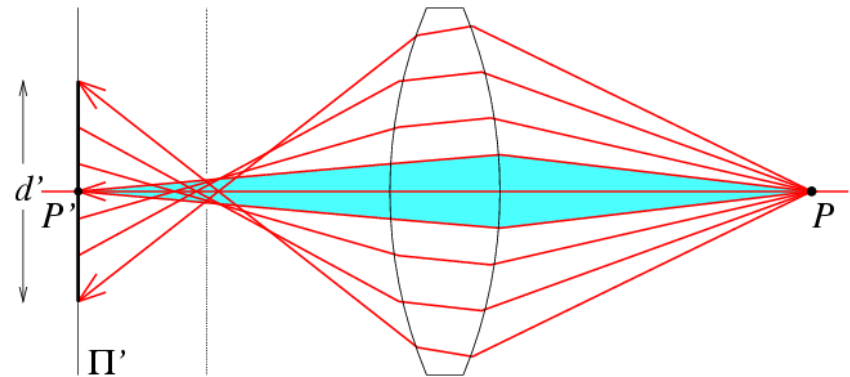
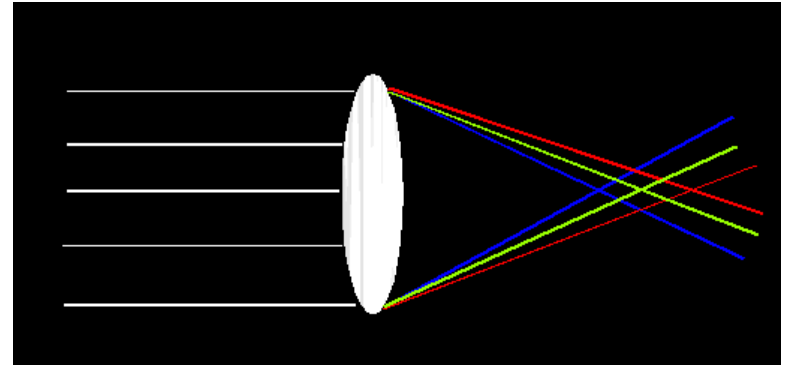


$$\mathcal{P} \stackrel{\text{def}}{=} \begin{matrix} i \\ \begin{pmatrix} \mathbf{P}_1^T & \mathbf{0}^T & -u_1 \mathbf{P}_1^T \\ \mathbf{0}^T & \mathbf{P}_1^T & -v_1 \mathbf{P}_1^T \\ \dots & \dots & \dots \\ \mathbf{P}_n^T & \mathbf{0}^T & -u_n \mathbf{P}_n^T \\ \mathbf{0}^T & \mathbf{P}_n^T & -v_n \mathbf{P}_n^T \end{pmatrix} \end{matrix}$$

- P_i 's cannot lie on the same plane!
- Points cannot lie on the intersection curve of two quadric surfaces

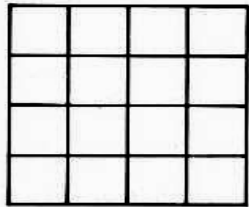
Taking lens distortions into account

- Chromatic Aberration
- Spherical aberration
- Radial Distortion

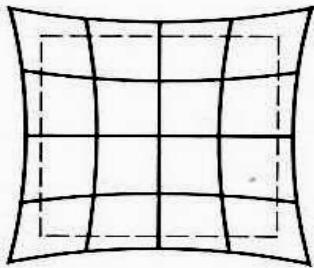


Radial Distortion

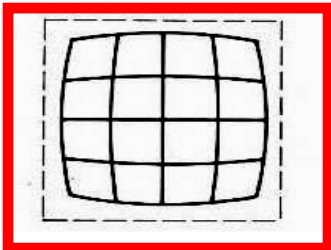
- Caused by imperfect lenses
- Deviations are most noticeable for rays that pass through the edge of the lens



No distortion



Pin cushion

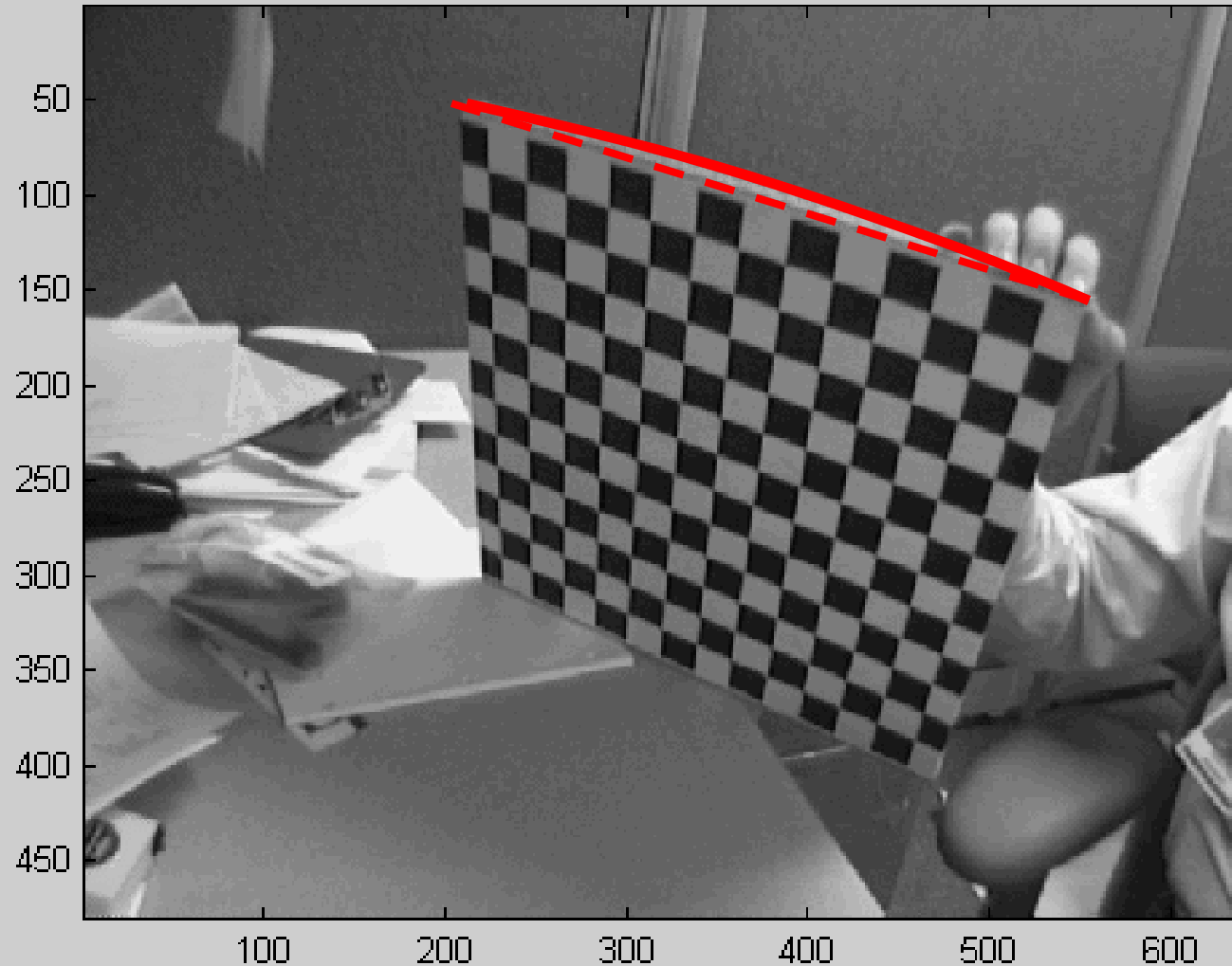


Barrel

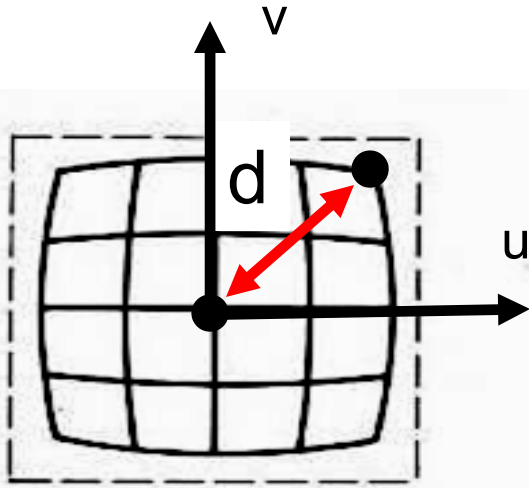


Radial Distortion

Click on the four extreme corners of the rectangular pattern...



Radial Distortion



$$d^2 = a u^2 + b v^2 + c u v$$

$$\begin{bmatrix} \frac{1}{\lambda} & 0 & 0 \\ 0 & \frac{1}{\lambda} & 0 \\ 0 & 0 & 1 \end{bmatrix} \mathbf{M} \mathbf{P}_i \rightarrow \begin{bmatrix} u_i \\ v_i \end{bmatrix} = \mathbf{p}_i$$

$$\lambda = 1 \pm \sum_{p=1}^3 \kappa_p d^{2p}$$

$\sum_{p=1}^3 \kappa_p d^{2p}$

 Distortion coefficient

}

 Polynomial function

Radial Distortion

Estimating m_1 and $m_2 \dots$

$$\mathbf{p}_i = \begin{bmatrix} \mathbf{u}_i \\ \mathbf{v}_i \end{bmatrix} = \frac{1}{\lambda} \begin{bmatrix} \frac{\mathbf{m}_1 P_i}{\mathbf{m}_3 P_i} \\ \frac{\mathbf{m}_2 P_i}{\mathbf{m}_3 P_i} \end{bmatrix}$$

How to do that?

Radial Distortion

Estimating m_1 and $m_2 \dots$

$$\mathbf{p}_i = \begin{bmatrix} \mathbf{u}_i \\ \mathbf{v}_i \end{bmatrix} = \frac{1}{\lambda} \begin{bmatrix} \frac{\mathbf{m}_1 P_i}{\mathbf{m}_3 P_i} \\ \frac{\mathbf{m}_2 P_i}{\mathbf{m}_3 P_i} \end{bmatrix}$$

How to do that?

$$\begin{cases} v(\mathbf{m}_1 P_1) - u(\mathbf{m}_2 P_1) = 0 \\ v(\mathbf{m}_1 P_i) - u(\mathbf{m}_2 P_i) = 0 \\ \vdots \\ v(\mathbf{m}_1 P_n) - u(\mathbf{m}_2 P_n) = 0 \end{cases}$$

$$Q \mathbf{n} = 0 \quad \mathbf{n} = \begin{bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \end{bmatrix}$$

Tsai technique [87]

Radial Distortion

Estimating m_1 and $m_2 \dots$

$$\mathbf{p}_i = \begin{bmatrix} \mathbf{u}_i \\ \mathbf{v}_i \end{bmatrix} = \frac{1}{\lambda} \begin{bmatrix} \frac{\mathbf{m}_1 P_i}{\mathbf{m}_3 P_i} \\ \frac{\mathbf{m}_2 P_i}{\mathbf{m}_3 P_i} \end{bmatrix}$$

How to do that?

\mathbf{m}_3 is non linear function of

- \mathbf{m}_1
- \mathbf{m}_2
- λ

Radial Distortion

$$\begin{bmatrix} 1 & 0 & 0 \\ \lambda & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & 1 \end{bmatrix} \mathbf{M} \mathbf{P}_i \rightarrow \begin{bmatrix} \mathbf{u}_i \\ \mathbf{v}_i \end{bmatrix} = \mathbf{p}_i \quad \mathbf{Q} = \begin{bmatrix} \mathbf{q}_1 \\ \mathbf{q}_2 \\ \mathbf{q}_3 \end{bmatrix}$$

Q

Non-linear system of equations

$$\mathbf{p}_i = \begin{bmatrix} \mathbf{u}_i \\ \mathbf{v}_i \end{bmatrix} = \begin{bmatrix} \frac{\mathbf{q}_1 \mathbf{P}_i}{\mathbf{q}_3 \mathbf{P}_i} \\ \frac{\mathbf{q}_2 \mathbf{P}_i}{\mathbf{q}_3 \mathbf{P}_i} \end{bmatrix} \rightarrow \begin{cases} \mathbf{u}_i \mathbf{q}_3 \mathbf{P}_i = \mathbf{q}_1 \mathbf{P}_i \\ \mathbf{v}_i \mathbf{q}_3 \mathbf{P}_i = \mathbf{q}_2 \mathbf{P}_i \end{cases}$$

General Calibration Problem

$$X = f(P) \quad f(\cdot) \text{ is nonlinear}$$

measurement \nearrow \nwarrow parameter

-Newton Method

-Levenberg-Marquardt Algorithm

- Iterative, starts from initial solution
- May be slow if initial solution far from real solution
- Estimated solution may be function of the initial solution
- Newton requires the computation of J, H
- Levenberg-Marquardt doesn't require the computation of H

General Calibration Problem

$$X = f(P) \quad f(\cdot) \text{ is nonlinear}$$

measurement \nearrow X \nwarrow parameter P

A possible algorithm

1. Solve linear part of the system to find approximated solution
2. Use this solution as initial condition for the full system
3. Solve full system using Newton or L.M.

General Calibration Problem

$$X = f(P) \quad f(\cdot) \text{ is nonlinear}$$

measurement \nearrow \nwarrow parameter

Typical assumptions:

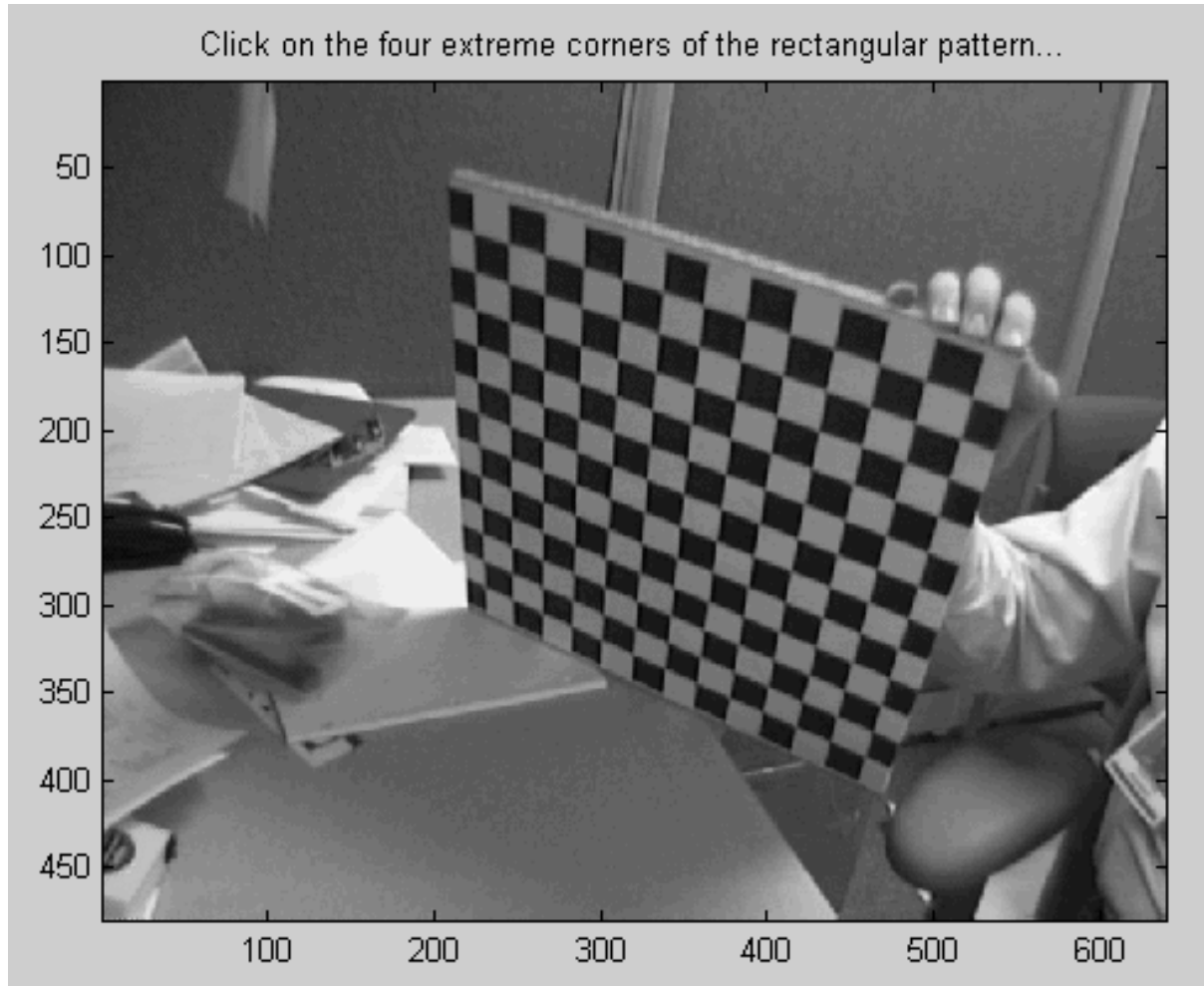
- zero-skew, square pixel
- $u_o, v_o =$ known center of the image
- no distortion



Just estimate f
and R, T

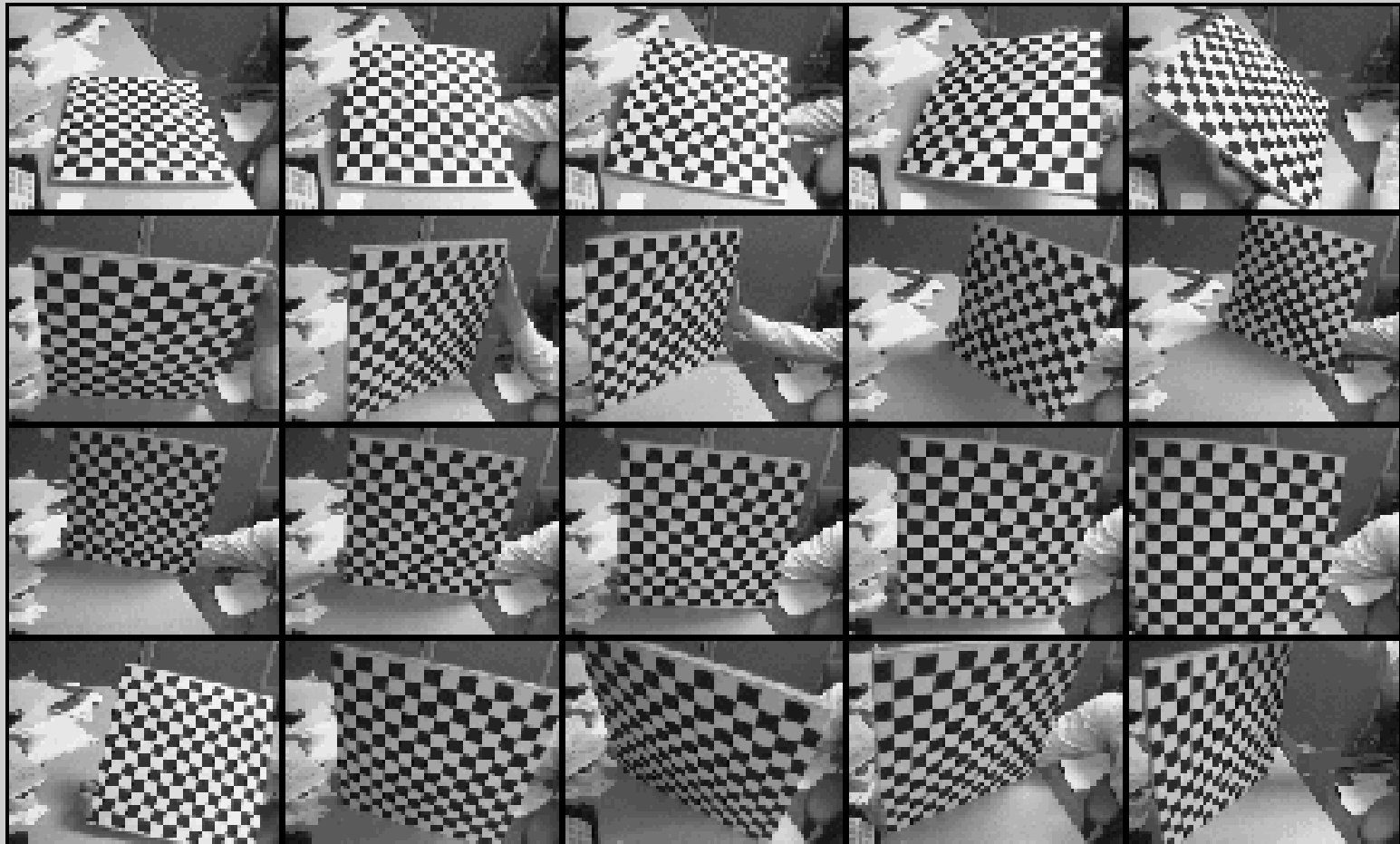
Calibration Procedure

Camera Calibration Toolbox for Matlab
J. Bouquet – [1998-2000]



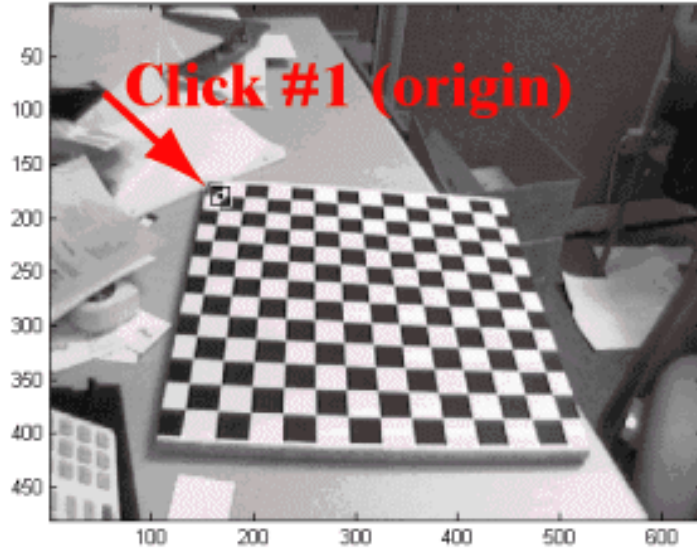
Calibration Procedure

Calibration images

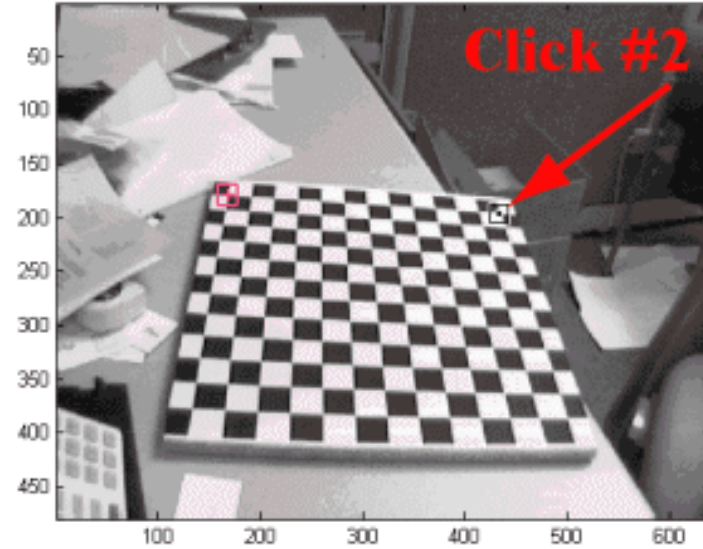


Calibration Procedure

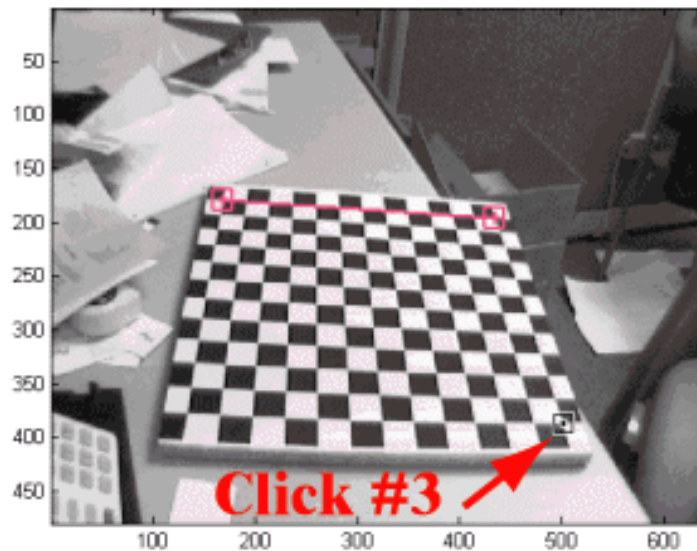
Click on the four extreme corners of the rectangular pattern (first corner = origin)... Image 1



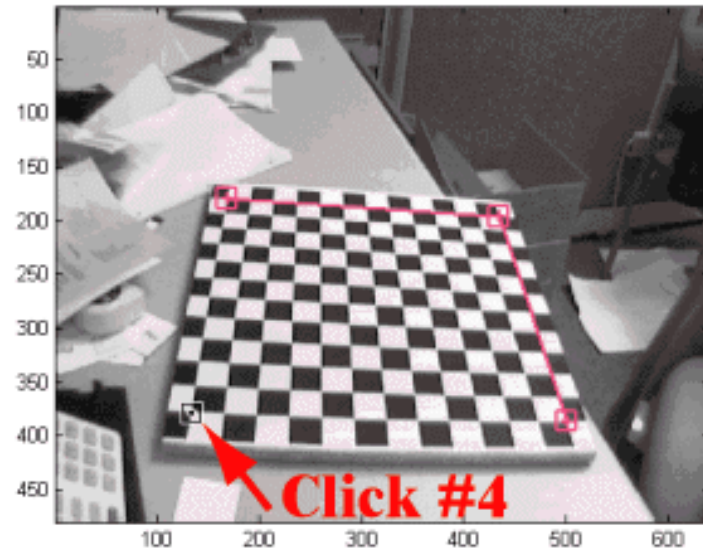
Click on the four extreme corners of the rectangular pattern (first corner = origin)... Image 1



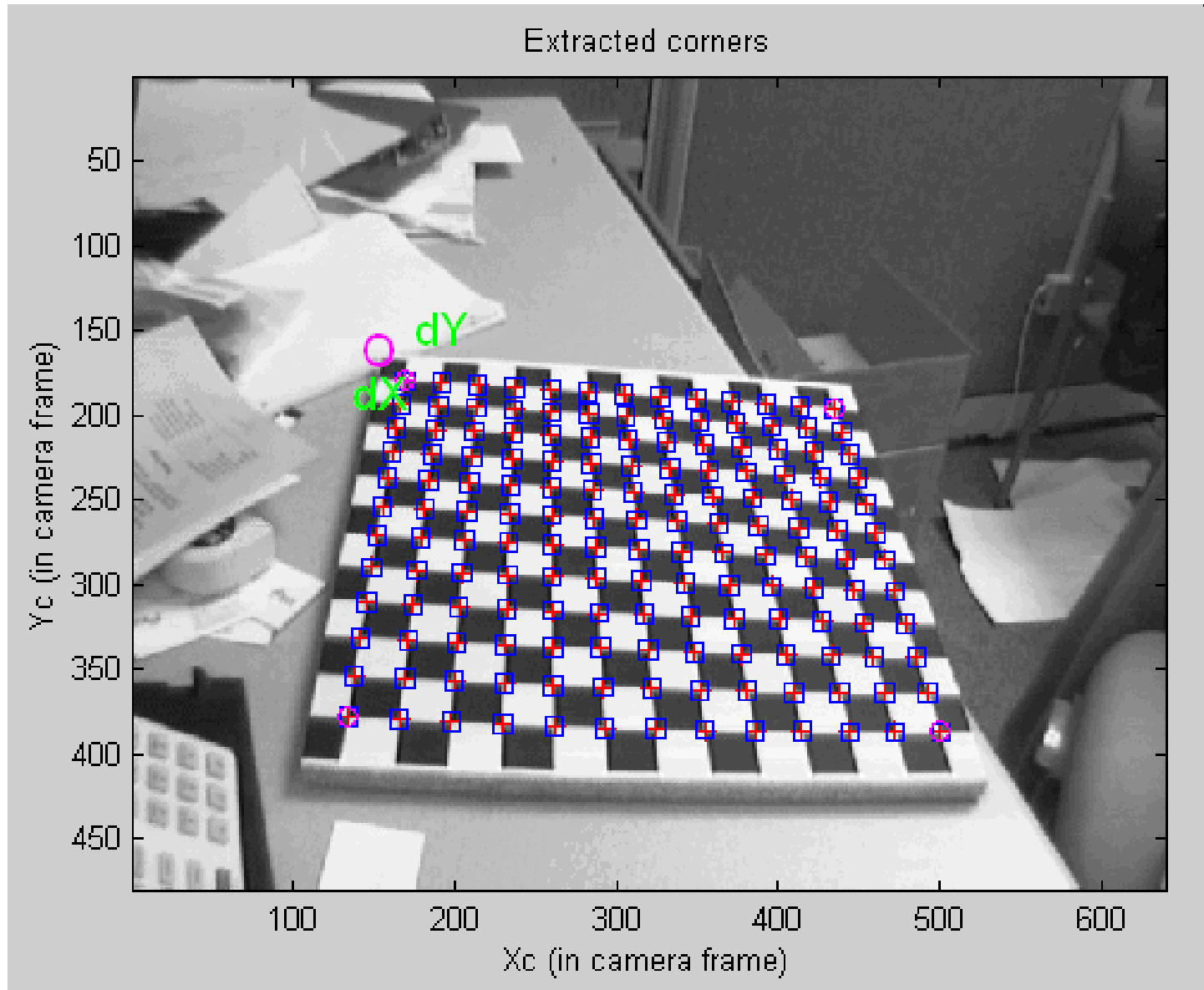
Click on the four extreme corners of the rectangular pattern (first corner = origin)... Image 1



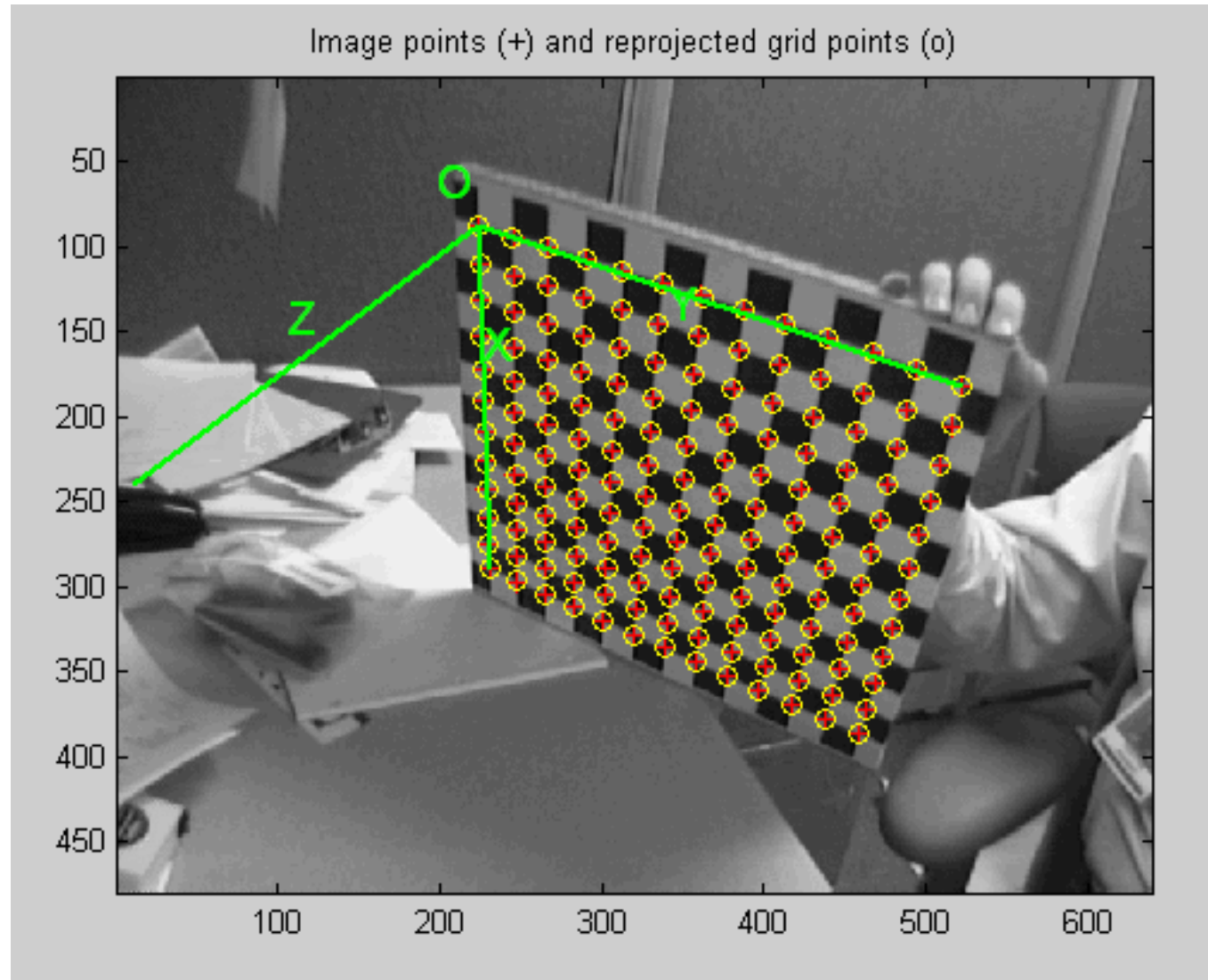
Click on the four extreme corners of the rectangular pattern (first corner = origin)... Image 1



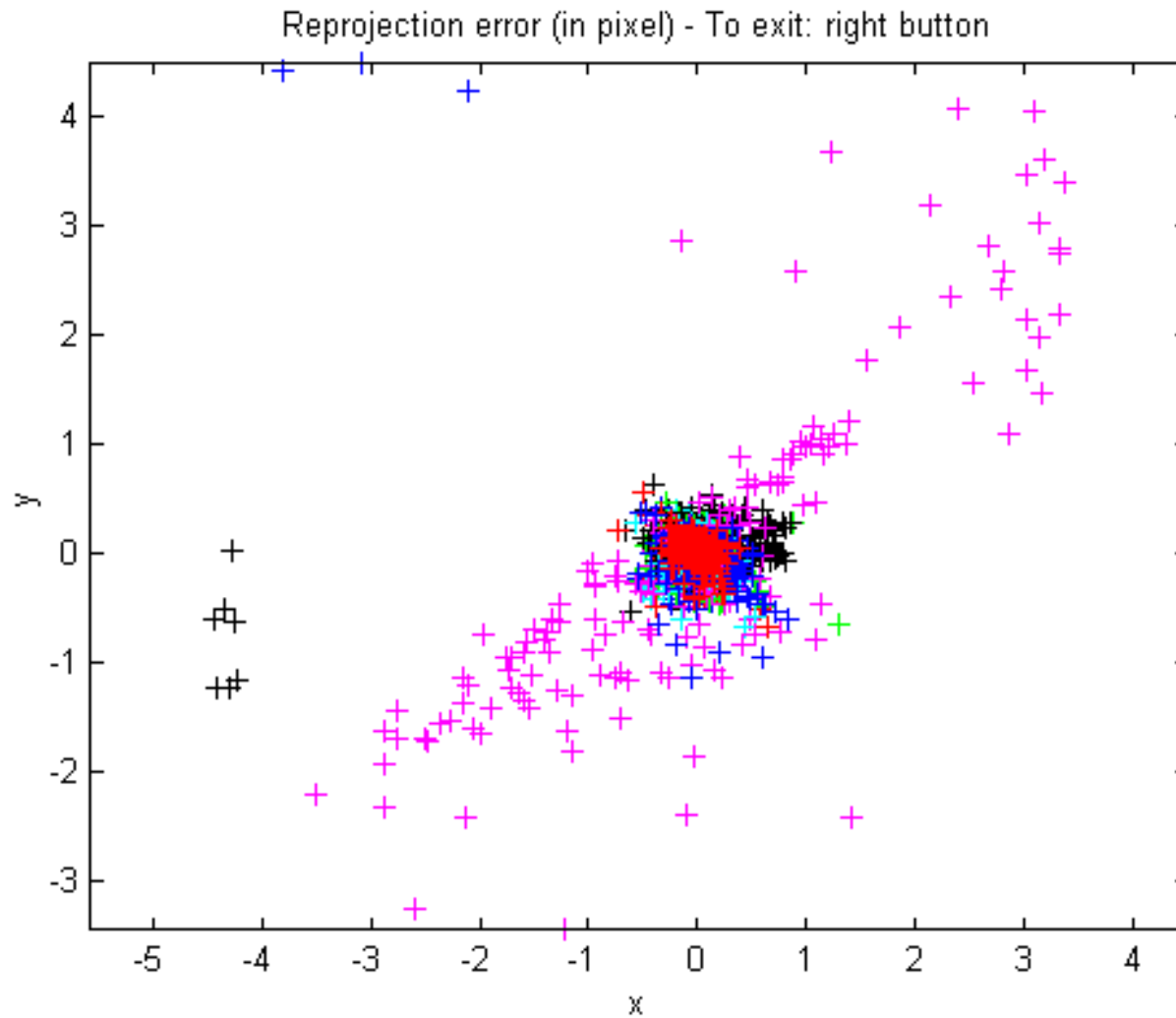
Calibration Procedure



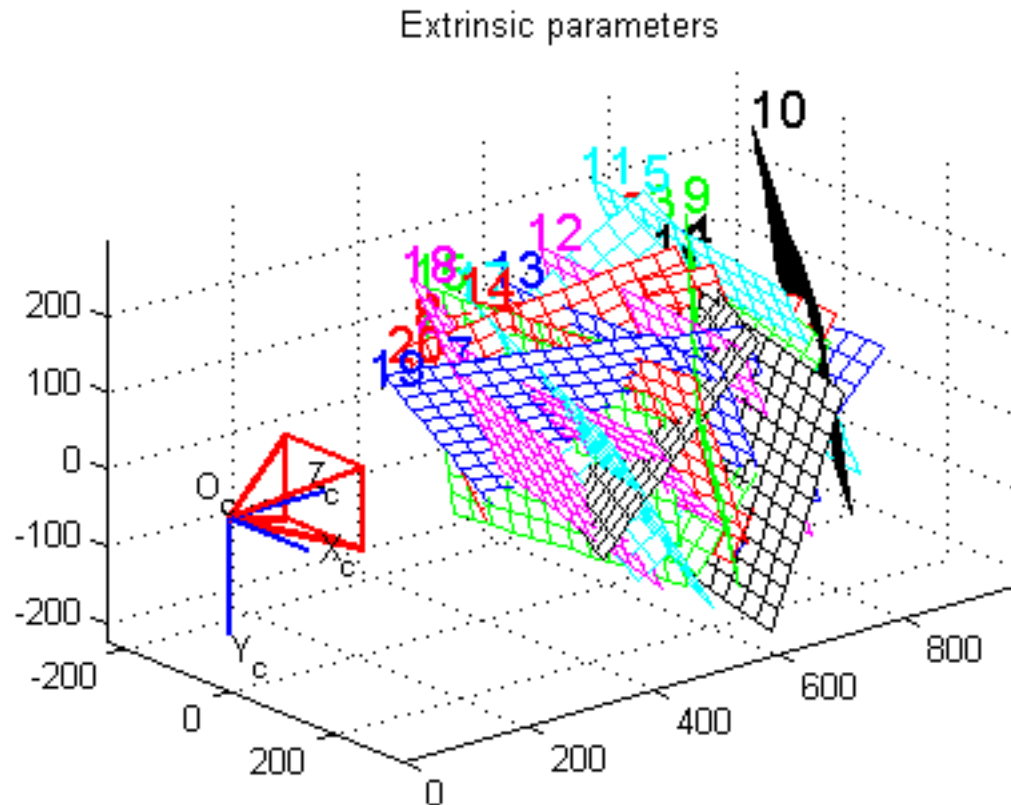
Calibration Procedure



Calibration Procedure

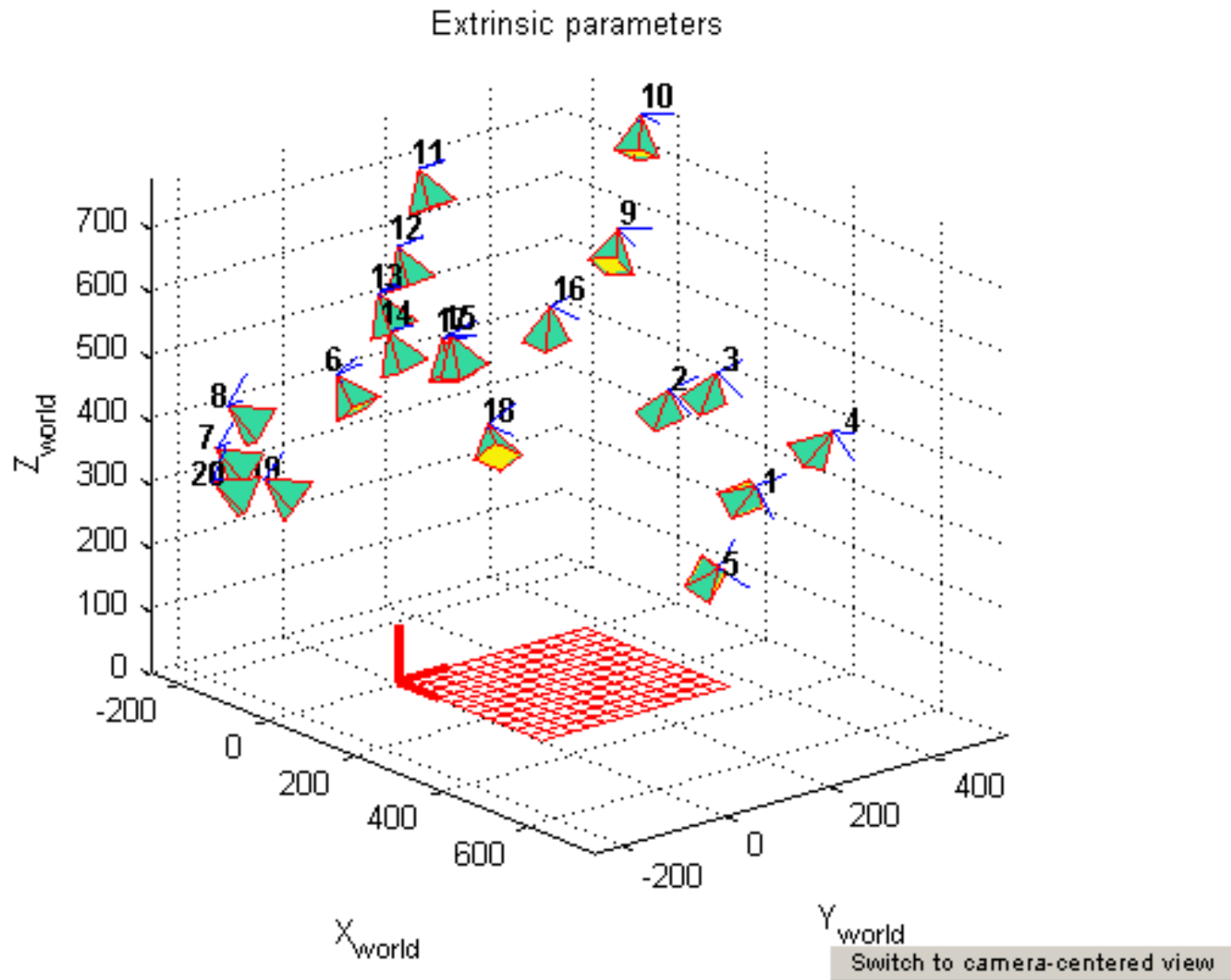


Calibration Procedure

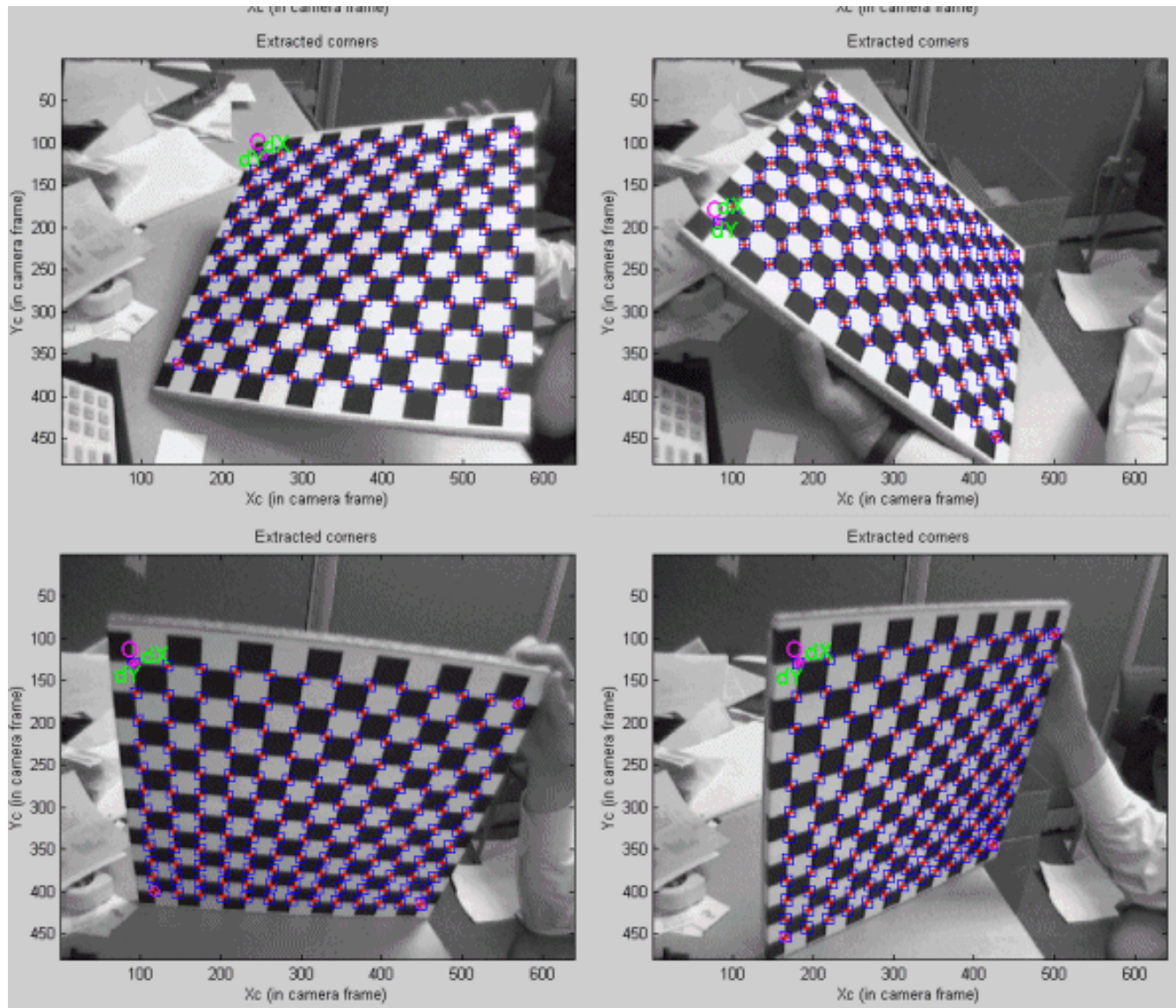


Switch to world-centered view

Calibration Procedure



Calibration Procedure



Next lecture

- Single view reconstruction
- Homework will be online tomorrow night
(syllabus page)