Effect of Canopy-Air Interface Roughness on HF-VHF Wave Propagation in Forest

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Abstract

The problem of wave propagation in forest is revisited. In particular, the effect of the non-planar interface between the air and the canopy on lateral waves is examined. An analytical formulation is obtained for the mean field when both the transmitter and receiver are within the foliage. This formulation is based on distorted Born approximation and is shown that compared to a planar interface, the field of a dipole in a canopy with rough interface is significantly reduced.

1 Introduction

A generally adopted model of a forest at HF through VHF, that attributes wave propagation in forest to a lateral wave, was first developed by Tamir [1]. In this approach, the forest is modeled by a homogeneous half-space dielectric medium with a planar interface. The field of a dipole within this medium was then evaluated at an observation point near the interface using the asymptotic form of the integral involved. This solution shows that the field at the observation point is dominated by the so called lateral wave that travels along the flat canopy-air interface. In reality, however, the interface between forest canopies and air is not flat, hence it is not clear as to what happens to the lateral wave or whether it can even be excited or not. In this paper, the effect of roughness of interface between canopy and air on the wave propagation in forest areas is investigated. An analytical solution is obtained using the volumetric integral equation in conjunction with the distorted Born approximation.

2 Analytical Formulation

Geometry of the diffraction problem is shown in Fig.1 where the dipole located at heights h and h' inside a canopy with effective dielectric constant ε_1. The envelope of the canopy-air interface is denoted by $d(x,y)$ The permittivity of the upper medium(air) is denoted by ε_2. We modify the problem by extending the canopy to $z = 0$ plane and assume that there exists a volumetric polarization current $\bar{\mathbf{J}} = ik_1Y_1(\varepsilon_2 - \varepsilon_1)\bar{\mathbf{E}}^t$, where $\bar{\mathbf{E}}^t = \bar{\mathbf{E}}^i + \bar{\mathbf{E}}^r + \bar{\mathbf{E}}^s$. Here $\bar{\mathbf{E}}^i$ is the incident field, $\bar{\mathbf{E}}^r$ is the reflected wave from the planar interface and $\bar{\mathbf{E}}^s$ is the scattered field generated by $\bar{\mathbf{J}}$ itself. To the first order in $(\varepsilon_2 - \varepsilon_1)$ the Born approximation can be used to the scattered field

$$\bar{\mathbf{E}}^s = k_0{}^2(\varepsilon_2 - \varepsilon_1)\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-d(x,y)}^{0}\bar{\bar{G}}(\bar{r},\bar{r}')\cdot[\bar{\mathbf{E}}^i(\bar{r}') + \bar{\mathbf{E}}^r(\bar{r}')]dx'dy'dz' \quad (1)$$

where $\bar{\bar{G}}$ is the dyadic Green's function of the half space dielectric medium [2, 3].
In (1) $d(x,y)$ is a two dimensional random process describing the interface between the canopy and air and is assumed to be Gaussian with a mean value of m(a positive number) and standard deviation of σ. Distorted Born approximation provides a more accurate solution for $\bar{\mathbf{E}}^s$. In this approximation, a phase correction term is

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incorporated into the expressions for the internal field to account for the difference in the propagation constant between the air and the canopy as seen in the following expression of incident and reflected waves.

$$\begin{aligned}\vec{\mathbf{E}}^i &= \hat{\mathbf{P}}(k_{1z}^i)e^{i[k_x^i x'+k_y^i y'+k_z^i z']}\cdot e^{i(k_{2z}^i-k_{1z}^i)m}\\ \vec{\mathbf{E}}^r &= \hat{\mathbf{P}}(-k_{1z}^i)Re^{i[k_x^i x'+k_y^i y'-k_z^i z']}\cdot e^{i(k_{2z}^i-k_{1z}^i)m}\end{aligned} \tag{2}$$

where $\hat{\mathbf{P}}(k_{1z})$ is a unit vector ($\hat{\mathbf{P}} = \hat{\mathbf{v}}$ or $\hat{\mathbf{h}}$) as is defined in [2, 3] and $R_{v,h}$ is the Fresnel reflection coefficient for vertical or horizontal polarization.

Since the layer thickness is not uniform, the phase correction terms in (2) are chosen for a layer with a uniform thickness m. Substituting 2-D Fourier transform of $\bar{\bar{G}}$ in (1) and after performing the integration with respect to z', evaluation of $\langle\vec{\mathbf{E}}^s\rangle$ would require computation of the term like $\langle e^{\pm jsd}\rangle$ which for a Gaussian d are found to be

$$\begin{aligned}\langle e^{jsd}\rangle &= \frac{e^{sm}}{2}[e^{-\frac{\sigma^2 s^2}{2}}erfc(-j\frac{\sigma^2 s}{\sqrt{2}}) + e^{-\frac{\sigma^2 s^{*2}}{2}}erfc(j\frac{\sigma^2 s^*}{\sqrt{2}})]\\ \langle e^{-jsd}\rangle &= \frac{e^{-sm}}{2}[e^{-\frac{\sigma^2 s^{*2}}{2}}erfc(-j\frac{\sigma^2 s^*}{\sqrt{2}}) + e^{-\frac{\sigma^2 s^2}{2}}erfc(j\frac{\sigma^2 s}{\sqrt{2}})]\end{aligned} \tag{3}$$

where s can be either $s_1 = k_{2z}^i + k_{1z}^i$ or $s_2 = k_{2z}^i - k_{1z}^i$.

The integration with respect to x' and y' can now be carried out analytically which results in $\delta(k_x-k_x^i)\cdot\delta(k_y-k_y^i)$. This in turn simplifies the integration with respect to k_x and k_y. Thus the final result is readily obtained as,

$$\vec{\mathbf{E}}^t = \vec{\mathbf{E}}^r + \langle\vec{\mathbf{E}}^s\rangle \approx (R_{ref} + R_{Born}^{(1)})e^{i[k_{1x}^i x+k_{1y}^i y-k_{1z}^i z]} \tag{4}$$

where R_{ref} is the Fresnel reflection coefficient for the canopy-air boundary at z = 0.

$R_{ref} + R_{Born}^{(1)}$ is accurate enough for horizontal polarization, but it cannot accurately predict the Brewster angle for vertical polarization. To rectify this deficiency, higher order solutions must be obtained but it is sufficient to use only $-\hat{z}\hat{z}\frac{\delta(\bar{r}-\bar{r}')}{k_2{}^2}$ term of the dyadic Green's function. Thus the partial second order solution can be obtained and is given by

$$\vec{\mathbf{E}}^{s(1.5)} = -k_0{}^2\frac{(\varepsilon_2-\varepsilon_1)^2}{\varepsilon_2}\int_v \bar{\bar{G}}\cdot[\hat{z}\hat{z}\cdot(\vec{\mathbf{E}}^i+\vec{\mathbf{E}}^r)]dv \tag{5}$$

The ensemble average of $\vec{\mathbf{E}}^{s(1.5)}$ can be obtained in a manner similar to what was used in computation of $\langle\vec{\mathbf{E}}^{s(1)}\rangle$. After some algebraic manipulations, the reflection coefficients for the mean field are obtained and given by

$$\begin{aligned}R_{Born(h)}^{(1)} =& \frac{k_0{}^2(\varepsilon_2-\varepsilon_1)}{2k_{1z}^i}\{\frac{1}{s_1}[R_h{}^2(\langle e^{is_1d}\rangle - 1) + 1 - \langle e^{-is_1d}\rangle]\\ &+ \frac{R_h}{s_2}(\langle e^{is_1d}\rangle - \langle e^{-is_2d}\rangle)k_0{}^2\}e^{is_2m}\end{aligned}$$

$$\begin{aligned}R_{Born(v)}^{(1.5)} =& \frac{(\varepsilon_2-\varepsilon_1)}{2k_{1z}^i\varepsilon_1}\{\frac{R_v}{s_2}(\langle e^{is_1d}\rangle - \langle e^{-is_2d}\rangle)\cdot(\frac{\varepsilon_1}{\varepsilon_2}k_\rho{}^2 + k_{1z}^i{}^2)\\ &+ \frac{1}{s_1}(1 - R_v{}^2 + R_v\langle e^{is_1d}\rangle - \langle e^{-is_1d}\rangle)\cdot(\frac{\varepsilon_1}{\varepsilon_2}k_\rho{}^2 - k_{1z}^i{}^2)\}e^{is_2m}\end{aligned}$$

where $k_\rho{}^2 = k_x^i{}^2 + k_y^i{}^2$.

Using the above result, the solution for an infinitesimal dipole embedded within the foliage can be obtained by expanding the field of the dipole in terms of a continuous spectrum of plane waves. Assuming that a dipole, whose orientation is denoted by a unit vector $\hat{l}$, is located at $\vec{r_0} = -(h+m)\hat{z}$, and using superposition, the mean scattered field can be computed from the coherent sum of all reflected plane waves. That is

$$\langle\bar{\mathbf{E}}_{Born}\rangle = -\frac{Ik_0Z_0}{8\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{1}{k_{1z}^i}[(\hat{l}\cdot\hat{v}_i)R_{Bv}^{(1.5)} + (\hat{l}\cdot\hat{h}_i)R_{Bh}^{(1)}]e^{-ik_{1z}^i(z-m-h)}e^{i(k_x^i x+k_y^i y)}dk_x^i dk_y^i$$

For more accurate computation, the mean field for a dielectric medium with a rough surface is calculated by,

$$\vec{\mathbf{E}}_{exact}(\sigma) \approx \vec{\mathbf{E}}_{exact}(\sigma = 0) + \vec{\mathbf{E}}_{Born}(\sigma) - \vec{\mathbf{E}}_{Born}(\sigma = 0)$$

where $\vec{\mathbf{E}}_{exact}(\sigma = 0)$ is the field of the dipole in the presence of the upper free-space medium with a planar interface at $z = -m$.

3 Numerical Simulation

In order to verify the accuracy of the distorted Born approximation, first, the approximate analytical solution for reflection coefficient of a planar boundary is compared with the exact Fresnel reflection coefficient when a plane wave is incident at the boundary. We consider a canopy with effective permittivity $\varepsilon = 1.03 + i0.001$ and the interface between the canopy and air is assumed planar at a distance m = 6[m] from the x-y plane of the reference coordinate system. Figure 2 and 3 show the comparison between the reflection coefficients as predicted by the distorted Born approximation and the exact solution for both parallel and perpendicular polarizations. A very good agreement is obtained for this example. Further sensitivity analysis show that the accuracy of the distorted Born approximation degrades. Similar behavior is obtained when m is kept fixed and dielectric constant of the dielectric layer is increased. Next we considered the field of dipole inside a dielectric layer with $\varepsilon = 1.03 + i0.001$ at a depth of 2[m] below the interface. The field is observed at a depth in 1[m] below the interface as a function of radial distance. The exact solution is compared with the distorted Born approximation for a chosen value of $m = 2$[m] at 30[MHz] in Fig.4. An excellent agreement is obtained. Close examination of these results indicates a maximum relative error of 2 % between the two solutions. As for the plane wave illumination simulations, the discrepancy between the distorted Born and the exact solution increases with increasing m.

With confidence in the distorted Born solution, the effect of the interface roughness on the field can now be examined. Fig.5 shows the variation of the field as function of radial distance between the transmitter and the receiver ($h = 2$[m], $h' = 1$[m]) for two cases. In the first case, $\varepsilon_1 = 1.01 + i0.6$ and $k\sigma = 3$ (roughness parameter) and the second case, $\varepsilon_1 = 1.03 + i0.6$ and $k\sigma = 2$. The results are also compared with those had $k\sigma = 0$ (flat interface). It is shown that these surface roughnesses reduce the field by a factor of 3-5dB. For these simulations, we used a p.d.f. for d of the following form $f_d(d) = \frac{2}{\sqrt{2\pi}\sigma} e^{-d^2/2\sigma^2}$ where d can only assume positive numbers.

4 Conclusions

Analytical formulation for the mean-field of a short dipole embedded in a forest is computed. In this formulation, the effect of the roughness of the air-canopy interface is taken into account. Distorted Born approximation is shown to provide a very accurate results for the limiting case when the interface roughness disappears. Simulated results indicate that the roughness of the interface reduces the contribution of the lateral waves significantly.

References

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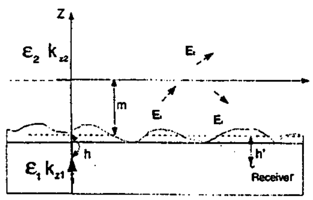

Figure 1. Geometry of a dipole with a canopy with rough interface

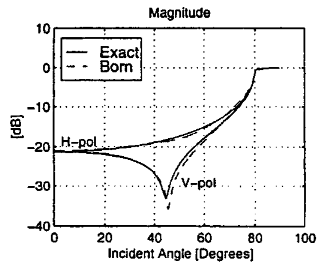

Figure 2. Comparison of the magnitude of reflection coefficients for a planar interface using exact and Born approximation

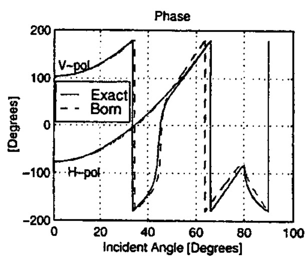

Figure 3. Comparison of the phase of reflection coefficients for a planar interface using exact and Born approximation

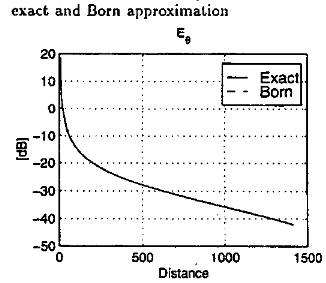

Figure 4. Comparison of exact versus approximate Born solution for the field in a dielectric with flat interface

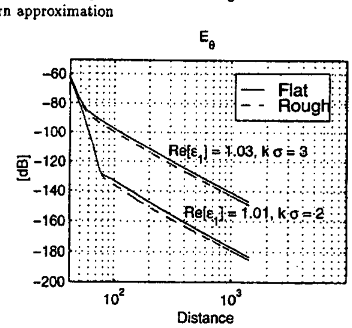

Figure 5. Field intensity under two different roughness and effective dielectric const.