A Coherent Scattering Model For Forest Canopies Based On Monte Carlo Simulation of Fractal Generated Trees

Yi-Cheng Lin and Kamal Sarabandi
Radiation Laboratory
Department of Electrical Engineering and Computer Science
The University of Michigan, Ann Arbor, MI 48109-2122
Tel :(313) 763-8162 FAX : (313) 747-2106
Email: yclin@umich.edu

Abstract – A coherent scattering model for tree canopies based on a Monte Carlo simulation of fractal generated trees is investigated in this study. In contrast to the incoherent models based on the radiative transfer theory, the present model is capable of preserving the relative phase of individual scatterer which give rise to the coherent effects and predicting the absolute phase of the backscattered field or equivalently the scattering phase center.

In the procedure for Monte Carlo simulation, first random generation of tree architectures is implemented by employing the Lindenmayer systems (L-systems), a convenient tool for creating fractal patterns of botanical structures. Since the generating code of tree structures is faithful in preserving the fine features of the simulated tree types, this study provides an efficient approach to examine the effects of tree structures on the radar backscatter. After generating a tree structure, the electromagnetic scattering problem is then treated by considering the tree structure as a cluster of scatterers comprised of cylinders (trunks and branches) and disks (leaves) with specified position, orientation, and size. The scattering solution is obtained by invoking the single scattering theory for a uniform plane wave illumination. In this solution scattering from individual tree components when illuminated by the mean field is computed and then added coherently. The mean field at a given point within the tree structure include the attenuation and phase change due to the scattering and absorption losses of vegetation particles. Finally, the backscattering coefficients are simulated at different frequencies based on the results of the Monte Carlo simulations obtained from a large number of independent trees.

INTRODUCTION

Radiative transfer theory [1] is the most widely used model for characterization of scattering from a forest canopy. When the medium consists of sparse scatterers that are small compared to the wavelength, RT theory can provide accurate statistics of radar backscatter including the phase difference of the scattering matrix elements. However, no information regarding the absolute phase can be extracted from a RT model. The other shortcoming of RT theory is its inability to account for the coherent effect that may exist between different scatterers or scattering mechanism. Moreover, recent investigations on scattering behavior of tree signatures have shown that both the backscattering and the attenuation are significantly influenced by tree architectures [2]. Therefore, a coherent scattering model seems crucial for the accurate estimation of radar behavior of forest canopies.

The purpose of this study is to develop an accurate coherent scattering model for tree canopies, which can preserve the effect of the tree structure on the radar response of forest canopies and can provide information about the absolute phase of the backscattered field. This model is comprised of three major components: (1) generation of tree structure, (2) evaluation of scattered fields, and (3) the Monte Carlo simulation. In the tree structure modeling, the L-systems [3] based on fractal theory, is employed to construct a tree-like structure as a tested target. The tree generating code is developed so that fine features of distinct tree structures can be preserved to any desired level of accuracy. In the scattering model, scattering from individual tree components when illuminated by the mean field is computed and then added coherently. The mean field at a given point within the tree structure, which accounts for the attenuation and phase change due to the scattering and absorption losses of vegetation particles, is calculated using Foldy's approximation [4]. Finally, a Monte Carlo simulation is performed on a large number of fractal generated trees to characterize the statistics of the backscattered signals.

RANDOM GENERATION OF TREES

Fractal theory is the most popular mathematical model used for relating the natural botanical structures to abstract geometries. Mandelbrot [5] defines the fractal as a set with Hausdorff-Besicovitch dimension strictly exceeding the topological dimension. In other words, the notion of fractal is defined only in the limit. However, in order to make the problem tractable, a finite curve can be considered an approximation of an infinite fractal as long as the significant properties of both are closely related. In the case of plant models, this distinctive feature is self-similarity which is kept through the derivation process.

The Lindenmayer systems (L-systems) is a versatile tool for plant pattern construction [3]. Its principal features lie on the easy generation of fractals and the realistic modeling of plant structures. The central concept of the L-systems is that of rewriting – a technique for defining a complex object using a set of simple patterns with rewriting rules or productions. Here,

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we apply the L-systems to the development of branching structures.

The final stage of tree modeling is to integrate botanical knowledge of the architecture for the trees of interest, including the growth model, the pattern and orientation of leaves and branches, and the profile occupied by the entire tree given the age and number density. In this study, we focus on a real tree type, red maple, which is faithfully characterized and encoded, and then decoded by a computer based tree-generating code.

COHERENT SCATTERING MODEL

In this section, a coherent scattering model is developed to calculate the radar backscatter from the fractal generated trees. Once a tree is created, it is treated as a cluster of scatterers comprised of cylinders (trunks and branches) and disks (leaves) with specific position, orientation, and geometric shape and size. The entire tree is then illuminated by a plane wave, and the scattered field in the far zone is evaluated. To the first order scattering approximation, the backscatter from the entire tree is calculated from the coherent addition of the individual scattering terms. Hence, neglecting the multiple scattering among the scatterers, the total scattered field can be written as

$$\mathbf{E}^s = \frac{e^{ikr}}{r} \sum_{n=1}^{N} e^{i\phi_n} \mathbf{S}_n \cdot \mathbf{E}_o^i, \tag{1}$$

where N is the total number of the scatterers; $\mathbf{S}_n$ is the individual scattering matrix of the n-th scatterer and ϕ_n is the phase compensation accounting for the shifting of the phase reference from the local to the global phase reference, and is given by $\phi_n = (\hat{k}_i - \hat{k}_s) \cdot \mathbf{r_n}$, where $\mathbf{r_n}$ is the position vector of the n-th scatterer with measured in the global coordinate system.

In order to compute the scattering matrix of the n-th particle $\mathbf{S}_n$, consider the isolated particle above a ground plane. Ignoring the multiple scattering between the scatterer and its mirror image, the scattering matrix can be decomposed to four components : (1) direct component $\mathbf{S}_n^t$, (2) ground-scatterer component $\mathbf{S}_n^{tg}$, (3) scatterer-ground component $\mathbf{S}_n^{gt}$, and (4) ground-scatterer-ground component $\mathbf{S}_n^{gtg}$. Therefore, the individual scattering matrix $\mathbf{S}_n$ can be written as

$$\mathbf{S}_n = \mathbf{S}_n^t + \mathbf{S}_n^{gt} + \mathbf{S}_n^{tg} + \mathbf{S}_n^{gtg}, \tag{2}$$

where

$$\begin{aligned}
\mathbf{S}_n^t &= \mathbf{T}_n^i \cdot \mathbf{S}_n^0(-\hat{k}_i, \hat{k}_i) \cdot \mathbf{T}_n^i, &(3)\\
\mathbf{S}_n^{gt} &= e^{i\tau_n} \mathbf{T}^t \cdot \mathbf{R} \cdot \mathbf{T}_n^r \cdot \mathbf{S}_n^0(-\hat{k}_r, \hat{k}_i) \cdot \mathbf{T}_n^i, &(4)\\
\mathbf{S}_n^{tg} &= e^{i\tau_n} \mathbf{T}_n^i \cdot \mathbf{S}_n^0(-\hat{k}_i, \hat{k}_r) \cdot \mathbf{T}_n^r \cdot \mathbf{R} \cdot \mathbf{T}^t, &(5)\\
\mathbf{S}_n^{gtg} &= e^{i2\tau_n} \mathbf{T}^t \cdot \mathbf{R} \cdot \mathbf{T}_n^r \cdot \mathbf{S}_n^0(-\hat{k}_r, \hat{k}_r) \cdot \mathbf{T}_n^r \cdot \mathbf{R} \cdot \mathbf{T}^t &(6)
\end{aligned}$$

with

$$\begin{aligned}
\hat{k}_r &= \hat{k}_i - 2\hat{n}_g(\hat{n}_g \cdot \hat{k}_i), &(7)\\
\tau_n &= 2k_0(\mathbf{r_n} \cdot \hat{n}_g)(\hat{n}_g \cdot \hat{k}_r). &(8)
\end{aligned}$$

In the above expressions, $\hat{n}_g$ is the unit vector normal to the ground plane which in general can be tilted with respect to the global coordinate system. The optical lengths τ_n accounts for the extra path length. $\mathbf{S_n^0}$ is the scattering matrix of the n-th scatterer isolated in free space. $\mathbf{R}$ is the reflection matrix of the ground plane accounting for the reflection and polarization transformation due to the tilt ground plane. $\mathbf{T}_n^i$, $\mathbf{T}_n^r$, and $\mathbf{T}^t$ are the transmission matrix accounting for the different extinction portion within the random media as shown in the Fig. 2.

Forest stand can be represented by a layered structure according to its particle distribution statistics. The particle distribution as a function of height is needed for the calculation of extinction and can be obtained directly from the fractal model.

MONTE CARLO SIMULATION AND DISCUSSION

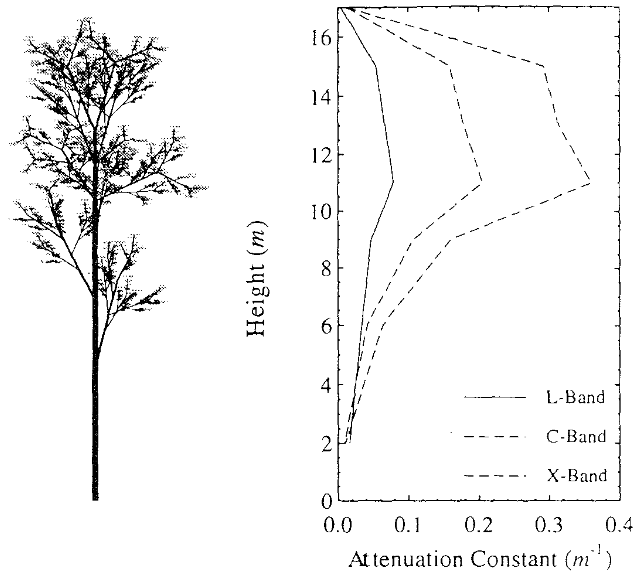
The radar backscatter from a distributed target is usually quantified in terms of the backscattering coefficient σ^0, define as the ratio of measured radar cross section to the illuminated area. To relate the total scattering matrix of a tree structure S to the backscattering coefficient σ^0, we assume no coherent correlation between trees, that i[illegible] addition of powers instead of fields is needed to compu[illegible] backscatter from a number of trees. Hence σ^0 is proportiona[illegible] the tree density D_t, i.e., $\sigma_{pq}^0 = 4\pi D_t < |S_{pq}|^2 >$.

A large number of fractal trees are created according to the prescribed tree features, such as leaf density, pdf of branch's orientation, and DBH distribution. Fig.1 shows a computer generated fractal tree based on the ground truth data collected at Raco Site, in Michigan's upper peninsula. In the traditional RT models, the forest canopies are simplified in terms of a two-layer structure. This treatment is not realistic in that location of the layer boundary between the crown and trunk layers is ambiguous and that the branches always have a non-uniform distribution in position, that is, the branches are located with larger diameter in lower region. Also the leaf density is usually higher in the upper layer of the canopy. In the present model, since the entire tree structure is well specified, it is easy to construct a multilayer model to characterize the vertical inhomogeneity of the tree structure. Fig. 1(b) shows the attenuation constants, α_m, profile of a seven layer canopy derived for the red maple forest stand. Using the Foldy's approximation, the expression for α_m is computed from :

$$\alpha_m = \frac{2\pi D_t}{k_0 d_m} Im \left(\sum_{n}^{layer-m} \mathbf{S}_n^0(\hat{k}, \hat{k}) \right) \tag{9}$$

where d_m is the thickness of the m-th layer; D_t is the tree density.

It is well known that the backscatter from forest canopies as a function of frequency is governed by various scattering



(a) (b)

Figure 1: A red maple fractal generated tree (a) and its extinction profile (b).

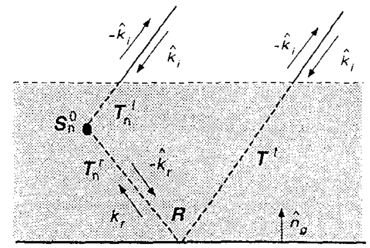


Figure 2: Extinction of a coherent wave in random media.

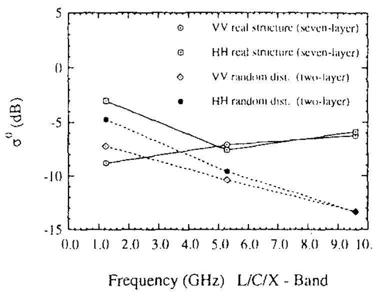


Figure 3: Comparison of the backscattering coefficients of the real structure andthe random distribution.

mechanisms. For instance, at lower frequencies the dominant backscatter term is the trunk-ground bounce, while at high frequencies the backscatter is mainly influenced by the direct scattering term from the crown layer. The difference between the backscatter from a forest stand using a multilayer model and a two-layer model is examined. In the two-layer model the forest is divided into a trunk layer (0-8 m) and a crown layer (8-17 m), and extinction parameters are averaged over the whole layer. Fig. 4 shows the results of this study where the backscattering coefficients are computed at different frequencies, L-band (1.25 GHz), C-band (5.3 GHz), and X-band (9,6 GHz). It is shown that at high frequencies the discrepancy between the two models increases.

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