Monte Carlo Simulation of Electromagnetic Scattering from a Heterogeneous Two-Component Medium

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ABSTRACT

In this paper a numerical solution for scattering from a random collection of long vertical cylinders and spheres above a ground plane is developed to examine the validity of the radiative transfer theory when applied to vegetation canopies. Numerical simulations, while frequently computationally intensive, give detailed statistical information, including the phase-coherence, directly related to the scattered fields themselves. Electromagnetic scattering properties of the heterogeneous medium, such as the backscattering coefficient and phase-statistics, are determined including interactions up to second-order. The second-order, near-field, sphere-cylinder interaction has been computed for non-uniform illumination of one particle by its pair using a novel technique based on reciprocity. The application of this reciprocity based approach is validated in this paper using the method of moments. The Monte Carlo simulation results are compared with corresponding analytical solutions obtained from RT theory for the purpose of examining the significance of limitations in the underlying assumptions of RT in formulating solutions for EM scattering from vegetation media.

INTRODUCTION

In the application of RT theory to the modeling of forests and other dense vegetation cover, several fundamental conditions necessary to the validity of the model have been ignored. The RT model is based on the single scattering properties of the particles that constitute the medium. That is, the assumption has been made that particles are in the far field of each other and that they are illuminated locally by plane-waves. A tree canopy usually contains particles such as, trunks and branches, which are much larger in dimension than the excitation wavelength at microwave frequencies. These large scatterers are usually in each other’s near-field zones. Smaller particles such as leaves, needles, and twigs are also in the near-field zone of the larger canopy components. These smaller canopy constituents may be present in large numbers and can have a significant effect on the electromagnetic properties of the medium, both as isolated scatterers and through multiple scattering interactions. In addition, it is to be expected that scatterers with vertical dimensions comparable with the height of the vegetation canopy will be illuminated with a fairly high degree of amplitude, phase, and polarization non-uniformity as a result of the finite coherence distance of the mean field in this type of random medium.

In this paper we construct a Monte Carlo simulation of a heterogeneous two-component random medium consisting of large vertical cylinders and small spheres above a ground plane. Using a recently developed technique [1], second-order interactions between cylinders and between spheres and cylinders are taken into account in the formulation of the scattering model, and all scattering terms used in the model have been validated using the method of moments technique. The backscattering properties of the medium obtained using this simulation technique are compared with those computed on the basis of the radiative transfer model. Second-order interaction effects in the medium are examined, and the validity of the layered RT approach is also investigated.

CONSTRUCTION OF SCATTERING TERMS

The heterogeneous two-component medium under consideration consists of vertical dielectric cylinders and small perfectly conducting spheres. Dielectric cylinders are employed to represent the strongly scattering stalks and trunks that exist in a typical vegetation canopy, and the spheres are representative of the smaller scatterers that comprise the upper (crown) layer in such media. The spheres are chosen to be perfectly conducting because this reduces the complexity of their scattering matrix and such reduction translates into less overall computation time in the Monte Carlo simulations. Since spheres have relatively small scattering cross-sections, it is also desirable to boost their albedo by making them perfectly conducting in order to bring them on par with typical small scatterers that might be found in the canopy.

The electromagnetic interaction between adjacent cylinders and between cylinders and spheres is computed to second-order, while the interaction between spheres is ignored. Our numerical studies indicate that multiple scattering between spheres is negligible for the number densities of small particles considered in this work. In any event, the cylinders are by far the stronger scatterers in the medium and, in most circumstances, are the dominant component of the return.

The cylinder-cylinder interaction has been discussed in detail in a previous study [2] and will not be considered here. It suffices to say that the scattering matrix for this interaction has been derived using the plane-wave expansion technique and is applicable to cylinders that are electromagnetically close with respect to their longitudinal dimension and, at the same time, are in each other’s far-zone with respect to the transverse dimension. The sphere-cylinder interaction has been derived using an analytical technique based on the reciprocity principle. The details are presented in [1] and will not be duplicated here.

HETEROGENEOUS CANOPY SIMULATION

For a given arrangement consisting of many cylinders and spheres, the solution of the scattering problem can be obtained to second order by computing the single and pairwise interactions for every particle in the ensemble, except that we ignore sphere-sphere...
interactions for the reasons given in Section 2. The principle of the Monte Carlo simulation based on second order interactions is as follows:

1. An ensemble of randomly positioned cylinders and spheres is generated using a random number generator. In this case the cylinder positions are uniformly distributed within a circular area, and the sphere positions are uniformly distributed within a cylindrical volume. The number of cylinders and spheres used is dependent on the specified numbers per unit area or volume and the dimensions of the cylindrical region that constitutes the medium.

2. The scattering is computed for all cylinders and spheres and between all pairs of particles within the ensemble up to second order excluding the interactions between spheres.

3. The ensemble is re-randomized and the scattering re-computed as discussed above. The number of independent samples is chosen so as to make the variance as small as possible within limits depending on the computing time. For the cases analyzed in this article the sample number was chosen to be two hundred except in the highest density case for which the sample number was one hundred.

4. The values of the scattering coefficients (\( \sigma_{vv}, \sigma_{ha} \)) are found from the ensemble average. The same is true for the co-polarized phase difference \( \phi \), and the degree of co-polarized phase correlation \( \rho \).

The simulations examined in this article utilized perfectly conducting spheres having diameters of 0.635 cm and characterized by number densities of 14,147 per cubic meter. The spheres were generated inside a cylindrical volume 0.1 meter in height and 0.6 meter in diameter. Identical cylinders 18 cm in height and 0.55 cm in diameter with a dielectric constant of \( \epsilon_r = 35 + i11 \) were uniformly distributed in the same 0.6 meter diameter circle used to generate the cylindrical volume for the spheres, but the cylinder bases were constrained to rest on the ground plane whereas the sphere layer could begin at any height above the plane. Simulations were performed for spheres alone, cylinders alone and combinations of cylinders and spheres together. Two basic heterogeneous canopy configurations were examined. In one case the sphere layer overlapped the upper 10 cm of the cylinder layer, while in the other case the sphere layer was distinct and existed separately above the cylinder layer. These configurations are shown in Figure 1. In all cases tested, the simulations converged to within about 0.5 dB for \( \sigma^2 \) and 5% for \( \rho \) in one hundred samples or less. The operational frequency is 9.25 GHz for all the examples considered in the following discussion.

REFERENCES


Figure 1: Configurations for the pure two-layer and mixed two-layer canopy models.

Figure 2: Heterogeneous canopy consisting of 106.1 cylinders per square meter in lower layer and 14,147 spheres per cubic meter in an upper layer 10cm thick. The cylinders are 18cm high and 0.55cm in diameter. The spheres have \( ka = 0.02 \). VV-polarized backscatter coefficient at 9.25 GHz.

Figure 3: Heterogeneous canopy consisting of 106.1 cylinders per square meter in lower layer and 14,147 spheres per cubic meter in an upper layer 10cm thick. The cylinders are 18cm high and 0.55cm in diameter. The spheres have \( ka = 0.02 \). HH-polarized backscatter coefficient at 9.25 GHz.
Figure 4: Comparison of Monte Carlo simulation of VH-polarized backscatter coefficient for a homogeneous canopy and a heterogeneous canopy. The homogeneous canopy has 106.1 cylinders per square meter. The heterogeneous canopy has the same number density of cylinders but has 14,147 spheres per cubic meter in a 10cm layer on top.

Figure 5: Effect of adding a layer of 14,147 small conducting spheres per cubic meter on the degree of copolarized phase correlation in a layer 10cm thick on top of the canopy of 106.1 cylinders per square meter.

Figure 6: Comparison of pure and mixed layer R.T. models for a canopy consisting of 106.1 cylinders per square meter and 14,147 spheres per cubic meter. The crown layer is 10cm in height in both cases. VV-polarized return at 9.25 GHz. The pure and mixed copolarized Monte Carlo simulations are almost identical.

Figure 7: Comparison of pure and mixed layer Monte Carlo simulations for a canopy consisting of 106.1 cylinders per square meter and 14,147 spheres per cubic meter. The crown layer is 10cm in height in both cases. VH-polarized return at 9.25 GHz.