HIGH FREQUENCY SCATTERING FROM CORRUGATED STRATIFIED CYLINDERS

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Abstract Interest in applying radar remote sensing for the study of forested areas led to the development of a model for scattering from corrugated stratified dielectric cylinders. The model is employed to investigate the effect of bark and its roughness on scattering from tree trunks and branches. The outer layer of the cylinder (bark) is assumed to be a low-loss dielectric material and to have a regular (periodic) corrugation pattern. The inner layers are treated as lossy dielectrics with smooth boundaries. A hybrid solution based on the moment method and the physical optics approximation is obtained. In the solution the corrugations are replaced with polarization currents that are identical to those of the local tangential periodic corrugated surface, and the stratified cylinder is replaced with equivalent surface currents. New expressions for the equivalent physical-optics currents are employed which are more convenient than the standard ones. It is shown that the bark layer and its roughness both reduce the radar cross section and it is also demonstrated that the corrugations can be replaced by an anisotropic layer.

Introduction

The literature concerning the problem of scattering from cylinders with rough surfaces is relatively scarce. To our knowledge the first treatment of a problem of this sort was given by Clemmow [1959] where a perturbation solution to an eigen function-expansion was obtained for a perfectly conducting cylinder with almost circular cross section, and only the E polarization case was considered. This technique is restricted to very smooth and small roughness functions. Other perturbation techniques for perfectly conducting cylinders with very small roughness have also been developed [Cabayan and Murphy, 1973; Tong 1974]. None of the existing techniques can handle dielectric rough cylinders, particularly when the roughness height is on the order of the wavelength.

Study of this problem is motivated by the fact that a tree trunk can be viewed as a multi-layer dielectric cylinder with a rough outer layer. The outer layer has almost a periodic pattern and consists of dead cells with almost no water content; hence, its dielectric constant is low and slightly lossy. The inner layers that carry high dielectric fluids have very high and lossy dielectric constants. In modeling a tree, the branches and trunk usually are considered to be homogeneous smooth cylinders [Durden et al, 1988]. In this paper the effect of bark and its roughness on scattering is studied.

Under the assumption that the bark roughness is a regular corrugation in only the angular direction and the radius of curvature of the cylinder is much larger than the wavelength and the period of corrugation, an approximate solution to the scattering problem is obtained. In this solution, each point on the surface of the cylinder is approximated by its tangential plane. Then the polarization current in the periodic tangential surface is obtained numerically. Once the polarization current in the corrugations is found, the scattered field due to the corrugations together with the scattered field from the smooth cylinder (when the corrugation is removed) give rise to the total scattered field. The scattered field for a smooth cylinder is obtained using new physical optics surface currents. It is shown that the corrugation on the surface can be replaced with an anisotropic layer which would extremely simplifies the problem. is derived.

Scattering from Periodic Corrugated Planar Dielectric Surface

In this section we seek a numerical solution for the total field (or polarization current) inside a periodic inhomogeneous layer lying over a stratified dielectric half-space illuminated by a plane wave. The geometry of the scattering problem is shown in Fig. 1.

The Hertz vector potential associated with an infinite current filament located at point (x', y') in free space with amplitude I_p and orientation $\hat{p}$ is of the form

$$\Pi_p(x,y) = \frac{-Z_0}{4k_0} H_0^{(1)}(k_0\sqrt{(x-x')^2+(y-y')^2}) I_p \quad p = x, y \text{ or } z.$$

By employing the identity

$$H_0^{(1)}(k_0\sqrt{(x-x')^2+(y-y')^2}) = \frac{1}{\pi}\int_{-\infty}^{+\infty} \frac{e^{ik_y|y-y'|-ik_x(x-x')}}{k_y} \mathrm{d}k_x \quad (1)$$

the resulting fields can be expressed in terms of continuous spectrum of plane waves. In (1) $k_y = \sqrt{k_0^2 - k_x^2}$ and the branch of the square root is chosen such that $\sqrt{-1} = i$.

In the presence of the dielectric half-space, when the current filament is in the upper half-space, each plane wave, is reflected at the air-dielectric interface according to Fresnel's law. It should be noted that the incidence angle of each plane wave, in general, is complex and is given by $\gamma = \arctan(\frac{k_x}{k_y})$. The net effect of the dielectric half-space on the radiated field can be obtained by superimposing all of the reflected plane waves. Now it can easily be shown that [Sarabandi, 1990] the total field in the upper half-space can be represented by

$$\mathbf{E} = \begin{bmatrix} G_{xx} & G_{xy} & 0 \\ G_{yx} & G_{yy} & 0 \\ 0 & 0 & G_{zz} \end{bmatrix} \begin{bmatrix} I_x \\ I_y \\ I_z \end{bmatrix}, \quad (2)$$

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where

$$\begin{aligned}
G_{xx} &= -\tfrac{k_0 Z_0}{4}(1+\tfrac{1}{k_0^2}\tfrac{\partial^2}{\partial x^2})[H_0^{(1)}(k_0\sqrt{(x-x')^2+(y-y')^2})-Q_H] \\
G_{xy} &= -\tfrac{Z_0}{4k_0}\tfrac{\partial^2}{\partial x \partial y}[H_0^{(1)}(k_0\sqrt{(x-x')^2+(y-y')^2})+Q_H] \\
G_{yx} &= -\tfrac{Z_0}{4k_0}\tfrac{\partial^2}{\partial y \partial x}[H_0^{(1)}(k_0\sqrt{(x-x')^2+(y-y')^2})-Q_H] \\
G_{yy} &= -\tfrac{k_0 Z_0}{4}(1+\tfrac{1}{k_0^2}\tfrac{\partial^2}{\partial y^2})[H_0^{(1)}(k_0\sqrt{(x-x')^2+(y-y')^2})+Q_H] \\
G_{zz} &= -\tfrac{k_0 Z_0}{4}[H_0^{(1)}(k_0\sqrt{(x-x')^2+(y-y')^2})+Q_E]
\end{aligned} \tag{3}$$

and

$$Q_{\substack{H\\E}}(x,y;x',y') = \frac{1}{\pi}\int_{-\infty}^{+\infty} R_{\substack{H\\E}}(\gamma)\frac{e^{ik_y(y+y')-ik_x(x-x')}}{k_y}\mathrm{d}k_x],$$

Consider an inhomogeneous dielectric layer of thickness t on top of a stratified half-space dielectric medium as shown in Fig. 1. The permittivity of the inhomogeneous layer is represented by $\epsilon(x,y)$ which is a periodic function of x with period L. Suppose this structure is illuminated by a plane wave whose angle of incidence is ϕ_0. A polarization current distribution is induced in the inhomogeneous layer. The polarization current is proportional to the total field within the inhomogeneous layer. The total field is comprised of the incident field $\mathbf{E}^i$, the reflected field $\mathbf{E}^r$ which would have existed in the absence of the inhomogeneous layer, and the scattered field. Therefore a set of integral equations for polarization currents can be obtained. Since there is no closed-form representation for the kernel of these integral equations, finding the solution, even numerically, seems impossible. But by employing Floquet's theorem the integral equations can be reduced to a form which is amenable to numerical solution. The fact that the permittivity of the inhomogeneous layer is periodic in x, excluding a phase factor, all the field quantities are required to be periodic in x. Now by breaking the integral into multiples of a period, using appropriate change of variable, and employing the Poisson summation formula the elements of the periodic dyadic Green's function($\mathbf{G}^p(x,y;x',y')$) are found to be

$$\begin{aligned}
G^p_{xx}(x,y;x',y') &= (1+\tfrac{1}{k_0^2}\tfrac{\partial^2}{\partial x^2})P^-_H(x,y;x',y'), \\
G^p_{xy}(x,y;x',y') &= \tfrac{1}{k_0^2}\tfrac{\partial^2}{\partial x\partial y}P^+_H(x,y;x',y'), \\
G^p_{yx}(x,y;x',y') &= \tfrac{1}{k_0^2}\tfrac{\partial^2}{\partial y\partial x}P^-_H(x,y;x',y'), \\
G^p_{yy}(x,y;x',y') &= (1+\tfrac{1}{k_0^2}\tfrac{\partial^2}{\partial y^2})P^+_H(x,y;x',y'), \\
G^p_{zz}(x,y;x',y') &= P^+_E(x,y;x',y')
\end{aligned} \tag{4}$$

where

$$P^{\pm}_{\substack{H\\E}}(x,y;x',y') = -\frac{k_0 Z_0}{2L}\sum_{n=-\infty}^{+\infty}[e^{ik_{ny}|y-y'|}\pm R_{\substack{H\\E}}(\gamma_n)e^{ik_{ny}(y+y')}]\frac{e^{-ik_{nx}(x-x')}}{k_{ny}}$$

$$k_{nx} = \frac{2\pi n}{L} - k_0\sin\phi_0, \quad k_{ny} = \sqrt{k_0^2 - k_{nx}^2}\ , \quad \gamma_n = \arctan(\frac{k_{nx}}{k_{ny}}).$$

The integral equations for polarization currents now take the following form

$$\mathbf{J}(x,y) = -ik_0Y_0(\epsilon(x,y)-1) + \int_0^t\int_{-L/2}^{+L/2}\mathbf{G}^p\cdot\mathbf{J}(x',y')\mathrm{d}x'\mathrm{d}y' \tag{5}$$

Far away from the surface ($y \gg \lambda_0$), contribution of only a few terms of the summations in (4) are observable. These terms correspond to values of n such that k_{ny} is real and they are known as the Bragg modes. Solution to these equations can be obtained using the standard moment method with point matching technique.

High Frequency Scattering from Stratified Cylinders

For dielectric cylinders with large radii of curvature, physical optics may be used to obtain the scattered field provided the dielectric has sufficient loss to prevent significant penetration through the cylinder. The dielectric loss also suppresses the effects of creeping waves which enhances the physical optics results. If the dielectric cylinder is stratified, the physical optics approximation could still be used if the radius of curvature of all the interface contours are much larger than the wavelength. New physical optics surface currents [Sarabandi et al 1990] that are more convenient to use than the standard ones [Beckmann 1968] are examined. These currents can be obtained by noting that the reflected plane wave from a dielectric interface can be generated by equivalent electric and magnetic current sheets. Suppose the incident field is given by

$$\mathbf{E}^i = \mathbf{E}_0 e^{ik_0\hat{k}_i\cdot\mathbf{r}}\ , \qquad \mathbf{H}^i = \mathbf{H}_0 e^{ik_0\hat{k}_i\cdot\mathbf{r}}$$

and the normal to the cylinder surface is represented by the unit vector $\hat{n}$. The unit vector normal to the plane of incidence is

$$\hat{t} = \frac{\hat{n}\times\hat{k}_i}{|\ \hat{n}\times\hat{k}_i\ |},$$

in terms of which the new physical optics electric and magnetic currents are given by

$$\mathbf{J}_e = -2Y_0(\mathbf{E}_0\cdot\hat{t})\cos\phi_i R_E(\phi_i)e^{ik_0\hat{k}_i\cdot\mathbf{r}}\hat{t} \tag{6}$$

$$\mathbf{J}_m = -2Z_0(\mathbf{H}_0\cdot\hat{t})\cos\phi_i R_H(\phi_i)e^{ik_0\hat{k}_i\cdot\mathbf{r}}\hat{t} \tag{7}$$

where $\phi_i = \arccos(-\hat{n}\cdot\hat{k}_i)$ is the local angle of incidence.

Suppose a stratified cylinder with arbitrary cross section is illuminated by a plane wave travelling in $-x$ direction ($\hat{k}_i = -\hat{x}$). The outer surface of the cylinder is described by a smooth function $\rho(\phi)$. For E-polarized wave ($\mathbf{E}_0 = \hat{z}$) only electric current and for H-polarized wave ($\mathbf{H}_0 = Y_0\hat{z}$) only magnetic current is induced on the cylinder surface as given by (6) and (7). It can easily be shown that in the far zone of the cylinder in a direction denoted by ϕ_s, the far field amplitudes for E and H polarization respectively are given by

$$\mathbf{S}_E = \hat{z}\cos(\frac{\phi_s}{2})R_E(\frac{\phi_s}{2})\frac{\rho(\phi_{SP})}{\cos(\phi_{SP}-\phi_s/2)}\sqrt{\frac{k_0\pi}{2\,|\,g\,|}} \cdot e^{-ik_0\rho(\phi_{SP})(\cos\phi_{SP}+\cos(\phi_{SP}-\phi_s))}\cdot e^{-i\mathrm{sgn}(g)\frac{\pi}{4}}\ , \tag{8}$$

$$\mathbf{S}_H = \hat{\phi}\cos(\frac{\phi_s}{2})R_H(\frac{\phi_s}{2})\frac{\rho(\phi_{SP})}{\cos(\phi_{SP}-\phi_s/2)}\sqrt{\frac{k_0\pi}{2\,|\,g\,|}} \cdot e^{-ik_0\rho(\phi_{SP})(\cos\phi_{SP}+\cos(\phi_{SP}-\phi_s))}\cdot e^{-i\mathrm{sgn}(g)\frac{\pi}{4}}\ , \tag{9}$$

where ϕ_{SP} is the stationary point on the cylinder and

$$g = \frac{\mathrm{d}^2}{\mathrm{d}\phi'^2}[\rho(\phi')\,(\cos\phi' + \cos(\phi'-\phi_s))]_{\phi'=\phi_{SP}}$$

To verify the validity of the physical optics expressions let us consider a two-layer cylinder with inner and outer radii of $a_1 = 10cm$ and $a = 10.5cm$ respectively. The dielectric constant of the inner and outer layers respectively are $15+i7$ and $4+i1$. These values are so chosen to simulate a tree with smooth bark. Figure

2 compares the normalized backscattering cross section ($\sigma/\pi a$) of the cylinder for E polarization using physical optics expressions and exact series solution. In these figures the cross section of the cylinder in absence of the outer layer (bark) is also plotted to demonstrate the effect of the bark on reducing the cross section of the cylinder. For frequencies above 2 GHz ($k_0 a = 4.2$) the agreement between the two solution is excellent. The bark layer plays the role of an impedance transformer which reduces the cross section of the cylinder by 14 dB around $k_0 a = 16$.

Scattering from Corrugated Cylinder

Consider a corrugated dielectric cylinder with arbitrary cross section as shown in Fig. 3. Assume the corrugation geometry is such that the humps are identical and of equal distance L from each other. If the radius of curvature at each point is much larger than the wavelength and L, each point on the cylinder surface can be replaced, approximately, by a periodic corrugated surface. The accuracy of this approximation is in the order of physical optics approximation for smooth cylinders.

Let us denote the tangential coordinate at the center of each hump by (x', y') where $\hat{y}'$ coincides with the outward normal unit vector ($\hat{n}(\phi)$). If the origin of the prime coordinate system corresponding to the m^{th} hump is located at (ρ_m, ϕ_m) we have

$$\phi_m = \sum_{\ell=1}^{m} \Delta\phi_\ell - \frac{\pi}{2}, \qquad \Delta\phi_{\ell+1} = \frac{L}{\sqrt{\rho^2(\phi_\ell) + \rho'^2(\phi_\ell)}},$$

where $\Delta\phi_1$ is a known quantity. The local incidence angle at the m^{th} hump is $\phi_m^i = \arccos(\hat{n}(\phi_m) \cdot \hat{x})$ and the induced current in the m^{th} hump can be approximated by that of the periodic corrugated surface when the incidence angle is ϕ_m^i. The scattering direction is denoted by ϕ_s as before and the scattering direction for the m^{th} local coordinate is given by $\phi_m^s = \phi_s - \phi_m^i$. The far field due to the m^{th} hump ($\mathbf{S}_m$), depending on the polarization, can be obtained using (2) and we note that those humps with $|\phi_m^s| > \pi/2$ do not contribute to the far field. The total contribution of the cylinder corrugation to the far field is the vector sum of the fields due to each hump modified by a phase factor to correct for the relative positions of the humps. Therefore

$$\mathbf{S}_c = \sum_m \mathbf{S}_m e^{-ik_0 x_m} e^{-ik_0(x_m \cos\phi_s + y_m \sin\phi_s)}, \qquad (10)$$

where $x_m = \rho_m \cos\phi_m$, $y_m = \rho_m \sin\phi_m$. The total scattered field may now be obtained from

$$\mathbf{S} = \mathbf{S}_c + \mathbf{S}_s,$$

where $\mathbf{S}_s$ is the far field amplitude of the smooth cylinder.

Numerical Results

To examine the effect of surface corrugation on scattering from corrugated cylinders, we consider a two-layer circular cylinder with uniform corrugation. The pertinent parameters are chosen as follows: each hump is a $\lambda_0/8 \times \lambda_0/8$ square with dielectric constant $\epsilon_1 = 4+i1$, the distance between humps is $L = \lambda_0/4$, the thickness and dielectric constant of outer layer are $\lambda_0/2$ and $4+i1$ respectively, and the radius and the dielectric constant of inner layer are $10\lambda_0$ and $15 + i7$ respectively. For the corresponding periodic surface all the components of the induced current in each hump are obtained by the moment method. Once the induced current versus angle is obtained the bistatic scattered field can be computed from (10). Figures 4 and 5 show the radar cross section due to the corrugation (σ_c) and smooth cylinder (σ_s) and the total radar cross section ($\sigma_c + \sigma_s$). To examine the role of the outer layer, the radar cross section of the cylinder (1-layer) when the outer layer is removed is also plotted. It is seen that the smooth bark reduces the scattered field by 3 dB and the corrugation on the bark further reduces the scattered field by another 8 dB. In Fig. 6 the radar cross section of the corrugated cylinder, for both polarizations, are compared with a smooth cylinder when corrugation is replaced by an equivalent uniaxial dielectric layer [Sarabandi 1990]. The thickness and dielectric tensor elements respectively are $t = \lambda_0/8$, $\epsilon_y = \epsilon_z = 2.6 + i0.58$, and $\epsilon_x = 1.81 + i0.78$. Excellent agreement is obtained.

Conclusions

A hybrid solution based on the moment method and physical optics approximation is obtained for corrugated layered cylinders. The only restriction on the physical dimensions is the radius of curvature (r) of the cylinder where we require $r \gg \lambda_0$. Also new physical optics expressions for the equivalent surface current on the dielectric structure is introduced. Also it is shown that when period of the corrugation is smaller than the half a wavelength the corrugation can be modeled by an uniaxial dielectric layer which extremely simplifies the problem.

This method is employed to investigate the effect of bark and its roughness on the scattering from tree trunks and branches. It is shown that the bark and its roughness both reduce the radar cross section. The low contrast dielectric bark layer manifest its effect more significantly at higher frequencies where the bark thickness and its roughness are a considerable fraction of the wavelength.

References

[1] Beckmann, P., The Depolarization of Electromagnetic Waves, Boulder, Co: The Golem Press, 1968.

[2] Cabayan, H.S., and R.C. Murphy, "Scattering of electromagnetic waves by rough perfectly conducting circular cylinders," *IEEE Trans. Antennas Propag.*, 21, pp. 893-895, 1973.

[3] Clemmow, P.C., and V.H. Weston, "Studies in radar cross section XXXVI- Diffraction of a plane wave by an almost circular cylinder," *Radiation Laboratory Report No. 2871-3-T*, The University of Michigan, Sept. 1959.

[4] Sarabandi, K., "Study of periodic dielectric surfaces: Simulation with anisotropic layers and application to high frequency scattering from corrugated convex bodies.", *Radiation Laboratory Report No. 024557-2-T* ,The University of Michigan, February 1990.

[5] Sarabandi, K., F.T. Ulaby, and T.B.A. Senior, "Millimeter wave scattering model for a leaf", *Radio Sci.*, accepted for publication.

[6] Tong, T.C., "Scattering by a slightly rough cylinder and a cylinder with impedance boundary condition," *Int. J. Electron.*, 36, pp. 767-772, 1974.

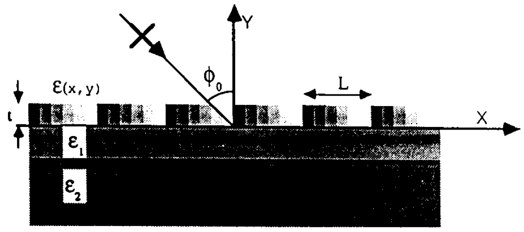

Figure 1: Geometry of a periodic inhomogeneous dielectric layer over a stratified dielectric half-space.

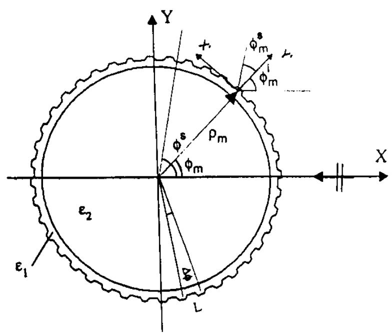

Figure 2: A corrugated cylinder geometry.

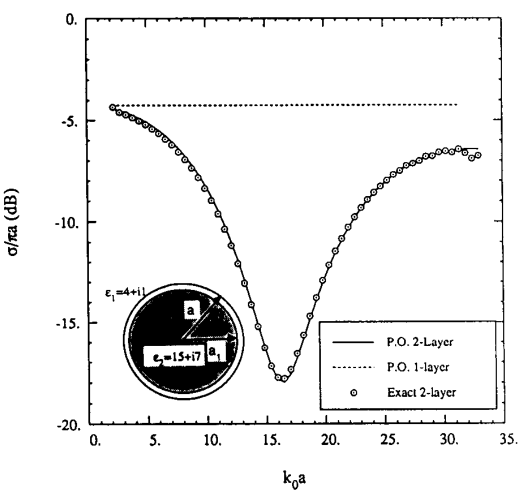

Figure 3: Normalized backscattering cross section ($\frac{\sigma}{\pi a}$) of a two-layer dielectric cylinder with $a = 10.5cm$, $a_1 = 10cm$, $\epsilon_1 = 15 + i7$, $\epsilon_2 = 4 + i1$ versus k_0a for TM case.

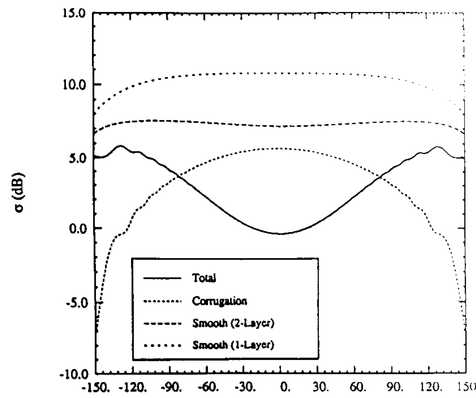

Figure 4: The radar cross section of a corrugated cylinder for TM case with $a = 10.5\lambda_0$, $a_1 = 10\lambda_0$, $L = \lambda_0/4$, $\epsilon_1 = 4 + i1$, $\epsilon_2 = 15 + i7$, and $t = d = \lambda_0/8$.

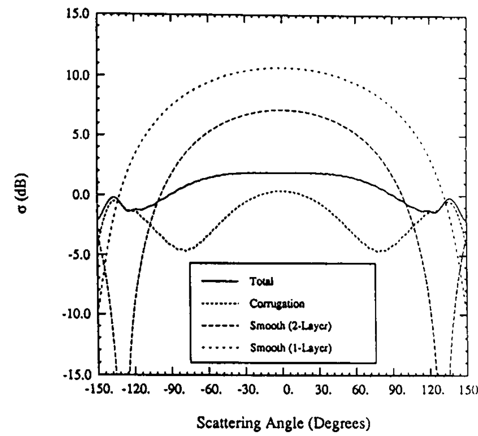

Figure 5: The radar cross section of a corrugated cylinder for TE case with $a = 10.5\lambda_0$, $a_1 = 10\lambda_0$, $L = \lambda_0/4$, $\epsilon_1 = 4 + i1$, $\epsilon_2 = 15 + i7$, and $t = d = \lambda_0/8$.

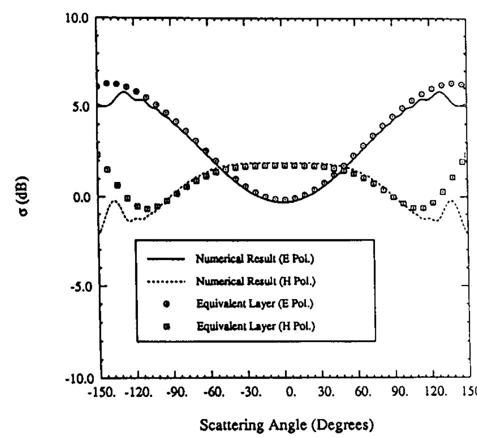

Figure 6: The radar cross section of the corrugated cylinder for TE and TM cases using the numerical and the equivalent dielectric methods.