A Calibration Technique for Polarimetric Coherent-on-Receive Radar Systems

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***Abstract*— A new technique for calibrating a coherent-on-receive polarimetric radar system is proposed. A coherent-on-receive polarimetric radar is capable of measuring the Mueller matrix of point or distributed targets directly by transmitting at least four independent polarizations and measuring the vertical and horizontal components of the backscatter signal simultaneously. The technique requires the use of two calibration targets, a target with known scattering matrix (such as a metallic sphere or a trihedral corner reflector) and any depolarizing target (for which knowledge of its scattering matrix is not required) to determine the system distortion parameters. The system distortion parameters, which include the channel imbalances, the cross-talk factors of both the transmit and the receive antennas, and the phase shifts and amplitude variations of the transmitter polarizers, are determined by measuring the calibration targets for four different transmit polarizations. The validity of the new calibration technique is examined by measuring the scattering matrices of spheres and cylinders as test targets using a coherent-on-receive radar operating at 34.5 GHz. Excellent agreement between the theoretical and the measured scattering matrices for the test targets are obtained.**

I. Introduction

Interest in retrieving information about the biophysical parameters of natural targets from their backscattering properties has prompted intensive investigation in the use of polarimetric radar techniques. Advances in polarimetric radar techniques and the development of polarimetric imaging SAR's in the past decade has opened many opportunities in the area of microwave and millimeter wave remote sensing. Measurement accuracy and precision are critical elements of any meaningful analysis or interpretation of the measured data. For radar systems, it is customary to use external calibration techniques to determine the overall system transfer function. By measuring the backscatter from a set of calibration targets with known scattering matrices, the radar distortion parameters, which include channel imbalances, antenna gain, and antenna distortion parameters, can be determined. Then, by applying an inverse algorithm to the data measured for a target of interest, the errors introduced by these parameters can be removed.

The applicability of a calibration technique depends on the type and design of the polarimetric radar under consideration. In general, polarimetric radars can be categorized into three major groups: 1) coherent radars, 2) incoherent radars, and 3) coherent-on-receive radars. With coherent radars, which are capable of measuring both the magnitude and the phase of the scattered signal, the scattering matrix of a target can be determined directly. This is accomplished by measuring the scattered waves from the target using two receive channels with orthogonal polarizations for two successive transmissions at each of the two orthogonal polarizations. Several techniques are available for calibrating coherent polarimetric radar systems, including those proposed by Barnes [1], Sarabandi *et al.* [4], and Whitt *et al.* [8] and [9]. Although capable of measuring only the magnitude of the scattered wave, incoherent radars can be used to determine the Mueller matrix of a target by measuring the power received in the co-polarized channels for each of six different, independent, transmit polarization states [2]. At microwave frequencies, coherent radar systems are often used because the coherence requirements of the measurement technique can be met by available microwave technology. The incoherent technique, on the other hand, is used mostly at optical wavelengths. Neither of these two types of radar systems is of interest to the present study.

This paper is concerned with the third type of polarimetric radars, namely the coherent-on-receive configuration, which is intermediate between the other two techniques in terms of the coherence requirements imposed by the measurement technique. In this configuration, which has been used successfully at millimeter wavelengths ([3], [7]), the receiver has two orthogonal polarization channels and is capable of measuring simultaneously the magnitudes and phases of the two signals. With this type of radar, the Mueller matrix of a target can be obtained from four receiver measurements corresponding to four different transmit polarization states. System calibration is performed in two steps. First, the receiver is calibrated using a wire grid placed in front of the receive antenna at three different orientations. Then the polarization states of the four desired transmitted waves are determined by measuring the wave backscattered from a corner reflector with the calibrated receiver. In this technique calibration errors caused by the wire grid itself are not considered. The sources of error introduced by the wire grid include: 1) the polarization purity (axial ratio) of the wire grid, which is on the order of that of the receiving antenna (20–30 dB), 2) the proximity of the wire grid to the receiving antenna, which affects the antenna radiation characteristics, and 3) errors associated with the grid orientation angles.

This paper proposes an alternate technique with which calibration can be performed in two steps and requires measuring the backscatter response of a metallic sphere (or trihedral

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reflector) in addition to any depolarizing target for four and two transmitted wave polarizations, respectively. In this algorithm the knowledge of the scattering matrix of the depolarizing target is not required. By applying this technique, it is possible to determine the polarization states of the transmitted waves, the antennas cross-talk factors and receiver channel imbalances, while avoiding the use of the wire grid.

II. Coherent-on-Receive Polarimetric Radars

The main advantage of the coherent-on-receive radars is in the measurement of random fluctuating targets or when the radar platform is not stable. In this section we briefly introduce the basic concepts of this system to guide the reader in understanding the calibration procedure.

By defining a set of orthogonal directions $(\hat{v},\ \hat{h})$ in a plane perpendicular to the direction of propagation, the components of the scattered field $\mathbf{E}^s$ from a given target can be related to the components of the incident wave $\mathbf{E}^i$ through the scattering matrix of the target, i.e.,

$$\mathbf{E}^s = \frac{e^{-ik_0 r}}{r}\begin{bmatrix} S_{vv} & S_{vh} \\ S_{hv} & S_{hh}\end{bmatrix}\mathbf{E}^i \tag{1}$$

where k_0 is the propagation constant and r is the range from the target to the receive antenna. In general, the polarization state of the transmitted wave can be any arbitrary elliptical polarization. An elliptically polarized wave can be characterized by two angles known as the rotation angle (ψ) and ellipticity angle (χ) [6]. The modified Stokes vector $\mathbf{F}_m(\psi,\ \chi)$ provides an alternate but equivalent representation of wave polarization

$$\mathbf{F}_m(\psi,\chi) = \begin{bmatrix} |E_v|^2 \\ |E_h|^2 \\ 2\mathrm{Re}[E_v E_h^*] \\ 2\mathrm{Im}[E_v E_h^*]\end{bmatrix} = \begin{bmatrix} \frac{1}{2}(1+\cos 2\psi\cos 2\chi) \\ \frac{1}{2}(1-\cos 2\psi\cos 2\chi) \\ \sin 2\psi \cos 2\chi \\ \sin 2\chi \end{bmatrix}(|E_v|^2 + |E_h|^2). \tag{2}$$

Upon using (1) the scattered (received) modified Stokes vector $\mathbf{F}_m^r$ can be related to the incident (transmitted) Stokes vector via the modified Mueller matrix $\bar{\bar{\mathcal{L}}}_m$ by

$$\mathbf{F}_m^r = \frac{1}{r^2}\,\bar{\bar{\mathcal{L}}}_m \mathbf{F}_m^t \tag{3}$$

where $\bar{\bar{\mathcal{L}}}_m$ is a 4×4 matrix whose entries in terms of the scattering matrix elements are given by

$$\bar{\bar{\mathcal{L}}}_m = \begin{bmatrix} |S_{vv}|^2 & |S_{vh}|^2 & \mathrm{Re}(S_{vh}^* S_{vv}) & -\mathrm{Im}(S_{vh}^* S_{vv}) \\ |S_{hv}|^2 & |S_{hh}|^2 & \mathrm{Re}(S_{hh}^* S_{hv}) & -\mathrm{Im}(S_{hv} S_{hh}^*) \\ 2\mathrm{Re}(S_{vv}S_{hv}^*) & 2\mathrm{Re}(S_{vh}S_{hh}^*) & \mathrm{Re}(S_{vv}S_{hh}^* + S_{vh}S_{hv}^*) & -\mathrm{Im}(S_{vv}S_{hh}^* - S_{vh}S_{hv}^*) \\ 2\mathrm{Im}(S_{vv}S_{hv}^*) & 2\mathrm{Im}(S_{vh}S_{hh}^*) & \mathrm{Im}(S_{vv}S_{hh}^* + S_{vh}S_{hv}^*) & \mathrm{Re}(S_{vv}S_{hh}^* - S_{vh}S_{hv}^*) \end{bmatrix}. \tag{4}$$

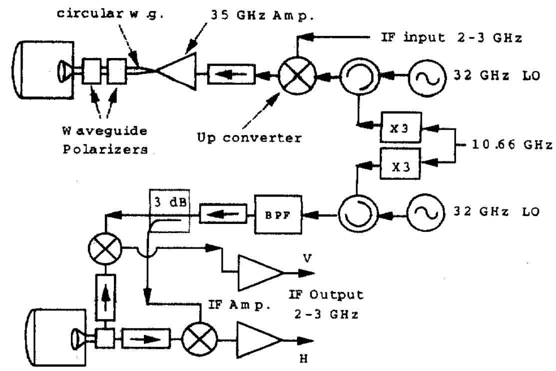

Fig. 1. Block diagram of the coherent-on-receive radar system operating at 34.5 GHz.

When dealing with natural targets, such as soil surfaces and vegetation canopies, the quantity of interest is $\langle\bar{\bar{\mathcal{L}}}_m\rangle$, the ensemble average of $\bar{\bar{\mathcal{L}}}_m$. Given $\langle\bar{\bar{\mathcal{L}}}_m\rangle$, the technique of polarization synthesis can be used to compute the polarization response of the target under consideration [6, Chapter 2]. With a coherent polarimetric radar, the process starts by measuring the scattering matrix for many statistically independent samples of the target. Each scattering matrix is converted to its corresponding modified Mueller matrix $\bar{\bar{\mathcal{L}}}_m$, and then all the $\bar{\bar{\mathcal{L}}}_m$ matrices are averaged together. With incoherent and coherent-on-receive polarimetric radars, $\langle\bar{\bar{\mathcal{L}}}_m\rangle$ is measured directly. To examine how the coherent-on-receive radar functions, consider the two-antenna system shown in Fig. 1. The transmitter part of the system consists of an upconverter followed by an RF amplifier with a rectangular waveguide as its output port. The rectangular waveguide is connected to a circular waveguide through a transition, followed by two waveguide polarizers which are connected to the antenna structure. The waveguide polarizers are independently rotatable in a plane perpendicular to the direction of propagation. The dielectric card inside a waveguide polarizer enforces two different wave velocities for waves with polarization vectors parallel and perpendicular to the card's surface. The position of the dielectric card with respect to the polarization of the incoming wave determines the polarization of the outgoing wave from the waveguide polarizer. To minimize reflection by the card, the dielectric constant of the card must be chosen to be relatively small. The dielectric cards are designed such that the phase difference between two outgoing waves corresponding to two incoming waves whose electric fields are parallel and perpendicular to the card is 90 degrees. This feature allows the generation of any polarization configuration of interest including vertical (V), 45-degree linear (45), left-hand circular (LHC), and right-hand circular (RHC), which together are used to obtain the elements of the Mueller matrix.

The receiver part of the radar includes a dual polarized antenna capable of receiving the vertical and horizontal polarization components of the scattered wave simultaneously. After down-converting the frequency of the received signals, the two IF signals are measured in both magnitude and phase. The Stokes vector corresponding to the transmitted polarization is computed from the coherent measurement of the scattered field components as given by (2). For coherent systems, phase coherence must be maintained during the measurement of the scattering matrix elements. Therefore, if the relative position of the radar platform with respect to the target is not fixed to within a very small fraction of a wavelength, the phase information will be lost. This is not the case for coherent-on-receive systems, however, since each column of the Mueller matrix is completely determined from a single pulse.

To measure the Mueller matrix with 16 unknowns, we are required to perform at least four measurements. The entries of the Mueller matrix can easily be obtained by transmitting four different polarizations, namely, vertical, 45-degree linear, right-hand circular, and left-hand circular, whose modified Stokes vectors are given by

$$\mathbf{F}_v^t = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{F}_{45}^t = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \\ 1 \\ 0 \end{bmatrix},$$

$$\mathbf{F}_{\mathrm{LHC}} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{F}_{\mathrm{RHC}} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \\ 0 \\ -1 \end{bmatrix}. \tag{5}$$

The received Stokes vectors can also be computed using the measured E_v^r and E_h^r in (2). By denoting the ith column of the modified Mueller matrix by $\mathcal{L}_m^i$ it is a straightforward matter to show that

$$\begin{aligned}
\mathcal{L}_m^1 &= \frac{1}{r^2}\mathbf{F}_V^r \\
\mathcal{L}_m^2 &= \frac{1}{r^2}[\mathbf{F}_{\mathrm{LHC}}^r + \mathbf{F}_{\mathrm{RHC}}^r - \mathbf{F}_V^r] \\
\mathcal{L}_m^3 &= \frac{1}{r^2}\left[\mathbf{F}_{45}^r - \frac{1}{2}(\mathbf{F}_{\mathrm{LHC}}^r + \mathbf{F}_{\mathrm{RHC}}^r)\right] \\
\mathcal{L}_m^4 &= \frac{1}{r^2}\left[\frac{1}{2}(\mathbf{F}_{\mathrm{LHC}}^r - \mathbf{F}_{\mathrm{RHC}}^r)\right]
\end{aligned} \tag{6}$$

where $\mathbf{F}_p^r$ represents the received Stokes vector corresponding to the transmit polarization p.

In case of distributed targets, measurements of $\mathbf{F}_p^r$ are repeated many times to estimate the expected value $\langle \mathbf{F}_p^r \rangle$. Then, $\langle \bar{\bar{\mathcal{L}}}_m \rangle$ can be determined from $\langle \mathbf{F}_p^r \rangle$ following the procedure outlined by (6), from which the radar cross section can be computed for any desired combination of transmit and receive antenna polarizations using the polarization synthesis technique [6].

This procedure of computing $\langle \bar{\bar{\mathcal{L}}}_m \rangle$ works well for an ideal radar system with the transmitted Stokes vectors as given in (5). In practice, the measurements required to compute $\langle \bar{\bar{\mathcal{L}}}_m \rangle$ are contaminated by two types of errors: systematic errors due to imperfections in the radar components and nonsystematic errors due to noise and finite sampling of the random process. The systematic errors can be removed using the calibration procedure described in Section IV, while the nonsystematic errors can be reduced by estimating $\langle \bar{\bar{\mathcal{L}}}_m \rangle$ using the received Stokes vectors corresponding to N (≥ 4) different transmit polarizations. One possible computation procedure to estimate $\langle \bar{\bar{\mathcal{L}}}_m \rangle$, as suggested in [3], is to use the least mean squared procedure for individual rows of the Mueller matrix.

Let $\bar{\bar{\mathbf{V}}}$ and $\bar{\bar{\mathbf{W}}}$ be $4 \times N$ matrices whose columns represent the received and transmitted Stokes vectors, respectively. It is easy to show from (3) that $\bar{\bar{\mathbf{V}}}$ and $\bar{\bar{\mathbf{W}}}$ are related to $\bar{\bar{\mathcal{L}}}_m$ through

$$\langle \bar{\bar{\mathbf{V}}} \rangle = \langle \bar{\bar{\mathcal{L}}}_m \rangle \, \bar{\bar{\mathbf{W}}}. \tag{7}$$

Here, the coefficient $(1/r^2)$ has been suppressed for simplicity. The error in the ith row is defined by

$$e_i = \sum_{j=1}^{N} \left(v_{ij} - \sum_{k=1}^{4} \mathcal{L}_{ik} w_{kj} \right)^2$$

and elements of $\mathcal{L}_{ik}$ are chosen such that e_i is minimized. This procedure ends up with the following expression for the measured Mueller matrix

$$\langle \bar{\bar{\mathcal{L}}}_m \rangle = [(\bar{\bar{\mathbf{W}}}\bar{\bar{\mathbf{W}}}^T)^{-1}\bar{\bar{\mathbf{W}}}\langle \bar{\bar{\mathbf{V}}} \rangle^T]^T.$$

A better estimation of the Mueller matrix from the measured data is perhaps the global minimization instead of minimization from individual rows. Moreover in backscattering there are only nine independent parameters in the Mueller matrix. The Mueller matrix in terms of the independent parameters can be represented by

$$\langle \bar{\bar{\mathcal{L}}}_m \rangle = \left\langle \begin{bmatrix} x_1 & x_3 & x_8 & -x_9 \\ x_3 & x_2 & x_6 & -x_7 \\ 2x_8 & 2x_6 & (x_4 + x_3) & -x_5 \\ 2x_9 & 2x_7 & x_5 & (x_4 - x_3) \end{bmatrix} \right\rangle$$

where $x_1 = |S_{vv}|^2$, $x_2 = |S_{hh}|^2$, $x_3 = |S_{hv}|^2$, $x_4 = \mathrm{Re}[S_{vv}S_{hh}^*]$, $x_5 = \mathrm{Im}[S_{vv}S_{hh}^*]$, $x_6 = \mathrm{Re}[S_{hv}S_{hh}^*]$, $x_7 = \mathrm{Im}[S_{hv}S_{hh}^*]$, $x_8 = \mathrm{Re}[S_{vv}S_{hv}^*]$, and $x_9 = \mathrm{Im}[S_{vv}S_{hv}^*]$. Using global minimization, elements of the Mueller matrix $(\mathcal{L}_{ik})$ are obtained such that the global error

$$E = \sum_{i=1}^{4}\sum_{j=1}^{N} \left(v_{ij} - \sum_{k=1}^{4} \mathcal{L}_{ik} w_{kj} \right)^2$$

is minimized. Basically, a linear system of equations for the nine unknown parameters is obtained by setting the partial derivatives of E (with respect to x_i) equal to zero.

III. System Distortion Model

The technique described in the preceding section for measuring the Mueller matrix is for a distortion-free radar system. It minimizes the error due to random variations and finite sample size, but it does not correct for system biases. In practice, however, the radar components are not perfect. For example, it is not possible to construct antennas that are totally free of polarization contamination (coupling between the orthogonal polarization ports of the antenna). Also, the polarization state

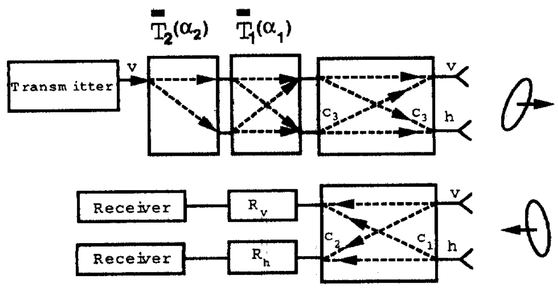

Fig. 2. Signal flow diagram of a coherent-on-receive radar.

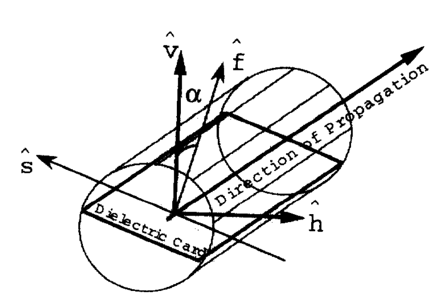

Fig. 3. Orientation of the principal axes of the dielectric card with respect to $\hat{v}$ and $\hat{h}$.

of the transmitted wave depends primarily on the phase-shift characteristics of the waveguide polarizers and the transmit antenna polarization contamination. Another factor that affects the measurement accuracy is the receive channel imbalances which include the variation in magnitude and phase of the system transfer function for different ports of the receiver. A simplified block diagram of such a system is depicted in Fig. 2. These imperfections in the radar system components can lead to serious errors in the measured Mueller matrix. The role of calibration is then to correct for these systematic errors in the measured target response before constructing the measured Mueller matrix.

In this section, the imperfections of the radar components will be modeled and a mathematical system distortion model will be constructed. This system distortion model will then form the basis of the new calibration technique discussed in the next section.

First, let us examine the effect of the phase shift introduced by the dielectric cards on the polarization state of the transmitted wave. As mentioned earlier the thickness and dielectric constant of the dielectric cards are chosen such that the reflected field is minimal and can be ignored. Suppose the transmission coefficient of the card for the waves whose electric fields are parallel (slow wave) and perpendicular (fast wave) to the dielectric card are, respectively, denoted by the complex quantities Υ_s, and Υ_f. Further assume that the card is oriented such that it makes an angle α with respect to a specified horizontal direction as shown in Fig. 3. The unit vectors parallel and perpendicular to the card are denoted by $\hat{s}$ and $\hat{f}$, which are given by

$$\begin{cases} \hat{s} = \sin\alpha\,\hat{v} - \cos\alpha\,\hat{h} \\ \hat{f} = \cos\alpha\,\hat{v} + \sin\alpha\,\hat{h} \end{cases}.$$

If the wave incident on the card, is denoted by $\mathbf{E}^i = E_v^i\hat{v} + E_h^i\hat{h}$, then by decomposing each component into components along $\hat{f}$ and $\hat{s}$ and multiplying the resultant components, by the corresponding transmission coefficients Υ_f and Υ_s, the outgoing field through the polarizer can be obtained from

$$\mathbf{E}^o = \bar{\bar{\mathbf{T}}}\mathbf{E}^i$$

where $\bar{\bar{\mathbf{T}}}$ is the polarization transformation matrix given by

$$\bar{\bar{\mathbf{T}}} = \begin{bmatrix} \cos^2\alpha\Upsilon_f + \sin^2\alpha\Upsilon_s & \sin\alpha\cos\alpha(\Upsilon_f - \Upsilon_s) \\ \sin\alpha\cos\alpha(\Upsilon_f - \Upsilon_s) & \cos^2\alpha\Upsilon_s + \sin^2\alpha\Upsilon_f \end{bmatrix}.$$

Now by cascading two polarizers and ignoring reflections from the cards, the transmitted wave $\mathbf{E}^t$ at the output of the second polarizer is related to the wave at the input of the first polarizer $\mathbf{E}^{\text{in}}$ by

$$\mathbf{E}^t = \bar{\bar{\mathbf{T}}}_1\bar{\bar{\mathbf{T}}}_2\mathbf{E}^{\text{in}}.$$

Here $\bar{\bar{\mathbf{T}}}_1$ and $\bar{\bar{\mathbf{T}}}_2$ are the transformation matrices of the first and second polarizers corresponding to the rotation angles α_1 and α_2, respectively. It is assumed, without loss of generality, that the electric field in the rectangular waveguide preceding the polarizers coincides with the specified vertical direction $\hat{v}\,(\mathbf{E}^{\text{in}} = T_t(1,\ 0))$. Introducing the phase-shift factor $\tau_i = \frac{\Upsilon_{si}}{\Upsilon_{fi}}$ $(i = 1, 2)$ the transmitted wave through the polarizers is given in terms of the phase-shift factors and orientation angles by

$$\mathbf{E}^t = T\begin{bmatrix} \{\cos(\alpha_1 - \alpha_2)(\cos\alpha_1\cos\alpha_2 + \tau_1\tau_2\sin\alpha_1\sin\alpha_2) \\ +\sin(\alpha_1 - \alpha_2)(\sin\alpha_1\cos\alpha_2\tau_1 - \cos\alpha_1\sin\alpha_2\tau_2)\} \\ \\ \{\cos(\alpha_1 - \alpha_2)(\sin\alpha_1\cos\alpha_2 - \cos\alpha_1\sin\alpha_2\tau_1\tau_2) \\ -\sin(\alpha_1 - \alpha_2)(\cos\alpha_1\cos\alpha_2\tau_1 + \sin\alpha_1\sin\alpha_2\tau_2)\} \end{bmatrix} \tag{8}$$

where $T = T_t\Upsilon_{f1}\Upsilon_{f2}$ includes the transmitter transfer function (T_t). The nominal value for the phase-shift factors (τ_i) is $e^{-i\frac{\pi}{2}}$ and for this value Table I gives the rotation angles for the desired polarizations. In general τ_i is not exactly $e^{-i\frac{\pi}{2}}$ and is a function of frequency. Using the above rotation angles, the actual polarizations of the waves transmitted through the polarizers are

$$\begin{aligned} \tilde{\mathbf{E}}^{tV} &= \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \tilde{\mathbf{E}}^{tH} = \begin{bmatrix} \frac{1+\tau_1\tau_2}{2} \\ \frac{1-\tau_1\tau_2}{2} \end{bmatrix}, \\ \tilde{\mathbf{E}}^{t45} &= \begin{bmatrix} \frac{1+\tau_2}{2} \\ \frac{\tau_1-\tau_1\tau_2}{2} \end{bmatrix}, \quad \tilde{\mathbf{E}}^{t135} = \begin{bmatrix} \frac{1+\tau_2}{2} \\ \frac{-\tau_1+\tau_1\tau_2}{2} \end{bmatrix}, \\ \tilde{\mathbf{E}}^{t\text{LHC}} &= \begin{bmatrix} \frac{1+\tau_1}{2} \\ \frac{1-\tau_1}{2} \end{bmatrix}, \quad \tilde{\mathbf{E}}^{t\text{RHC}} = \begin{bmatrix} \frac{1+\tau_1}{2} \\ \frac{-1+\tau_1}{2} \end{bmatrix} \end{aligned} \tag{9}$$

where $\tilde{\mathbf{E}}^{tp} = \mathbf{E}^{tp}/T$. Deviations of τ_1 and τ_2 from their nominal values lead to errors in the presumed polarization states.

TABLE I
WAVE PLATES ROTATION ANGLES FOR THE DESIRED POLARIZATION STATES OF THE TRANSMITTED WAVE

Rotation Angle	v	h	45°	135°	LHC	RHC
α_1	0	45	0	0	45	-45
α_2	0	45	45	-45	0	0

The physical structure of the antenna may modify the polarization of the bounded wave as the wave transforms into an unbounded wave and vice versa. The transmitted unbounded wave in terms of the bounded wave at the input is given by

$$\tilde{\mathbf{E}}_o^t = T\begin{bmatrix} 1 & c_3 \\ c_3 & 1 \end{bmatrix} \tilde{\mathbf{E}}^t \tag{10}$$

where c_3 ($|c_3| < 1$) is the transmit antenna cross-talk factor. The off-diagonal terms in (10) are assumed to be identical because of the circular symmetry of the transmit antenna. It is worth noting that the polarizers and the antenna structure are passive devices, thus the phase-shift factors τ_1 and τ_2 as well as the antenna cross-talk factor c_3 are time invariant.

Now we can turn to study the distortions caused by the receiver. In a recent study reported by Sarabandi and Ulaby [5], it was shown that for a dual polarized receiver the measured signal $\mathbf{E}^r$ is related to the wave incident upon the receive antenna by

$$\mathbf{E}^r = \begin{bmatrix} R_v & 0 \\ 0 & R_h \end{bmatrix}\begin{bmatrix} 1 & c_1 \\ c_2 & 1 \end{bmatrix}\tilde{\mathbf{E}}^r \tag{11}$$

where c_1 and c_2 are the receiver cross-talk factors and R_v and R_h are the vertical and horizontal system transfer functions of the receiver ports. It was also shown that c_1 and c_2 are only a feature of the passive devices in the antenna system and hence are not affected by instability of active components in the radar system. On the other hand, the unbounded transmitted and received waves at the antennas are related by

$$\tilde{\mathbf{E}}^r = \frac{e^{-2ik_0 r}}{r^2}\bar{\bar{\mathbf{S}}}\,\tilde{\mathbf{E}}_o^t \tag{12}$$

where r is the range between the radar antenna and the target. By inserting (10) and (12) into (11), we can relate the measured signal $\mathbf{E}^r$ to the bounded wave at the output of the polarizers $\tilde{\mathbf{E}}^t$ by

$$\mathbf{E}^r = \frac{e^{-2ik_0 r}}{r^2}\begin{bmatrix} R_1 & 0 \\ 0 & R_2 \end{bmatrix}\begin{bmatrix} 1 & c_1 \\ c_2 & 1 \end{bmatrix} \times \begin{bmatrix} S_{vv} & S_{vh} \\ S_{hv} & S_{hh} \end{bmatrix}\begin{bmatrix} 1 & c_3 \\ c_3 & 1 \end{bmatrix}\tilde{\mathbf{E}}^t \tag{13}$$

where we have included the transmitter transfer function into the receiver transfer functions, i.e., $R_1 = TR_v$ and $R_2 = TR_h$. Furthermore, let

$$\bar{\bar{\mathbf{R}}}_d = \begin{bmatrix} R_1 & 0 \\ 0 & R_2 \end{bmatrix}\begin{bmatrix} 1 & c_1 \\ c_2 & 1 \end{bmatrix}, \quad \bar{\bar{\mathbf{T}}}_d = \begin{bmatrix} 1 & c_3 \\ c_3 & 1 \end{bmatrix}. \tag{14}$$

The system distortion model for the coherent-on-receive radar shown in Figs. 1 and 2 can be expressed in matrix format by

$$\mathbf{E}^r = \frac{e^{-2ik_0 r}}{r^2}\bar{\bar{\mathbf{R}}}_d\,\bar{\bar{\mathbf{S}}}\,\bar{\bar{\mathbf{T}}}_d\,\tilde{\mathbf{E}}^t. \tag{15}$$

IV. CALIBRATION PROCEDURE

So far the system distortions have been modeled mathematically. The next step is to determine the distortion parameters and then, by applying a correction algorithm, calculate the actual Mueller matrix. The standard approach used to find the distortion parameters is to measure a target with a known scattering matrix. A metallic sphere is an excellent candidate for this purpose because its scattering matrix is orientation-independent and its radar cross section is known theoretically. As will become apparent later on, however, the measurements of only a metallic sphere with any combination of transmitted polarizations is not sufficient to determine the seven unknown distortion parameters of the radar system. The measurement of an additional independent depolarizing target is needed to determine all distortion parameters. Fortunately, the scattering matrix of the additional target need not be known. It should be noted that once the distortion parameters are determined carefully using the two calibration targets, a simpler procedure using only a sphere can be used to find the time variant parameters R_1 and R_2 in subsequent calibration of the radar.

The fields, detected at the V-channel of the receiver, that are scattered from a sphere with a radar cross section $\sigma = 4\pi|S^o|^2$ due to vertical, 45-linear, right-hand circular, and left-hand circular polarizations can be expressed in terms of the distortion parameters as follows

$$\begin{aligned}
E_v^{oV} &= R_1 S^o(1 + c_1 c_3) \\
E_v^{o\mathrm{LHC}} &= \frac{R_1 S^o}{2}[(1 + c_1 c_3)(1 + \tau_1) + (c_1 + c_3)(1 - \tau_1)] \\
E_v^{o\mathrm{RHC}} &= \frac{R_1 S^o}{2}[(1 + c_1 c_3)(1 + \tau_1) - (c_1 + c_3)(1 - \tau_1)] \\
E_v^{o45^\circ} &= \frac{R_1 S^o}{2}[(1 + c_1 c_3)(1 + \tau_2) + \tau_1(1 - \tau_2)(c_1 + c_3)].
\end{aligned} \tag{16}$$

Here, the quantities on the left-hand side are the components of the received signal modified by $r_o^2 e^{2ik_0 r_o}$ and the first superscript denotes the sphere and the second one denotes the polarization of the transmitted wave. Solving (16) simultaneously provides the following expressions for the distortion parameters τ_1 and τ_2

$$\begin{aligned}
\tau_1 &= A_v + B_v - 1 \\
\tau_2 &= 2\frac{(A_v - 1) + (2 - A_v - B_v)D_v + \frac{B_v^2 - A_v^2}{2}}{(1 - A_v^2) + (1 - B_v)^2}
\end{aligned}$$

where

$$A_v = \frac{E_v^{o\mathrm{LHC}}}{E_v^{oV}}, \qquad B_v = \frac{E_v^{o\mathrm{RHC}}}{E_v^{oV}}, \qquad D_v = \frac{E_v^{o45}}{E_v^{oV}}. \tag{17}$$

The field $\tilde{\mathbf{E}}^t$ can now be determined from (9) for any given rotational angles α_1 and α_2 of the dielectric cards. The system distortion model given by (15) can be cast into a matrix equation by juxtaposing the measured responses due to two transmitted polarizations; for example for the transmit V and 45-degree linear polarizations

$$\bar{\bar{\mathbf{E}}}^r = \frac{e^{-2ik_0 r}}{r^2} \bar{\bar{\mathbf{R}}}_d \, \bar{\bar{\mathbf{S}}} \, \bar{\bar{\mathbf{T}}}_d \, \bar{\bar{\mathbf{E}}}^t \tag{18}$$

where $\bar{\bar{\mathbf{E}}}^t = [\tilde{\mathbf{E}}^{tV}, \tilde{\mathbf{E}}^{t45}]$ and $\bar{\bar{\mathbf{E}}}^r = [\mathbf{E}^{rV}, \mathbf{E}^{r45}]$. Noting that the scattering matrix of the sphere is diagonal, the following equation can be obtained from (18)

$$\bar{\bar{\mathbf{A}}} = \bar{\bar{\mathbf{E}}}^{r_o} (\bar{\bar{\mathbf{E}}}^t)^{-1} = \frac{e^{-2ik_0 r_o}}{r_o^2} S^o \, \bar{\bar{\mathbf{R}}}_d \, \bar{\bar{\mathbf{T}}}_d \tag{19}$$

where $\bar{\bar{\mathbf{E}}}^{r_o}$ is the matrix of the measured response of the sphere for two transmitted polarizations. Equation (19) shows that the sphere measurements can provide at most four independent equations. Since there are five unknowns in $\bar{\bar{\mathbf{R}}}_d$ and $\bar{\bar{\mathbf{T}}}_d$ matrices (R_1, R_2, c_1, c_2, and c_3), an additional independent equation is needed. This can be accomplished by measuring any depolarizing target for any two transmit polarizations and then enforcing the reciprocity theorem.

If $\bar{\bar{\mathbf{E}}}^{r_d}$ represents the matrix of the measured response of the depolarizing target for two given transmit polarizations, using (18) and defining $\bar{\bar{\mathbf{B}}} = \bar{\bar{\mathbf{S}}}^{r_d} (\bar{\bar{\mathbf{E}}}^t)^{-1}$, the scattering matrix of the depolarizing target can be obtained from

$$\bar{\bar{\mathbf{S}}}^d = r_d^2 \, e^{2ik_0 r_d} \, \bar{\bar{\mathbf{R}}}_d^{-1} \, \bar{\bar{\mathbf{B}}} \, \bar{\bar{\mathbf{T}}}_d^{-1}. \tag{20}$$

Obtaining $\bar{\bar{\mathbf{R}}}_d^{-1}$ from (19) and substituting in (20), the scattering matrix of the depolarizing target can be expressed by

$$\bar{\bar{\mathbf{S}}}^d = \frac{r_d^2}{r_o^2} e^{2ik_0(r_d - r_o)} \, S^o \, \bar{\bar{\mathbf{T}}}_d \, \bar{\bar{\mathbf{A}}}^{-1} \, \bar{\bar{\mathbf{B}}} \, \bar{\bar{\mathbf{T}}}_d^{-1} \tag{21}$$

which is only a function of c_3. Enforcing the reciprocity condition ($S_{vh}^d = S_{hv}^d$), the following quadratic equation for c_3 can be obtained

$$c_3^2 + 2\frac{\Omega_1}{\Omega_0} c_3 + 1 = 0 \tag{22}$$

where

$$\Omega_0 = a_{21}b_{11} - a_{11}b_{21} + a_{22}b_{12} - a_{12}b_{22}$$
$$\Omega_1 = a_{12}b_{21} - a_{22}b_{11} - a_{21}b_{12} + a_{11}b_{22}.$$

Parameters a_{ij} and b_{ij} $(i, j = 1, 2)$ are the elements of $\bar{\bar{\mathbf{A}}}$ and $\bar{\bar{\mathbf{B}}}$ matrices, respectively. The acceptable root of (22) is the one that satisfies the condition $|c_3| < 1$. The only restriction on choosing the depolarizing target is that its scattering matrix must have nonzero eigenvalues; otherwise c_3 cannot be computed from (22). Once c_3 is found, the other system distortion parameters can be obtained systematically from (19) and are given by

$$\begin{aligned}
c_1 &= \frac{c_3(a_{11}/a_{12}) - 1}{c_3 - (a_{11}/a_{12})} \\
c_2 &= \frac{c_3(a_{22}/a_{21}) - 1}{c_3 - (a_{22}/a_{21})} \\
R_1 &= R_1' \, r_o^2 e^{2ik_0 r_o}, \qquad R_1' = \frac{1}{S^o} \frac{a_{12}}{c_1 + c_3} \\
R_2 &= R_2' r_o^2 e^{2ik_0 r_o}, \qquad R_2' = \frac{1}{S^o} \frac{a_{21}}{c_2 + c_3}.
\end{aligned} \tag{23}$$

A. Special Cases

If the transmit antenna cross-talk factor is very small ($c_3 \simeq 0$), the depolarizing target is not required and the distortions can be determined from the sphere alone. The phase-shift factors τ_1 and τ_2 can be determined as before while the other parameters can be determined as follows

$$\begin{aligned}
c_1 &= \frac{A_v - B_v}{2 - A_v - B_v} \\
c_2 &= \frac{2 - A_v - B_v}{A_h - B_h} \\
R_1 &= \frac{E_v^{oV}}{S^o}, \qquad R_2 = \frac{E_h^{oV}}{S^o} \cdot \frac{A_h - B_h}{2 - A_v - B_v}
\end{aligned}$$

where

$$A_h = \frac{E_h^{o\mathrm{LHC}}}{E_h^{oV}}, \qquad B_h = \frac{E_h^{o\mathrm{RHC}}}{E_h^{oV}}.$$

If in addition, the receive antenna cross-talk factors are very small, then measurements of A_h and B_h are very noisy and unreliable. In such cases where $c_1 = c_2 \simeq 0$, (16) reduces to

$$\begin{cases}
E_v^{oV} &= R_1 S^o \\
E_v^{o\mathrm{LHC}} &= R_1 S^o \frac{1+\tau_1}{2} \\
E_h^{o\mathrm{LHC}} &= R_2 S^o \frac{1-\tau_1}{2} \\
E_v^{o45} &= R_1 S^o \frac{1+\tau_2}{2}.
\end{cases}$$

These equations can be solved to obtain

$$\begin{aligned}
\tau_1 &= 2\frac{E_v^{o\mathrm{LHC}}}{E_v^{oV}} - 1 \\
\tau_2 &= 2\frac{E_v^{o45}}{E_v^{oV}} - 1 \\
R_1 &= \frac{E_v^{oV}}{S^o}, \qquad R_2 = \frac{E_h^{o\mathrm{LHC}}}{(E_v^{oV} - E_v^{o\mathrm{LHC}})} \cdot \frac{E_v^{oV}}{S^o}.
\end{aligned}$$

B. Correction Schemes

Now we are in a position to correct for the distortions in the measurements of an unknown target. For a coherent-on-receive radar, two correction schemes are considered: 1) coherent and 2) incoherent. The coherent correction scheme can be used whenever the relative distance between the target and the radar remains constant during the measurements as it is usually the case for indoor measurements. In this case, the scattering matrix is determined from the scattered fields when the target is illuminated by two independent polarizations, for example V and 45-degree linear. The incoherent correction scheme must be used whenever the relative distance between the target and the radar changes during the measurements (unstable platform or moving target) as it is usually the case for outdoor measurements. In this case, the received stokes vectors for at least four different transmitted polarizations are used to construct the modified Mueller matrix directly, as was discussed in Section II.

In the coherent correction scheme the scattering matrix of the unknown target can be determined from (18) directly

$$\bar{\bar{\mathbf{S}}}^u = r_u^2 \, e^{2ik_0 r_u} \, \bar{\bar{\mathbf{R}}}_d^{-1} \, \bar{\bar{\mathbf{U}}} \, \bar{\bar{\mathbf{T}}}_d^{-1} \tag{24}$$

where $\bar{\bar{\mathbf{U}}} = \bar{\bar{\mathbf{E}}}^{ru} \cdot (\bar{\bar{\mathbf{E}}}^{t})^{-1}$, and $\bar{\bar{\mathbf{E}}}^{ru} = [\mathbf{E}_u^{rV}, \mathbf{E}_u^{r45}]$ is the measured target response and r_u is the range to the unknown target.

In the incoherent correction scheme the unbounded transmitted wave, $\tilde{\mathbf{E}}_o^t$, can be determined from (8) and (10) for any prescribed rotation angles α_1 and α_2 of the dielectric cards. Furthermore, the unbounded received wave, $\tilde{\mathbf{E}}^r$, can be obtained from the measured field $\mathbf{E}^r$ and is given by

$$\tilde{\mathbf{E}}^r = r_u^2 \, e^{2ik_0 r_u} \, \bar{\bar{\mathbf{R}}}_d^{-1} \, \mathbf{E}^r. \tag{25}$$

The transmitted and received Stokes vectors, needed to compute the modified Mueller matrix, can be determined by substituting $\tilde{\mathbf{E}}_o^t$ and $\tilde{\mathbf{E}}^r$, respectively, into the definition of the Stokes vector given by (2).

V. Comparison With Measured Data

The validity of the new calibration technique is examined by measuring the scattering matrices of cylinders and spheres as test targets using a coherent-on-receive network analyzer-based scatterometer operating at 34.5 GHz. A block diagram of the system is shown in Fig. 1. For each transmitted polarization, the network analyzer sweeps over a bandwidth of 1 GHz in 401 steps and collects simultaneously the received fields at the V and H channels. Using the time domain capability of the network analyzer, the target return can then be separated from the unwanted short-range returns and nearby objects. The received signal at the target range includes the target response in addition to disturbances due to the thermal and background noises. The effects of the thermal noise, which is a zero-mean random process, can be minimized by averaging over many samples and the effects of the background, which is due to returns adjacent to the target can be eliminated by subtracting the background response (in the absence of the target) from the response of the target and background.

Target orientation was facilitated by an elevation over azimuth stepper motor positioner. An 8.1 cm sphere and a finite array of parallel wires were used as the calibration targets. The test targets include a 4.45 cm sphere and a conducting cylinder with a diameter of 0.552 cm and a length of 6.045 cm. The cylinder is measured for two different orientations: vertical and 45 degrees in a plane normal to the direction of incidence. The measured scattering matrices using (24) are compared with those computed using the exact eigenfunction expansion solution and the method of moments in conjunction with the Fourier transform technique (body of revolution) for the sphere and cylinders, respectively.

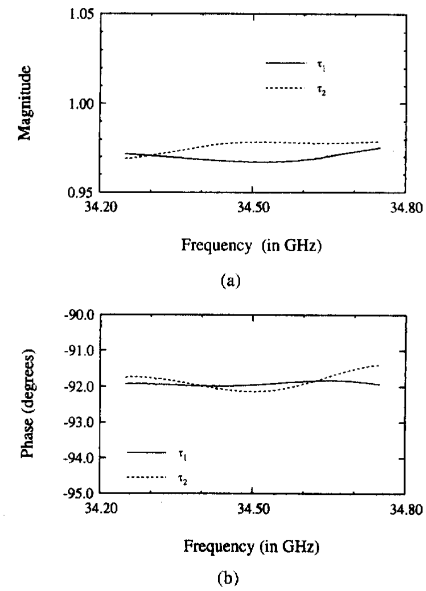

Fig. 4. The measured variations of the magnitude (a) and phase (b) of the phase-shift factors, τ_1 and τ_2, as function of frequency.

Using averaging and background subtraction, a signal-to-noise ratio of better than 65 dB was achieved. For targets with $S_{vh} = S_{hv} = 0$ (sphere and vertical cylinder), the signal-to-noise ratio in the cross-polarized channels was better than 40 dB.

Applying the calibration technique, the magnitude and phase of the phase-shift factors of the two polarizers, τ_1 and τ_2 are measured and shown in Fig. 4 as a function of frequency. As expected, both polarizers exhibit negligible transmission losses as the magnitudes of the transmission coefficients vary between 0.968 and 0.98 over the measured bandwidth. The phases of τ_1 and τ_2 are measured to be around -92 degrees, which is very close to the expected phase shift of -90.0 degrees. The measured antennas cross-talk factors of the system as computed by (22) and (23) are shown in Fig. 5.

Fig. 6 compares the theoretical and measured scattering matrix elements of the 4.45 cm sphere. The error in the magnitudes of co-polarized terms is less than 0.5 dB and the error in the phase-difference between the copolarized terms is less than 4 degrees. The cross-polarized term is shown in Fig. 6(a) and shows that an effective cross-polarization isolation of at least -40 dB is obtained which represents a 20 dB improvement in the system cross-polarization isolation.

Excellent agreement is also achieved between the theoretical and measured scattering matrix elements of the cylinder oriented at vertical and 45 degrees. The comparisons are shown in Figs. 7–8, which show a maximum discrepancy of about 0.4 dB in magnitude and 4 degrees in phase. In Figs. 7(a) and 8(a), the values of σ_{vv} and σ_{hh} are overlapped. The experiments were repeated many times and consistent results were achieved.

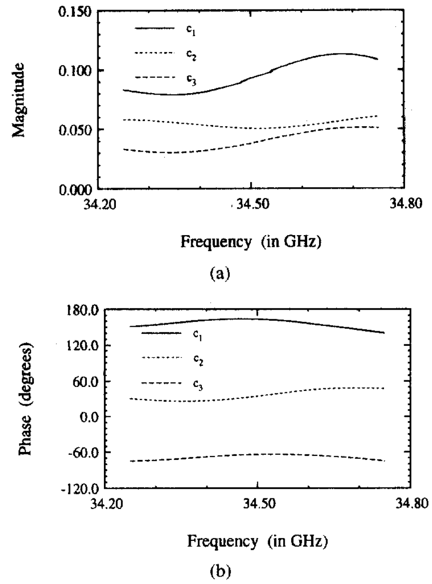

Fig. 5. The measured variations of the magnitude (a) and phase (b) of the antenna cross-talk factors, c_1, c_2, and c_3 as function of frequency.

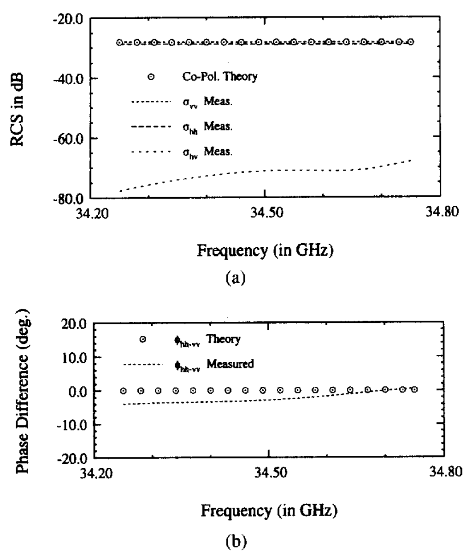

Fig. 6. The magnitude (a) and phase-difference (b) of the scattering matrix elements of a 4.45 cm sphere.

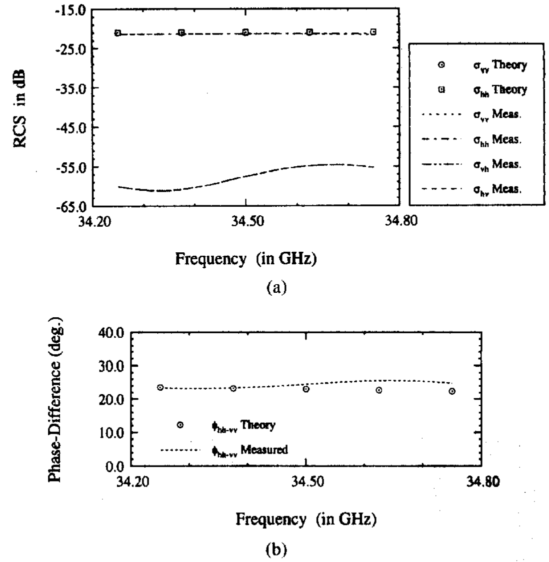

Fig. 7. The magnitude (a) and phase-difference (b) of the scattering matrix elements of a vertical cylinder.

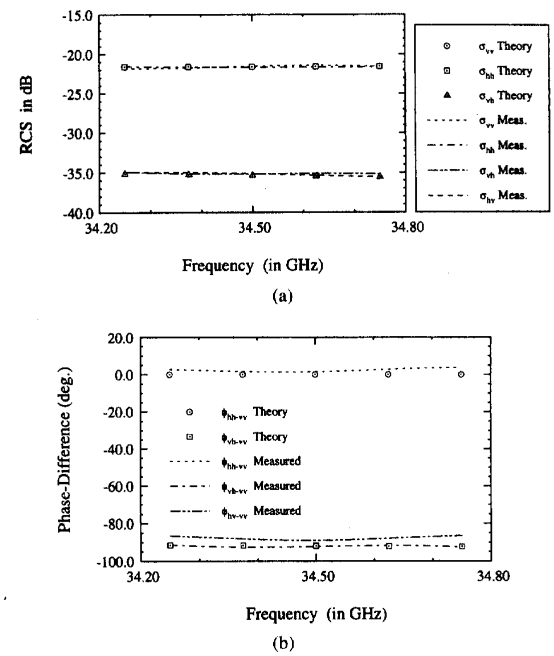

Fig. 8. The magnitude (a) and phase-difference (b) of the scattering matrix elements of a cylinder oriented at 45 degrees from vertical.

VI. CONCLUSION

Practical aspects of a calibration procedure for a coherent-on-receive polarimetric radar system are discussed. The polarization state of the transmitter is related to the transmit antenna cross-talk factor and the phase-shift factors of the waveguide polarizers in the radar system. Distortions in the receiver, such as channel imbalances and cross-talk factors, together with the transmitter distortion parameters are obtained from measurements of a metallic sphere and a depolarizing target with unknown scattering matrix for four and two transmit polarizations, respectively. The validity and accuracy of the calibration technique is verified by measuring the scattering matrix of independent point targets with known scattering matrices.

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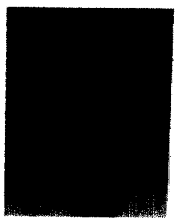

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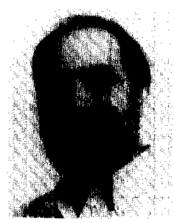

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