Propagation in a Two-Dimensional Periodic Random Medium with Inhomogeneous Particle Distribution

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Abstract—**The behavior of electromagnetic waves when propagating in a periodic random medium, such as a row-structured canopy, is considered. The semideterministic character of the particle distributions is represented by nonuniform extinction and phase matrices and the problem is formulated by the radiative transfer equation. Solution of the radiative transfer equation is pursued in two ways: 1) iteratively and 2) by using a new numerical technique based on the discrete–ordinate approximation and Taylor series (DOT) expansion. It is shown that the numerical solution for the periodic canopy is computationally efficient, and also a closed form for the first-order solution (iterative approach) of the radiative transfer equation is obtained for periodic cases. The analytical and numerical results are compared with transmission measurements at L- and C-band frequencies for a corn canopy. Measurements were performed for a variety of canopy conditions such as different densities, number of rows, and stalks only (without foliage). The agreement between measurements and model predictions is very good for all canopy conditions.**

I. Introduction

The problem of propagation in man-made random media usually involves some parameters that are deterministic in character in combination with others that are random in nature. An example of such a problem is a corn field where the plants are arranged in a row-structured fashion as shown in Fig. 1. This canopy structure is statistically periodic in the y direction and homogeneous in planes perpendicular to the periodic direction (i.e., in the x-z plane). Each period of this structure includes a slab of corn plants adjacent to a slab of air. The stalks are planted uniformly along the row direction (along the x axis) with the leaves filling the space around the stalks. The leaves may have a nonuniform distribution along the y direction. The objective of this paper is to characterize the propagation behavior of a plane wave propagating in any direction in the x-y plane and then investigate the beam broadening effect as observed by measurements.

In this paper we assume that there is no variation along the z direction and treat the problem as a two-dimensional (2-D) one. The stalks will be modeled by long circular cylinders and the leaves are modeled by infinitesimally thin dielectric strips.

The standard radiative transfer technique [1], [2] is used to obtain a solution for the propagation matrix of the medium. This technique is based on conservation of energy and single scattering properties of constituent particles in the medium (near-field interaction of particles are ignored). In radiative transfer, the quantity of interest is the specific intensity $\mathbf{I}(r, \hat{s})$, which is defined as the power per unit area and per unit solid angle propagating along $\hat{s}$, and which is a function of position in the random medium (r). In applying transport theory to the electromagnetic problem the specific intensity $\mathbf{I}(r, \hat{s})$ usually is defined by a four-component vector. For a monochromatic elliptically polarized plane wave with electric field vector

$$\mathbf{E} = \left(E_v \hat{v}_i + E_h \hat{h}_i\right) e^{i k_0 r}$$

with unit vertical and horizontal polarization vectors $\hat{v}_i, \hat{h}_i$, $\mathbf{I}$ is defined through the modified stokes parameters:

$$\mathbf{I} = \begin{bmatrix} I_v \\ I_h \\ U \\ V \end{bmatrix} = \frac{1}{\eta} \begin{bmatrix} |E_v|^2 \\ |E_h|^2 \\ 2\,\mathrm{Re}\,(E_v E_h^*) \\ 2\,\mathrm{Im}\,(E_v E_h^*) \end{bmatrix} \tag{1}$$

where η is the intrinsic impedance of the medium. The vector radiative transfer equation, which simply expresses the conservation of energy in a unit volume of the medium, is given by

$$\nabla \cdot (\mathbf{I}(r, \hat{s})\hat{s}) = -\boldsymbol{\kappa}(r, \hat{s})\mathbf{I}(r, \hat{s}) + \int_{\Omega} \mathbf{P}(r, \hat{s}, \hat{s}')\mathbf{I}(r, \hat{s}')\, d\Omega \tag{2}$$

where $\boldsymbol{\kappa}(r, \hat{s})$ and $\mathbf{P}(s, \hat{s}')$ are, respectively, the extinction and phase matrices of the medium. The extinction matrix represents the losses due to absorption and scattering by particles per unit volume and, in general, is a function of position and direction of propagation. The phase matrix accounts for the fraction of the intensity incident upon the unit volume along direction $\hat{s}'$ that is bistatically scattered by the unit volume into direction $\hat{s}$.

Referring to Fig. 1, for a plane wave incidence there would be no variations with respect to variables x and z,

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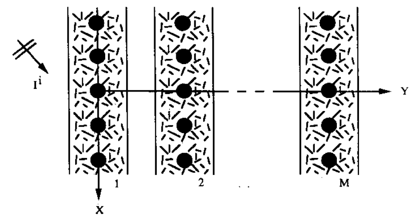


Fig. 1. Top view of a row structured periodic canopy. Each period consists of a row of vegetation next to a slab of air.

and therefore (2) is simplified to

$$\hat{s}\cdot\hat{y}\frac{d\mathbf{I}(y,\hat{s})}{dy} = -\boldsymbol{\kappa}(y,\hat{s})\mathbf{I}(y,\hat{s}) + \int_{\Omega}\mathbf{P}(y,\hat{s},\hat{s}')\mathbf{I}(y,\hat{s}')\,d\Omega. \quad (3)$$

Since scatterers are infinite in the z direction and the incidence direction is normal to $\hat{z}$, then

$$\hat{s} = \cos\phi\hat{x} + \sin\phi\hat{y} \qquad d\Omega = d\phi$$

noting that the intensity is redefined by power per unit length per unit angle. Thus, the radiative transfer equation for this problem takes the following form:

$$\sin\phi\frac{dI(y,\phi)}{dy} = -\boldsymbol{\kappa}(y,\phi)\mathbf{I}(y,\phi) + \int_0^{2\pi}\mathbf{P}(y,\phi,\phi')\mathbf{I}(y,\phi')\,d\phi'. \quad (4)$$

II. Phase and Extinction Matrices for Constituent Particles

In the formulation of the mean field intensity in a 2-D random medium by radiative transfer, the scattered intensity by a unit cross-sectional area of the random medium into direction ϕ_s is related to the intensity incident upon the unit area from direction ϕ_i by the phase matrix $\mathbf{P}(y,\phi_s,\phi_i)$. Also the extinction matrix $\boldsymbol{\kappa}(y,\phi)$ characterizes the loss of intensity due to absorption and scattering by the unit area. To relate the extinction and phase matrices of the medium to the constituent particles, we ignore near-field interactions between particles and only consider single-scattering interactions, which is a reasonable approach for low-density media (the volume fraction of vegetation material in a canopy seldom exceeds 1%).

The field scattered by a 2-D object whose generating axis is parallel to the z direction and is illuminated by a plane wave $\mathbf{E}^i$ is given by [Harrington, 1961]

$$\mathbf{E}^s = \frac{1}{\sqrt{\rho}}e^{ik_0\rho-(i\pi/4)}\mathbf{S}\mathbf{E}^i \quad (5)$$

where S is the scattering matrix of the object. In 2-D problems, transverse electric (TE) and transverse magnetic (TM) fields are decoupled and therefore the scattering matrix becomes diagonal, i.e.,

$$\mathbf{S} = \begin{bmatrix} S_{vv} & 0 \\ 0 & S_{hh} \end{bmatrix}.$$

Using a similar definition for the scattered Stokes vector as given by (1) (modified by $1/\rho$) and using (5), it can be shown that [3]

$$I_s = \frac{1}{\rho}\mathbf{L}I_i$$

where $\mathbf{L}$ is known as the Stokes matrix and is given by

$$\mathbf{L} = \begin{bmatrix} |S_{vv}|^2 & 0 & 0 & 0 \\ 0 & |S_{hh}|^2 & 0 & 0 \\ 0 & 0 & \mathrm{Re}\,[S_{vv}S_{hh}^*] & \mathrm{Im}\,[S_{vv}S_{hh}^*] \\ 0 & 0 & \mathrm{Im}\,[S_{vv}S_{hh}]^* & \mathrm{Re}\,[S_{vv}S_{hh}^*] \end{bmatrix}. \quad (6)$$

The radiative transfer formulation is based on conservation of energy and the Stokes vectors are added incoherently, thus the phase matrix must be obtained from the Stokes matrix averaged over the particle type, size, and orientation-angle distributions. If m types of particles exist in the canopy and each has N_j particles per unit area, then

$$\mathbf{P}(\phi,\phi') = \left\langle \sum_{j=1}^{m} N_j\mathbf{L}_j \right\rangle. \quad (7)$$

The extinction matrix of the canopy can also be obtained by applying the optical theorem for 2-D particles. Using (5) for the scattering matrix, it can be shown that the extinction cross section is related to the forward scattering amplitude [4]

$$\sigma_e^p = -4\sqrt{\frac{\pi}{2k_0}}\,\mathrm{Re}\left[S_{pp}(\phi,\phi)\right] \qquad p = v \text{ or } h.$$

Because no cross coupling occurs between v and h polarized waves propagating in 2-D media, the extinction matrix can be shown to have the following form (see [5]):

$$\kappa = \begin{bmatrix} \sum_{j=1}^{m} N_j\langle\sigma_e^v\rangle & 0 & 0 & 0 \\ 0 & \sum_{j=1}^{m} N_j\langle\sigma_e^h\rangle & 0 & 0 \\ 0 & 0 & \frac{1}{2}\sum_{j=1}^{m} N_j\langle\sigma_e^v+\sigma_e^h\rangle & \kappa_{34} \\ 0 & 0 & \kappa_{43} & \frac{1}{2}\sum_{j=1}^{m} N_j\langle\sigma_e^v+\sigma_e^h\rangle \end{bmatrix} \quad (8)$$

where

$$\kappa_{34} = -\kappa_{43} = -2\sqrt{\frac{\pi}{2k_0}} \sum_{j=1}^{m} N_j \langle \mathrm{Im}\,[S_{vv}(\phi, \phi) - S_{hh}(\phi, \phi)] \rangle.$$

From (6) and (8) it is obvious that the first two components of the Stokes vectors are decoupled from each other and from the other Stokes components. Hence the vector radiative transfer equation reduces to uncoupled scalar equations for I_v and I_h. Since the total energy is carried by the first two components only, we just consider the solution to scalar radiative transfer equation for these components. Before deriving the extinction and phase functions for particles of particular shape and orientation distributions, it is worth noting that the scattering matrix (for 2-D particles) is a function of ϕ_s, ϕ_i (where ϕ_s and ϕ_i are the incident and scattered azimuth angles) and the particle orientation ϕ_p. In the case of uniform particle distributions, the extinction coefficient of the medium becomes independent of the incidence and scattering angles, and the phase function becomes a function of $\phi_s - \phi_i$ only.

The geometry of interest for constituent particles of a corn canopy are infinitely long, vertically oriented, homogeneous cylinders representing stalks and infinitesimally thin dielectric strips representing the leaves.

A. Extinction and Phase Function for Stalks

The cylinder is one of the few geometries for which an exact electromagnetic scattering solution exists. If a plane wave propagating in a direction denoted by ϕ_i is illuminating an infinitely long dielectric cylinder with radius r_o whose axis coincides with the z axis, it can be shown that the bistatic scattering matrix elements can be obtained from [6]

$$S_{vv} = \sum_{n=0}^{\infty} C_n^E \cos n(\phi_s - \phi_i)$$
$$S_{hh} = \sum_{n=0}^{\infty} C_n^H \cos n(\phi_s - \phi_i) \qquad (9)$$

where ϕ_s denotes the scattering direction and

$$C_n^E = \sqrt{\frac{\pi k_0}{2}}(-1)^{(n+1)}\alpha_n \cdot \frac{J_n(x_0)J_n'(x_1) - \sqrt{\epsilon}J_n'(x_0)J_n(x_1)}{H_n^{(1)}(x_0)J_n'(x_1) - \sqrt{\epsilon}H_n^{(1)\prime}(x_0)J_n(x_1)}$$

$$C_n^H = \sqrt{\frac{\pi k_0}{2}}(-1)^{(n+1)}\alpha_n \cdot \frac{\sqrt{\epsilon}J_n(x_0)J_n'(x_1) - J_n'(x_0)J_n(x_1)}{\sqrt{\epsilon}H_n^{(1)}(x_0)J_n'(x_1) - H_n^{(1)\prime}(x_0)J_n(x_1)}.$$

In (9), ϵ is the relative dielectric constant of the cylinders,

$$x_0 = k_0 r_0, \quad x_1 = k_0\sqrt{\epsilon}\, r_0$$

and

$$\alpha_n = \begin{cases} 1, & n = 0 \\ 2, & \text{otherwise} \end{cases}.$$

Since the cylinders are vertically oriented and azimuthally symmetric, evaluation of the extinction and phase function does not involve particle orientation averaging. Using (9) in (6) and (8) gives the phase and extinction functions.

B. Extinction and Phase Functions for Leaves

Leaves in this model are considered as long thin dielectric strips. The thickness and dielectric constant of leaves are usually such that they can be modeled as resistive sheets at centimeter wavelengths [7], [9]. At high frequencies, where the width of a leaf is large compared to the wavelength, the physical optics approximation can be used to find the scattered field. Otherwise, numerical techniques such as the method of moments should be employed instead. Consider a resistive strip of width w illuminated by a plane wave as shown in Fig. 2 where the orientation and incidence angles, measured from the x axis, are denoted by ϕ_l and ϕ_i, respectively. Depending on the polarization, the incident field is assumed to have the following form

$$\mathbf{E}^{\mathrm{i}} = \hat{z}e^{ik_0(x\cos\phi_i + y\sin\phi_i)} \quad E\text{-polarization}$$
$$\mathbf{H}^{\mathrm{i}} = Y_0\hat{z}e^{ik_0(x\cos\phi_i + y\sin\phi_i)} \quad H\text{-polarization}.$$

Following the procedure outlined by Senior et al. [7] the physical optics currents for E- and H-polarized incident waves are given by

$$J_z = \Gamma^E\left(\frac{\pi}{2} - |\phi_i - \phi_l|\right)J_z^{\mathrm{PC}}$$
$$J_{x'} = \Gamma^H\left(\frac{\pi}{2} - |\phi_i - \phi_l|\right)J_{x'}^{\mathrm{PC}} \qquad (10)$$

Where J^{PC} refers to the physical optics current that would exist on the surface of the resistive sheet had it been a perfect conductor. Γ^E and Γ^H are the reflection coefficients for the resistive sheet for horizontal and vertical polarization and are given by

$$\Gamma^E\left(\frac{\pi}{2} - |\phi_i - \phi_l|\right) = \left[1 + \frac{2R}{Z_0}\cos\left(\frac{\pi}{2} - |\phi_i - \phi_l|\right)\right]^{-1}$$

$$\Gamma^H\left(\frac{\pi}{2} - |\phi_i - \phi_l|\right) = -\left[1 + \frac{2R}{Z_0}\sec\left(\frac{\pi}{2} - |\phi_i - \phi_l|\right)\right]^{-1}. \qquad (11)$$

In (11) Z_0 is the free-space characteristic impedance and R is the resistivity of the leaf given by

$$R = \frac{iZ_0}{k_0\tau(\epsilon - 1)}$$

where τ and ϵ are the thickness and dielectric constant of the leaf. Noting that

$$J_z^{\mathrm{PC}} = -2Y_0\sin(|\phi_i - \phi_l|)e^{ik_0\cos(\phi_i - \phi_l)x'} \quad E\text{-polarization}$$
$$J_{x'}^{\mathrm{PC}} = 2Y_0 e^{ik_0\cos(\phi_i - \phi_l)x'} \quad H\text{-polarization}$$

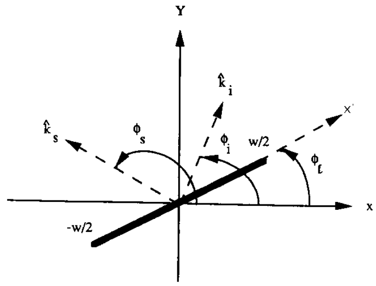


Fig. 2. Geometry of a resistive strip depicting the orientation, incidence, and scattered angles.

and using the far-field approximation, we can show that

$$E_z^s = \sqrt{\frac{k_0}{2\pi}}\,\frac{e^{ik_0\rho - i(\pi/4)}}{\sqrt{\rho}}\,\frac{\sin(|\phi_i - \phi_l|)}{\left(1 + \dfrac{2R}{Z_0}\sin(|\phi_i - \phi_l|)\right)} \cdot \left\{\frac{w\sin[k_0 w/2(\cos|\phi_i - \phi_l| - \cos|\phi_s - \phi_l|)]}{k_0 w/2(\cos|\phi_i - \phi_l| - \cos|\phi_s - \phi_l|)}\right\}$$

$$H_z^s = Y_0\sqrt{\frac{k_0}{2\pi}}\,\frac{e^{ik_0\rho - i(\pi/4)}}{\sqrt{\rho}}\,\frac{\sin(|\phi_s - \phi_l|)}{\left(1 + \dfrac{2R}{Z_0}\csc(|\phi_i - \phi_l|)\right)} \cdot \left\{\frac{w\sin[k_0 w/2(\cos|\phi_i - \phi_l| - \cos|\phi_s - \phi_l|)]}{k_0 w/2(\cos|\phi_i - \phi_l| - \cos|\phi_s - \phi_l|)}\right\}.$$

Hence,

$$\mathbf{S} = \sqrt{\frac{k_0}{2\pi}}\,w\begin{bmatrix} \dfrac{\sin|\phi_i - \phi_l|}{1 + \dfrac{2R}{Z_0}\sin|\phi_i - \phi_l|}\cdot\dfrac{\sin X}{X} & 0 \\ 0 & \dfrac{\sin|\phi_s - \phi_l|}{1 + \dfrac{2R}{Z_0}\csc|\phi_i - \phi_l|}\cdot\dfrac{\sin X}{X} \end{bmatrix} \tag{12}$$

with

$$X = \frac{k_0 w}{2}[\cos(\phi_s - \phi_l) - \cos(\phi_i - \phi_l)].$$

The averaging over the orientation angle ϕ_l involved in the derivations of the extinction and phase matrices cannot be evaluated analytically, hence, the integrals should be calculated numerically. The physical optics approximation is valid as long as the width of the strips is large compared to the wavelength, and if the material is lossy, this approximation provides reasonable results for values of w as small as λ_0.

For cases where the width of the strips is not large compared to the wavelength, numerical techniques, such as the method of moments, may be used to find the scattered field. However, use of such computational techniques to compute the phase and the extinction matrices from the scattered field would be prohibitive in terms of computational time. In what follows, we demonstrate an efficient procedure for numerical computation of the extinction and phase matrices. The integral equations for the induced currents on the resistive sheet, as shown in Fig. 2, for TM and TE cases, respectively, given by [8]:

$$RJ_z(x') = e^{ik_0\cos(\phi_i - \phi_l)x'} - \frac{k_0 Z_0}{4} \cdot \int_{-w/2}^{w/2} J_z(\zeta) H_0^{(1)}(k_0|x' - \zeta|)\,d\zeta$$

$$RJ_{x'}(x') = -\sin(|\phi_i - \phi_l|)e^{ik_0\cos(\phi_i - \phi_l)x'} - \frac{k_0 Z_0}{4}\int_{-w/2}^{w/2} J_{x'}(\zeta)\left(1 + \frac{1}{k_0^2}\frac{\partial^2}{\partial x'^2}\right)H_0^{(1)}(k_0|x' - \zeta|)\,d\zeta \tag{13}$$

where $H_0^{(1)}$ is the Hankel function of the first kind and zeroth order. These integral equations can be cast into a linear system of equation by applying the method of moments and point matching technique. By solving the linear system of equations an approximate solution for the currents can be obtained. Having found the induced currents, the scattered fields can be obtained from

$$E_z^s = -Z_0\sqrt{\frac{k_0}{8\pi}}\,\frac{e^{ik_0\rho - (i\pi/4)}}{\sqrt{\rho}}\int_{-w/2}^{w/2} J_z(\zeta)e^{-ik_0\zeta\cos(\phi_s - \phi_l)}\,d\zeta$$

$$H_z^s = \sqrt{\frac{k_0}{8\pi}}\sin|\phi_s - \phi_l|\int_{-w/2}^{w/2} J_{x'}(\zeta)e^{-ik_0\zeta\cos(\phi_s - \phi_l)}\,d\zeta.$$

The approximate form of the scattering matrix assuming that the strip is divided into M small sections takes the following form:

$$\mathbf{S} = -Z_0\sqrt{\frac{k_0}{8\pi}}\begin{bmatrix} \sum\limits_{m=1}^{M} J_m^z e^{ik_0 x_m\cos(\phi_s - \phi_l)} & 0 \\ 0 & \sin|\phi_s - \phi_l|\sum\limits_{m=1}^{M} J_m^{x'}(x_m)e^{ik_0 x_m\cos(\phi_s - \phi_l)} \end{bmatrix} \tag{14}$$

where x_m is the coordinate of the mth cell in the x' coordinate system and J_m is the value of the current for the mth cell. The matrix approximation of (13) can be written as

$$\mathbf{Z}_E \mathbf{J}_z = \mathbf{V}_E \quad \mathbf{Z}_H \mathbf{J}_{x'} = \mathbf{V}_H \tag{15}$$

where $\mathbf{Z}_{E,H}$ are the impedance matrices and $\mathbf{V}_E$ and $\mathbf{V}_H$ are the excitation vectors whose elements are

$$V_m^E = e^{ik_0 \cos(\phi_i - \phi_l) x_m},$$

$$V_m^H = -\sin|\phi_i - \phi_l| e^{ik_0 \cos(\phi_i - \phi_l) x_m}. \tag{16}$$

From (15) and (16) we have

$$J_m^z = \sum_{l=1}^{M} [\mathbf{Z}_E^{-1}]_{ml} e^{ik_0 \cos(\phi_i - \phi_l) x_l}$$

$$J_m^{x'} = -\sum_{l=1}^{M} [\mathbf{Z}_H^{-1}]_{ml} \sin|\phi_i - \phi_l| e^{ik_0 \cos(\phi_i - \phi_l) x_l}$$

and

$$S_{vv}(\phi_i, \phi_s, \phi_l) = -Z_0 \sqrt{\frac{k_0}{8\pi}} \sum_{m=1}^{M} \sum_{l=1}^{M} [\mathbf{Z}_E^{-1}]_{ml} \times e^{ik_0[\cos(\phi_i - \phi_l)x_l - \cos(\phi_s - \phi_l)x_m]}$$

$$S_{hh}(\phi_i, \phi_s, \phi_l) = -Z_0 \sqrt{\frac{k_0}{8\pi}} \sum_{m=1}^{M} \sum_{l=1}^{M} [\mathbf{Z}_H^{-1}]_{ml} \sin|\phi_s - \phi_l| \sin|\phi_i - \phi_l| e^{ik_0[\cos(\phi_i - \phi_l)x_l - \cos(\phi_s - \phi_l)x_m]}.$$

Suppose the orientation angle distribution of the leaves is uniform in interval $(0, 2\pi)$, then the ensemble average of S_{vv} and S_{hh} in the forward direction ($\phi_s = \phi_i$) can be obtained from

$$\langle S_{vv} \rangle = -Z_0 \sqrt{\frac{k_0}{8\pi}} \sum_{m=1}^{M} \sum_{l=1}^{M} [\mathbf{Z}_E^{-1}]_{ml} J_0(k_0|x_l - x_m|)$$

$$\langle S_{hh} \rangle = -Z_0 \sqrt{\frac{k_0}{8\pi}} \sum_{m=1}^{M} \sum_{l=1}^{M} [\mathbf{Z}_H^{-1}]_{ml} \cdot \left[\frac{1}{2} J_0(k_0|x_l - x_m|) \pm \frac{1}{2} J_2(k_0|x_l - x_m|)\right] \tag{17}$$

where J_0 and J_2 are the Bessel function of the zeroth and second order, respectively, and the plus or minus sign must be used according to the sign of $x_l - x_m$. Therefore the extinction coefficient of leaves for vertical and horizontal polarization, respectively, are

$$\kappa_L^v = Z_0 N_L \sum_{m=1}^{M} \sum_{l=1}^{M} \mathrm{Re}\left[[\mathbf{Z}_E^{-1}]_{ml}\right] J_0(k_0|x_l - x_m|)$$

$$\kappa_L^h = \frac{Z_0 N_L}{2} \sum_{m=1}^{M} \sum_{l=1}^{M} \mathrm{Re}\left[[\mathbf{Z}_H^{-1}]_{ml}\right] \cdot [J_0(k_0|x_l - x_m|) \pm J_2(k_0|x_l - x_m|)] \tag{18}$$

where N_L is the number of the leaves per unit area. Inspection of (18) reveals that the extinction coefficients are independent of the incidence angle.

For evaluation of the phase matrix we note that

$$\langle |S_{vv}|^2 \rangle = Z_0^2 \frac{k_0}{8\pi} \sum_m \sum_l \sum_n \sum_j [\mathbf{Z}_E^{-1}]_{ml} [\mathbf{Z}_E^{-1}]_{nj}^* \cdot \frac{1}{2\pi} \int_0^{2\pi} e^{ik_0[\cos(\phi_i - \phi_l)(x_l - x_n) - \cos(\phi_s - \phi_l)(x_m - x_j)]} \, d\phi_l.$$

Noting that the integrand is a periodic function of ϕ_l we have

$$\frac{1}{2\pi} \int_0^{2\pi} e^{ik_0[\cos(\phi_i - \phi_l)(x_l - x_n) - \cos(\phi_s - \phi_l)(x_m - x_j)]} \, d\phi_l = \frac{1}{2\pi} \int_0^{2\pi} e^{ik_0 q \cos(\phi_l - \gamma)} \, d\phi_l$$

where

$$q = \sqrt{(x_l - x_n)^2 + (x_m - x_j)^2 - 2(x_l - x_n)(x_m - x_j)\cos(\phi_i - \phi_s)}$$

$$\gamma = \tan^{-1} \frac{(x_l - x_n)\sin(\phi_s - \phi_l)}{(x_l - x_n)\cos(\phi_s - \phi_i) - (x_m - x_j)}.$$

After some algebraic manipulation we can show that

$$P_{vv} = N_L Z_0^2 \frac{k_0}{8\pi} \sum_m \sum_l \sum_n \sum_j [\mathbf{Z}_E^{-1}]_{ml} [\mathbf{Z}_E^{-1}]_{nj}^* J_0(k_0 q) \tag{19}$$

$$\begin{aligned} P_{hh} = N_L Z_0^2 \frac{k_0}{32\pi} \sum_m \sum_l \sum_n \sum_j [\mathbf{Z}_H^{-1}]_{ml} [\mathbf{Z}_H^{-1}]_{nj}^* &\cdot [1/2 \cos 2(\phi_s - \phi_i) J_0(k_0 q) \\ &+ 2\cos(\phi_s - \phi_i)\cos(2\gamma - (\phi_s - \phi_i)) J_2(k_0 q) \\ &+ \cos(4\gamma - 2(\phi_s - \phi_i)) J_4(k_0 q)]. \end{aligned} \tag{20}$$

III. Solution of the Radiative Transfer Equation

The radiative transfer equation is an integro-differential equation with nonhomogeneous boundary conditions. If we assume that the air-canopy boundaries are diffuse, there would be no reflections at boundaries and the intensity would be continuous across each boundary. Also the intensity should approach zero as y approaches infinity. Solution to this equation in analytical form, is very difficult, if not impossible. However, under certain conditions, approximate solutions or efficient numerical solutions are attainable. Here we present two approaches to this problem: 1) an iterative solution and 2) a numerical solution.

A. Iterative Solution

For a canopy condition where extinction is dominated by absorption, that is the albedo $\alpha = \sigma_s/\sigma^{ext} \ll 1$, the iterative method is the standard approach used to solve the radiative transfer equations (RTE). In this approach, first the contribution of the phase function to the intensity is ignored (i.e., $P(\phi, \phi') = 0$) so the equation reduces to a first-order homogeneous ordinary differential equation that can be solved easily. Then the solution of this equation (the zeroth-order solution) will be substituted back into the RTE to obtain the first-order solution. By continuing this process, solutions to any desired order can be obtained in principle. To set the formulation in a form amenable for boundary conditions, common practice is to separate the intensity in the random medium into two functions $I^+(y, \phi)$ and $I^-(y, \phi)$ corresponding to positive- and negative-going intensities, respectively. In this form ϕ ranges from 0 to π, and the RTE becomes

$$\sin\phi \frac{dI^+(y,\phi)}{dy} = -\kappa(y)I^+(y,\phi) + \int_0^\pi [P(y,\phi,\phi')I^+(y,\phi') + P(y,\phi,\phi'+\pi)I^-(y,\phi'+\pi)]\,d\phi'$$

$$-\sin\phi \frac{dI^-(y,\phi)}{dy} = -\kappa(y)I^-(y,\phi) + \int_0^\pi [P(y,\phi+\pi,\phi')I^+(y,\phi') + P(y,\phi+\pi,\phi'+\pi)I^-(y,\phi'+\pi)]\,d\phi'. \tag{21}$$

But $P(\phi, \phi') = P(\phi - \phi')$ which is a periodic function with period 2π, i.e., $P(\alpha) = P(\alpha + 2\pi)$, thus

$$P(\phi+\pi, \phi'+\pi) = P(\phi-\phi') \triangleq P^+(\phi-\phi'), \quad 0 \le \phi, \phi' \le \pi$$

$$P(\phi+\pi, \phi') = P(\phi, \phi'+\pi) = P(\pi+\phi-\phi') \triangleq P^-(\phi-\phi'), \quad 0 \le \phi, \phi' \le \pi.$$

To get the zeroth-order solution, $P(\phi - \phi')$ must be set to zero, then

$$\sin\phi \frac{dI_0^+(y,\phi)}{dy} = -\kappa(y)I_0^+(y,\phi)$$

$$-\sin\phi \frac{dI_0^-(y,\phi)}{dy} = -\kappa(y)I_0^-(y,\phi). \tag{22}$$

If the incident wave is a plane wave in direction ϕ_i, then

$$I^+(0,\phi) = I^i\delta(\phi-\phi_i) \tag{23}$$

and the solution of (22) is given by

$$I_0^+(y,\phi) = I^i e^{-\csc\phi_i \tau(y)}\delta(\phi-\phi_i), \qquad I_0^-(y,\phi) = 0 \tag{24}$$

where

$$\tau(y) = \int_0^y \kappa(\zeta)\,d\zeta.$$

Now (24) can be substituted back into (21) to obtain the first-order solution which is given by

$$I_1^+(y,\phi) = \left\{ e^{-\csc\phi_i\tau(y)}\delta(\phi-\phi_i) + \csc\phi\, e^{-\csc\phi\tau(y)} \int_0^y P^+(y,\phi-\phi_i) e^{(\csc\phi - \csc\phi_i)\tau(y')}\,dy' \right\} I^i \tag{25}$$

Also the solution for $I_1^-(y, \phi)$ can be obtained by noting that $I_1^-(Y_0, \phi) = 0$, where Y_0 is a distance after which there is no canopy, thus

$$I_1^-(y,\phi) = \left\{ \csc\phi\, e^{\csc\phi\tau(y)} \int_y^{Y_0} e^{(\csc\phi-\csc\phi_i)\tau(y')} \cdot P^+(y',\phi,\phi_i)\,dy' \right\} I^i. \tag{26}$$

Equations (25) and (26) can be simplified by noting that $\kappa(y)$ and $P(y, \phi - \phi')$ are periodic in y. Let us define the integral over one period L of $\kappa(\zeta)$ by T, i.e.,

$$T = \int_0^L \kappa(\zeta)\,d\zeta. \tag{27}$$

Also assume that the observation point is in the $(N + 1)$th row, then by subdividing the integral in (25) into summation of integrals over multiples of a period, and noting the fact that

$$\int_0^{nL} \kappa(\zeta)\,d\zeta = nT,$$

$$P^\pm(y+nL, \phi-\phi') = P^\pm(y, \phi-\phi')$$

we can show that

$$I_1^+(y,\phi) = \left\{ e^{-(NT+\csc\phi_i\tau(y-NL))}\delta(\phi-\phi_i) + \csc\phi\, e^{-\csc\phi\tau(y-NL)} \left[\frac{e^{-NT\csc\phi} - e^{-NT\csc\phi_i}}{1-e^{-T(\csc\phi-\csc\phi_i)}} \cdot F^+(L,\phi,\phi_i) + e^{-NT\csc\phi_i} \cdot F^+(y-NL, \phi-\phi_i) \right] \right\} I^i. \tag{28}$$

In a similar manner the negative going intensity can be obtained by assuming that Y_0 is some integer multiple of a period (i.e, $Y_0 = ML$) and is given by

$$I_1^-(y,\phi) = \csc\phi\, e^{\csc\phi\tau(y-NL)} \left\{ e^{-(\csc\phi-(N+1)\csc\phi_i)T} \cdot \frac{1-e^{-(\csc\phi-\csc\phi_i)(M-N)T}}{1-e^{-(\csc\phi-\csc\phi_i)T}} \cdot F^-(L,\phi,\phi_i) + e^{\csc\phi_i NT} \cdot [F^-(L,\phi,\phi_i) - F^-(y-NL,\phi,\phi_i)] \right\} I^i \tag{29}$$

where

$$F^{\pm}(x, \phi, \phi_i) = \int_0^x P^{\pm}(\zeta, \phi - \phi_i) e^{(\pm \csc \phi - \csc \phi_i)\tau(\zeta)}\, d\zeta, \quad 0 \le x \le L.$$

Higher order solutions can be obtained by substituting (28) and (29) into (21). However, obtaining the higher order solutions in a closed form seems difficult. Therefore the solutions for problems with large albedo must be computed numerically.

B. Numerical Procedure

If the albedo of the medium is not much smaller than one, the iterative solution does not converge rapidly. In such a case we have to resort to an appropriate numerical method. One such technique is the discrete ordinate eigenanalysis method in which the continuum propagation direction of the intensity within the medium is discretized into finite number of directions [2]. By this approximation the integro-differential equation can be cast into a system of first-order differential equations. For cases where the medium has homogeneous extinction and phase functions the system of linear differential equations can be solved by the eigenanalysis method. For the general case, however, this method cannot be applied. The extinction and phase functions of the row-structured canopy as discussed earlier are periodic and an alternative approach must be pursued.

In what follows we introduce a new numerical technique for the solution of the radiative transfer equation with inhomogeneous extinction and phase functions. This method is specifically very efficient for problems with periodic extinction and phase functions. We discretize the direction of propagation into $2n$ directions and subdivide each row into many thin slabs, then we relate the input–output intensity of each thin slab by approximating the elements of the transmission matrix by linear functions of the incremental width parameter. Finally by multiplying the resultant transmission matrix of individual slabs, the overall transmission matrix can be obtained. This method henceforth will be referred to as discrete ordinate Taylor (DOT) expansion method.

Starting from (21) and approximating the integrals, the following coupled system of differential equations can be obtained:

$$\begin{aligned}\frac{d\mathbf{I}^+(y)}{dy} &= -\mathbf{K}(y)\mathbf{I}^+(y) + \mathbf{P}^+(y)\mathbf{I}^+(y) + \mathbf{P}^-(y)\mathbf{I}^-(y)\\ \frac{d\mathbf{I}^-(y)}{dy} &= \mathbf{K}(y)\mathbf{I}^-(y) - \mathbf{P}^-(y)\mathbf{I}^+(y) - \mathbf{P}^+(y)\mathbf{I}^-(y)\end{aligned} \tag{30}$$

where $\mathbf{P}^{\pm}(y)$ is a $2n \times 2n$ matrix whose elements are defined using the following approximation

$$\csc \phi_i \int_0^{\pi} P^{\pm}(y, \phi_i, \phi') I^{\pm}(y, \phi')\, d\phi' \simeq \sum_{j=-n}^{n} [\mathbf{P}^{\pm}]_{ij}(y) I^{\pm}(y, \phi_j)$$

and $\mathbf{K}(y)$ is a diagonal matrix with

$$[\mathbf{K}]_{ii}(y) = \csc \phi_i \kappa(\phi_i, y).$$

Equation (30) is valid for all of the thin slabs. Suppose thickness of each slab (Δ) is chosen thin enough so that the functions in (30) can be approximated by their Taylor series expansion around the values of y at the left boundary of the slabs up to the first term. That is, for the mth slab we have

$$\begin{aligned}\mathbf{I}^{\pm}(y) &\simeq \mathbf{a}_m^{\pm} + \mathbf{b}_m^{\pm}(y - y_m),\\ \mathbf{K}(y) &\simeq \mathbf{K}_m + \mathbf{K}'_m(y - y_m)\\ \mathbf{P}^{\pm}(y) &= \mathbf{P}_m^{\pm} + \mathbf{P}_m'^{\pm}(y - y_m).\end{aligned} \tag{31}$$

After substituting (31) into (30) and rearranging the terms of equal power in y, the following algebraic equations for the coefficients are obtained:

$$\begin{aligned}\mathbf{b}_m^+ &= -\mathbf{K}_m\mathbf{a}_m^+ + \mathbf{P}_m^+\mathbf{a}_m^+ + \mathbf{P}_m^-\mathbf{a}_m^-\\ \mathbf{b}_m^- &= \mathbf{K}_m\mathbf{a}_m^- - \mathbf{P}_m^-\mathbf{a}_m^+ - \mathbf{P}_m^+\mathbf{a}_m^-.\end{aligned} \tag{32}$$

The intensity in the $(m+1)$th slab is related to the intensity of the mth slab by (see Fig. 3)

$$\mathbf{a}_{m+1}^{\pm} = \mathbf{a}_m^{\pm} + \mathbf{b}_m^{\pm}\Delta. \tag{33}$$

Using (32) in (33), the intensity on the right-hand side can be related to the left-hand side of the mth slab by

$$\begin{bmatrix}\mathbf{a}_{m+1}^+\\ \mathbf{a}_{m+1}^-\end{bmatrix} = \begin{bmatrix}\mathscr{I} - \Delta(\mathbf{K}_m - \mathbf{P}_m^+) & \Delta\mathbf{P}_m^-\\ -\Delta\mathbf{P}_m^- & \mathscr{I} + \Delta(\mathbf{K}_m - \mathbf{P}_m^+)\end{bmatrix} \cdot \begin{bmatrix}\mathbf{a}_m^+\\ \mathbf{a}_m^-\end{bmatrix} \tag{34}$$

where $\mathscr{I}$ is the unit matrix. The matrix in (34), which relates the input–output of the thin slab ($\mathbf{T}_m$), will be referred to as the transmission matrix. Δ must must be chosen small enough, so that the intensity, extinction, and phase functions behave as linear functions over any Δ interval of the medium. The rate of change of the intensity in the medium in the worst case ($P = 0$) is exponential ($I \approx e^{-\kappa y}$) and since extinction and phase function are known, a lower estimate for Δ can be chosen to satisfy the linearity conditions. Higher order approximations in Δ can also be obtained by retaining more terms in the Taylor series expansion. For example, if we keep the expansion terms up to the second order in Δ, the trans-

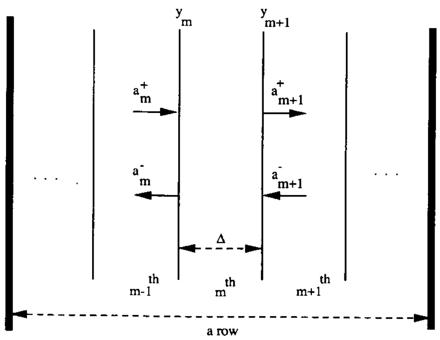


Fig. 3. Configuration showing a row of vegetation consisting of M thin slabs. Also depicting the incidence and reflected vector intensities for the mth slab of thickness Δ.

mission matrix for the mth slab becomes

$$\mathbf{T}_m = \begin{bmatrix} \mathscr{I} + (-\mathbf{K}_m + \mathbf{P}_m^+)\left[\Delta\mathscr{I} + \frac{\Delta^2}{2}(-\mathbf{K}_m + \mathbf{P}_m^+)\right] + \frac{\Delta^2}{2}\left[-\mathbf{K}_m' + \mathbf{P}_m^{+'} - (\mathbf{P}_m^-)^2\right] & \left[\Delta\mathbf{P}_m^- + \frac{\Delta^2}{2}\mathbf{P}^{-'}\right]\left[\mathscr{I} + (-\mathbf{K}_m + \mathbf{P}_m^+)\left[\Delta\mathscr{I} + \frac{\Delta^2}{2}(-\mathbf{K}_m + \mathbf{P}_m^+)\right] + \frac{\Delta^2}{2}\left[-\mathbf{K}_m' + \mathbf{P}_m^{+'} - (\mathbf{P}_m^-)^2\right]\right] \\ -\left[\Delta\mathbf{P}_m^- + \frac{\Delta^2}{2}\mathbf{P}_m^{-'}\right] & \end{bmatrix}$$

where superscript prime in $\mathbf{K}_m'$ and $\mathbf{P}_m^{\pm'}$ denotes derivative with respect to y. In principle, the approximation of transmission matrix can be obtained to any order in Δ, at the expense of complexity of the matrix entries. The transmission matrix of a row comprised of M thin slabs therefore is given by

$$\mathbf{T}_S = \prod_{m=1}^{M} \mathbf{T}_m \tag{35}$$

which relates the input and output intensities of a single row. If there are N rows of the canopy under consideration the overall transmission matrix can be obtained from

$$\mathbf{T} = [\mathbf{T}_S]^N \triangleq \begin{bmatrix} \mathbf{T}_{11} & \mathbf{T}_{12} \\ \mathbf{T}_{21} & \mathbf{T}_{22} \end{bmatrix}. \tag{36}$$

The overall transmission matrix for large values of N can be calculated by diagonalizing the matrix. Suppose $\mathbf{T}_S$ can be diagonalized, i.e.,

$$\mathbf{T}_S = \mathbf{Q}\mathbf{\Lambda}\mathbf{Q}^{-1}$$

where $\mathbf{Q}$ is a matrix whose columns are eigenvectors of $\mathbf{T}_S$, and $\mathbf{\Lambda}$ is a diagonal matrix whose entries are the eigenvalues of $\mathbf{T}_S$. Thus

$$[\mathbf{T}_S]^N = \mathbf{Q}\mathbf{\Lambda}^N\mathbf{Q}^{-1}$$

where $\mathbf{\Lambda}^N$ is a diagonal matrix whose entires are the eigenvalues of $[\mathbf{T}_S]$ raised to the Nth power. Noting that there is no intensity incident on the $(N+1)$th boundary, $(\mathbf{a}_{N+1}^- = 0)$, the reflected and transmitted intensities, respectively, are given by

$$\mathbf{I}^r = -[\mathbf{T}_{22}]^{-1}[\mathbf{T}_{21}]\mathbf{I}^i$$

$$\mathbf{I}^t = \left[\mathbf{T}_{11} - \mathbf{T}_{12}[\mathbf{T}_{22}]^{-1}\mathbf{T}_{21}\right]\mathbf{I}^i.$$

IV. Comparison With Experimental Results

A corn canopy is an example of a periodic vegetation structure under investigation in this study. Measurements of the magnitude of wave patterns transmitted horizontally through a corn canopy were made at 1.5 (L-band) and 4.75 (C-band) GHz for both vertical and horizontal polarizations. Detailed experimental procedure is given by Tavakoli *et al.* [10]. Briefly, the receivers were kept stationary at the height of 1.2 m above the ground, and the transmitters were made to glide along a rail system at the same height as the transmitters. Vertical and horizontal polarizations were transmitted simultaneously and the detected power of the received fields as a function of distance were recorded using an HP-8510 network analyzer. The measurements were performed for a full (stalks and leaves) canopy as well as a defoliated (stalks only) canopy. At the end of the experiment, the corn plants were cut and removed and then direct line-of-sight measurements were conducted to obtain the reference pattern. A summary of the canopy parameters are given in Table I . Before proceeding with the comparison of the theory with experimental data, it is useful to examine the behavior of the phase function of cylinders and resistive strips. Using the data of Table I, the phase function of a single cylinder and a single resistive strip at C-band were calculated as a function of $(\phi_s - \phi_i)$ and are represented in Fig. 4. It can be seen that considering the number of leaves and stalks in the canopy, the phase function of leaves becomes comparable in magnitude to the phase

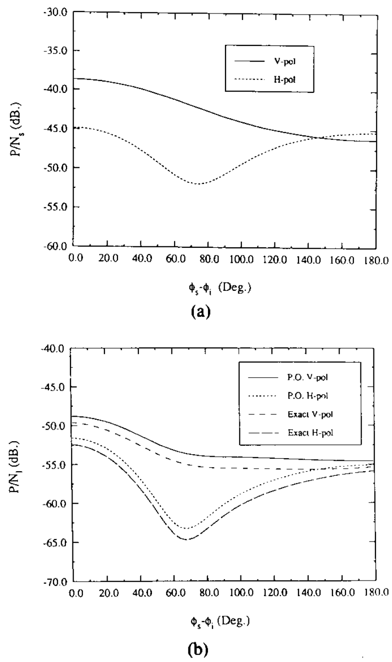


Fig. 4. Calculated phase function of a single (a) cylinder, (b) resistive strip as a function of $\phi_s - \phi_i$ at 4.75 GHz for the parameters listed in Table I.

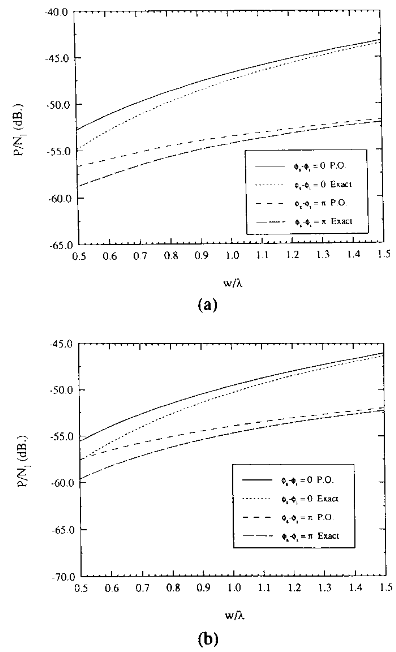


Fig. 5. Calculated phase function of a single strip as a function of w/λ, using the iterative and DOT methods with the parameters listed in Table I for (a) V-polarization and (b) H-polarization.

TABLE I
CANOPY PARAMETERS USE IN THE CALCULATION OF THE THEORETICAL RESULTS

	λ	ϵ_l	ϵ_s
f = 1.5 GHz	20 cm	28 + i8	33 + i10
f = 4.75 GHz	6.3 cm	26 + i8	31 + i10
Leaf: w = 5 cm τ = 0.25 mm			
Stalk: d = 1.75 cm			
Row spacing = 80 cm			
Number of plants per square meter = 6.2			
Number of leaves per plan = 13			

function of stalks at 4.75 GHz. Hence, leaves play as important a role in the scattering process as stalks, which was also observed in the experimental data. Referring to Fig. 4(b), it can be concluded that the physical-optics is in good agreement with the exact solution derived by the method of moments at 4.75 GHz. It should be mentioned that in computation of the phase function by the method of moments, the size of the cells (segments) does not have to be as small as the size that is required for accurate computation of the scattering amplitude (usually $\lambda/15$). This is due to the fact that the averaging process over the magnitude of the scattering amplitude in the calculation of the phase function washes out the fine features of the scattering amplitude.

Fig. 5 shows comparisons between the phase function of a single resistive strip calculated by the physical optics method and the result obtained by using the method of moments. The phase function is calculated as a function of w/λ using the parameters of Table I. It can be seen that the physical-optics solution is a good approximation for calculation of the leaves' phase function at C-band. But, at lower frequencies, the exact solution derived by the method moments must be used instead (i.e., at L-band). Therefore, in the calculation of the phase function for leaves, the exact solution at 1.5 GHz, and the physical optics solution at 4.75 GHz are used. For a plane wave incident on seven rows of a full corn canopy, the bistatic scattering coefficient was calculated and is presented in Fig. 6. The result of the iterative and DOT solutions are comparable at L-band for H-polarization where scattering in the medium is negligible. However, the first-order solution underestimated the DOT solution by up to 3 dB for vertical polarization at both L- and C-band. The coherent component of the transmitted wave is also calculated using the iterative and DOT methods and are given in Table II. The polarization and frequency dependency of the coherent solutions is similar to the that of the diffused scattering.

Measurements of the magnitude of wave patterns transmitted horizontally through seven rows of a corn canopy were made at 1.5 and 4.75 GHz for both vertical and horizontal polarizations. We can now compare the simulated results for the iterative and DOT solutions with the experimental data. In calculation, each row is subdivided into sections of 1.95 cm wide slabs for L-band and 1.77 cm wide slabs at C-band were stalks are only present in the central slab and leaves are uniformly distributed in all slabs. Figs. 7 and 8 compare the result of simulation with

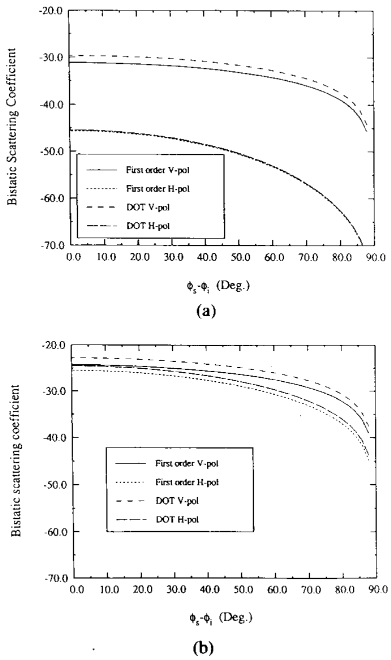


Fig. 6. Calculated bistatic scattering coefficient for a plane wave incident normally on seven rows of a corn canopy, using both the iterative and DOT methods at (a) L-band (b) C-band.

TABLE II
COHERENT COMPONENT OF THE TRANSMITTED WAVE (IN dB), WHEN A PLANE WAVE OF UNIT MAGNITUDE IS INCIDENT ON SEVEN ROWS OF A CORN CANOPY

	V-polarization		H-polarization	
	First order	DOT	First order	DOT
L band	−24.1	−22.7	−2.7	−2.2
C band	−30.8	−27.3	−18.6	−16.8

the experimental data for stalks at 1.5 and 4.75 GHz, respectively. It should be noted that the radiative transfer technique produces statistical average of the transmitted wave pattern and the experimental wave patterns are not averaged over many measurements. It can be seen that both the first-order iterative solution and the DOT solution produce satisfactory wave patterns at both frequencies. Figs. 9 and 10 depict similar results when leaves are also present. Good agreement between the experimental data and the theory is obtained for a vertically polarized wave, but poor agreement is achieved for the horizontal polarization. The model underestimates the propagation loss for horizontally polarized waves. This shortcoming is due to the fact that leaves are modeled as vertically oriented, infinitely long strips, whereas in reality, leaves are not perfectly vertical. Therefore, the simulated propagation loss is not accurate for H-polarization.

V. CONCLUSION

In this paper, a 2-D radiative transfer model for horizontal wave propagation through a periodic random medium was developed. Solutions of the radiative transfer

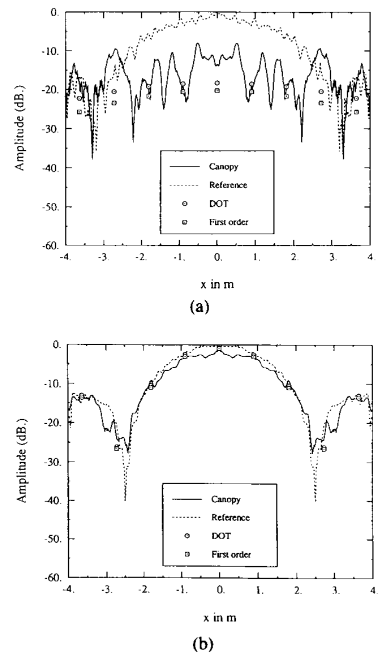


Fig. 7. Comparison between the theoretical and experimental transmitted wave patterns for seven rows of stalks at L-band for (a) V-polarization and (b) H-polarization.

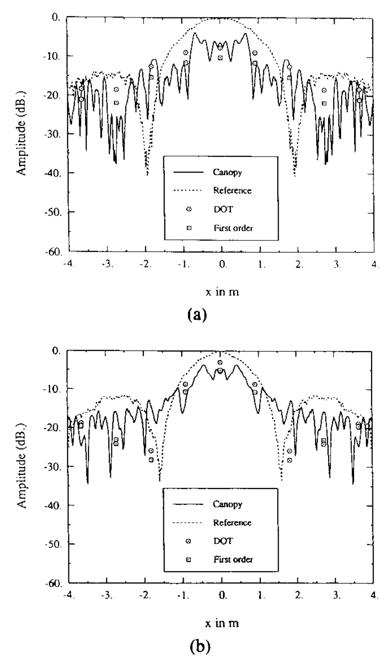
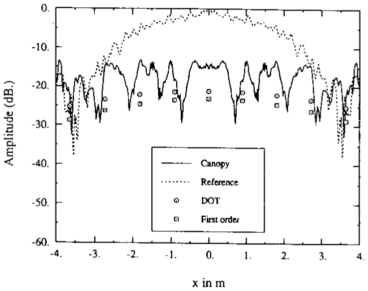
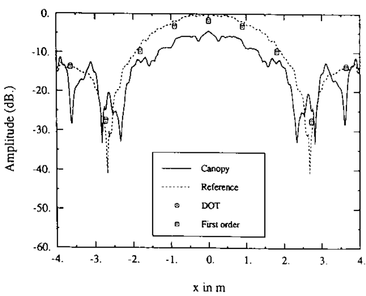


Fig. 8. Comparison between the theoretical and experimental transmitted wave patterns for seven rows of stalks at C-band for (a) V-polarization and (b) H-polarization.

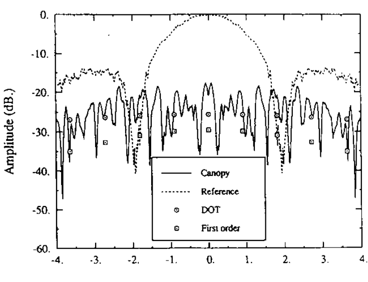


(a)

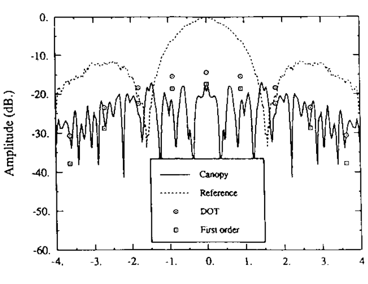


(b)

Fig. 9. Comparison between the theoretical and experimental transmitted wave patterns for seven rows of stalks and leaves at L-band for (a) V-polarization and (b) H-polarization.



(a)



(b)

Fig. 10. Comparison between the theoretical and experimental transmitted wave patterns for seven rows of stalks and leaves at C-band for (a) V-polarization and (b) H-polarization.

equation was pursued both iteratively and by a new method based on the discrete-ordinate approximation and the Taylor series expansion (DOT). As an example, a corn canopy was considered where stalks were represented as infinitely long, vertically oriented dielectric cylinders, and leaves were represented by vertically oriented thin resistive strips. An efficient numerical procedure was developed to compute the extinction and phase functions of particles with arbitrary cross section. This model was used to compute the extinction and phase function at L-band and the physical optics approximation was used to calculate the extinction and phase function of the leaves at C-band. It is shown that the models agree with the experimental data for a defoliated canopy at both polarizations. The model also predicts the propagation characteristics of a vertically polarized transmitted wave through a full corn canopy, but underestimated the horizontal propagation loss due to the fact that the leaves are not perfectly vertical. Overall, the model can be used to study the electromagnetic interaction with a random medium with inhomogeneous particle distribution of arbitrary cross section when the longitudinal dimension is much larger than the cross sectional dimensions of the particles.

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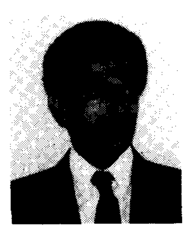

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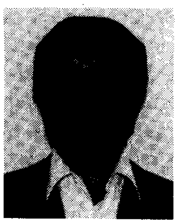

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