

Random Matrices and Applications

June 17 – 19, 2013

Venue

The conference is being held at the University of Michigan's Central Campus in 335 West Hall.

For additional details consult the conference website

SCHEDULE

Monday, June 17

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|---------------------|--|
| 8:45 –9:15 | Refreshments (coffee and pastries) |
| 9:15 –10:00 | Govind Menon
<i>How long does it take to compute the eigenvalues of a random symmetric matrix?</i> |
| 10:00 –10:45 | Paul Bourgade
<i>Universality for beta ensembles</i> |
| 10:45 –11:00 | Break |
| 11:00 –11:45 | Craig Tracy
<i>Asymmetric Simple Exclusion Process with an Open Boundary</i> |
| 12:00 –14:00 | Lunch at the Museum of Art |
| 14:00 –14:45 | Dong Wang
<i>Uniform slow decorrelation and TASEP with general initial conditions</i> |
| 14:45 –15:30 | Mark Newman
<i>Community discovery in networks and the limits of detectability</i> |
| 15:30 –16:00 | Break |
| 16:00 –16:45 | Sasha Sodin
<i>Fluctuations of the Green function associated with random Schroedinger operators</i> |
| 16:45 –17:30 | Ioana Dumitriu
<i>Fluctuations of spectra of overlapping Wishart matrices</i> |
| 18:00 –20:30 | Group Dinner at <i>Jolly Pumpkin Bar on Main Street</i> |

Tuesday, June 18

- 8:45 –9:15** Refreshments (coffee and pastries)
- 9:15 –10:00** Sheehan Olver
Numerical Complex Analysis and Random Matrix Theory
- 10:00 –10:45** Ivan Corwin
Integrable particle systems and Macdonald processes - part I
- 10:45 –11:00** Break
- 11:00 –11:45** Alexei Borodin
Integrable particle systems and Macdonald processes - part II
- 12:00 –14:00** Lunch at the Museum of Art
- 14:00 –14:45** Alan Edelman
Progress in “Ghosts and Shadows” for Beta Ensembles
- 14:45 –15:30** Benedek Valko
Operator limits of random matrices
- 15:30 –16:00** Break
- 16:00 –16:45** Karl Liechty
Random matrices and the six-vertex model
- 18:00 –20:30** Group Dinner at *Sava’s Restaurant on State Street*

Wednesday, June 19

8:45 –9:15	Refreshments (coffee and pastries)
9:15 –10:00	Liza Levina <i>Mixed and covariate-dependent graphical models</i>
10:00 –10:45	Gregory Schehr <i>Large deviations of the top eigenvalue of large Cauchy random matrices</i>
10:45 –11:00	Coffee Break
11:00 –11:45	Patrick Desrosiers <i>Averaged products of characteristic polynomials in beta-ensembles</i>
11:45	Workshop Ends

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ABSTRACTS

(in alphabetic order by speaker surname)

Speaker: **Alexei Borodin** (MIT) & **Ivan Corwin** (MIT & Microsoft)

Title: *Integrable particle systems and Macdonald processes (part I and II)*

Abstract: A large class of one dimensional stochastic particle systems are predicted to share the same universal long-time/large-scale behavior. By studying certain integrable models within this (Kardar-Parisi-Zhang) universality class we access what should be universal statistics and phenomena. In this talk we focus on two different integrable exclusion processes: q -TASEP and ASEP. Using them as a prompt, we will describe the theory of Macdonald processes which unites integrability in various areas of probability including directed polymers, interacting particle systems, growth processes, and random matrix theory.

Speaker: **Paul Bourgade** (Harvard)

Title: *Universality for beta ensembles*

Abstract: I will report on a joint work with L. Erdős and H.-T. Yau, which yields universality for log-gases at arbitrary temperature at the microscopic scale. A main step consists in the optimal localization of the particles, and the involved techniques include a multiscale analysis and a local logarithmic Sobolev inequality.

Speaker: **Patrick Desrosiers** (Universidad de Talca)

Title: *Averaged products of characteristic polynomials in beta-ensembles*

Abstract: We first review recent results on the expectation values of products of characteristic polynomials for random matrices in three classical beta-ensembles: Hermite, Laguerre, and Jacobi. In the bulk and at the edge of the spectrum, the limiting expectation values are explicitly given as special multivariate functions of trigonometric and Airy type, whose exact expansions in terms of Jack polynomials are known. We then turn our attention to the Hermite beta-ensemble perturbed by an external source of finite rank. A duality formula, relating two different Hermite ensembles with external sources, allows the asymptotic evaluation of the averaged products of characteristic polynomials. At the edge of the spectrum, we find a phase transition à la Baik-Ben Arous-Péché. Joint work with Dang-Zheng Liu

Speaker: **Ioana Dumitriu** (U. Washington)

Title: *Fluctuations of spectra of overlapping Wishart matrices*

Abstract: We start with an array of i.i.d. centered variables (with real, complex, or quaternion entries) with fourth moment condition, from which we extract a finite number of rectangular, overlapping submatrices B_i , and form the Wishart matrices $W_i = B_i^* B_i$. The centered traces of powers of such matrices turn out to converge jointly to a Gaussian vector with an interesting covariance structure, which can be described in terms of the height function, in a similar way as in the work of Borodin on Wigner matrices. The resulting connection leads to the classical zero-boundary Gaussian Free Field on the upper half plane. This is joint work with Elliot Paquette.

Speaker: **Alan Edelman** (MIT)

Title: *Progress in "Ghosts and Shadows" for Beta Ensembles*

Abstract: The method of "Ghosts and Shadows," proposes treating β ensembles as first class matrices in some sort of β dimensional space, where β can be any positive real number. One major goal of the method is to derive ideally efficient numerical algorithms for β ensembles suitable for computation, Another goal is to understand how far the β -dimensional structure can be pushed geometrically. In this talk, we show a few recent examples:

1. Wishart matrices with arbitrary covariance
2. Corner Growth Models as arrow or broken-arrow eigenvalue, singular value, and GSVD problems and
3. We derive a Jacobian on the Grassmann manifold, which when raised to the beta-power gives the beta-Jacobi density. Along the way, we streamline the derivation of Jacobi ensembles.

Lastly we ask notationally if β acts dimensionally, should we be rewriting formulas involving β to always appear multiplicatively? (i.e, $\beta \cdot m$ is good, $\beta \cdot m - 1$ may be less desirable)

Speaker: **Liza Levina** (Michigan)

Title: *Mixed and covariate-dependent graphical models*

Graphical models are a popular tool for understanding dependency structure of multivariate data, and sparse graphical models can provide an informative and interpretative summary of high-dimensional data. The commonly used graphical models are usually one of two types: Gaussian graphical models (for continuous data) and the Ising models or Markov networks (for binary and discrete data). However, in practice both types of variables are frequently present in the same dataset, creating the need for mixed graphical models that can represent dependence relationships between continuous and discrete variables at the same time. Some models for these were developed in the earlier literature, but none of them scale to high dimensions. We propose a novel graphical model for mixed data, which is simple enough to be suitable for high-dimensional data, yet flexible enough to represent all possible graph structures, and develop a computationally efficient algorithm for fitting the model. We will also discuss another extension of the graphical model that allows the graph to be observation-specific by allowing dependence on additional covariates, and apply it to data on genetic instability in tumor samples.

Speaker: **Karl Liechty** (Michigan)

Title: *Random matrices and the six-vertex model*

Abstract: The six-vertex model, or the model of square ice, is an integrable model in two-dimensional statistical physics. When considered with domain wall boundary conditions, the partition function for the six-vertex model is the same as the partition function for a random matrix ensemble with nonpolynomial interaction. The asymptotics of this partition function can be studied via the Riemann–Hilbert method. I will discuss this analysis and results in different regions of the six-vertex phase diagram.

Speaker: **Govind Menon (Brown)**

Title: *How long does it take to compute the eigenvalues of a random symmetric matrix?*

Abstract: My former student Christian Pfrang carried out an extensive numerical investigation of the behavior of several eigenvalue algorithms (QR with and without shifts, Toda, matrix-sign) on ensembles of random matrices. My talk is primarily a report on these experiments.

Our main finding is an intriguing form of "universality of fluctuations in computation". I'll also discuss some context for our work, in particular a common Hamiltonian framework for the algorithms.

This is joint work with Percy Deift and Christian Pfrang.

Speaker: **Mark Newman** (Michigan)

Title: *Community discovery in networks and the limits of detectability*

Abstract: There has in recent years been considerable interest in understanding the structure of graphs and networks as they occur in the real world – networks such as the Internet, the web, social networks, biological networks, and others. Of particular interest is the large-scale structure of networks, which can be quantified using the spectral properties of the adjacency matrix or other matrix representations of graphs. In this talk I will discuss one particular type of structure, community or group structure, in which networks divide into modules of some kind. The problem of detection and quantification of communities is often posed as an optimization problem, typically making use of the objective function known as modularity. We show how this problem can be addressed using a standard spectral relaxation approach on the so-called modularity matrix of a network, and how random matrix theory allows us to calculate the expected performance of the resulting algorithm on benchmark problems in the limit of large network size. In particular, we show that there exists a sharp threshold below which we fail to detect planted structure. We draw connections between this threshold, known limitations on statistical inference on networks, and the fundamental limits of computation.

Speaker: **Sheehan Olver** (U. Sydney)

Title: *Numerical Complex Analysis and Random Matrix Theory*

Abstract: Complex analytical tools – Cauchy transforms, their inverses and Riemann-Hilbert problems – have served a central role in the analysis of random matrices. By developing numerical versions of these tools, it is possible to calculate random matrix statistics to high accuracy. This talk gives an overview of recent advances in this direction. The results of this work include: computation of limiting spectral densities of algebraic manipulations of random matrices (numerical free probability) and computing finite-dimensional spectral densities and gap probabilities for invariant ensembles. Combining these two approaches leads to a new conjecture on the relationship between statistics of algebraic manipulations of random matrices and invariant ensembles.

Speaker: **Gregory Schehr** (Paris SUD and CNRS)

Title: *Large deviations of the top eigenvalue of large Cauchy random matrices*

Abstract: In this talk, I will present an exact computation of the large deviation tails of the probability density function (pdf) of the top eigenvalue λ_{\max} in rotationally invariant and heavy-tailed Cauchy ensembles of $N \times N$ matrices for any Dyson index $\beta > 0$, where $\beta = 1, 2, 4$ correspond respectively to orthogonal, unitary and symplectic ensembles. These large deviation tails flank a central non-Gaussian regime for $\lambda_{\max} \sim \mathcal{O}(N)$ on both sides. By matching these tails with the central regime, one obtains the exact leading asymptotic behaviors for any β of the pdf in the central regime, which generalizes the Tracy-Widom distribution known for Gaussian ensembles.

Speaker: **Sasha Sodin** (Princeton)

Title: *Fluctuations of the Green function associated with random Schroedinger operators*

Abstract: We shall discuss several questions and conjectures pertaining to the long-distance fluctuations of the Green function associated with random Schroedinger operators on the lattice, and the connection to first passage percolation.

Speaker: **Craig Tracy** (UC Davis)

Title: *Asymmetric Simple Exclusion Process with an Open Boundary*

Abstract: In previous work the authors considered the asymmetric simple exclusion process (ASEP) where particles are confined to the nonnegative integers $\mathbb{Z}^+ = \{0, 1, 2, \dots\}$. Each particle waits exponential time, and then with probability p it moves one set to the right if the site is unoccupied, otherwise it does not move; and with probability $q = 1 - p$ a particle not at 0 moves one step to the left if the site is unoccupied, otherwise it does not move. For n -particle ASEP a possible configuration is

$$\mathbf{x} = \{x_1, \dots, x_n\}, \quad (0 \leq x_1 < \dots < x_n).$$

The x_i are the occupied sites. We denote by \mathcal{X}_n the set of all possible configurations for the n -particle ASEP.

In this talk we consider the ASEP on \mathbb{Z}^+ with an open boundary at zero. The *stationary measure* for ASEP on the finite lattice $[1, L]$ or on the semi-infinite lattice \mathbb{Z}^+ with boundaries connected to reservoirs has been the subject of much research starting with Derrida *et. al.*. (See the recent work of Sasamoto and Williams for an up-to-date account of these developments.) Here we consider the *time-dependent* properties of ASEP on \mathbb{Z}^+ with an open boundary. Specifically, the point o is connected to a reservoir where a particle is *injected* into site 0 from the reservoir at rate α , assuming that the site 0 is empty, and a particle at site 0 is *ejected* into the reservoir at rate β . Now the number of particles is not conserved and for ASEP with open boundary the configuration \mathbf{x} may lie in \mathcal{X}_n while \mathbf{y} may line in \mathcal{X}_m with $m \neq n$. We give formulas for the transition probabilities $\mathbf{y} \rightarrow \mathbf{x}$ as a function of time. (This is joint work with Harold Widom.)

Speaker: **Benedek Valko** (U. Wisconsin - Madison)

Title: *Operator limits of random matrices*

Abstract: By the Hilbert-Polya conjecture the critical zeros of the Riemann zeta function correspond to the eigenvalues of a self adjoint operator. By a conjecture of Dyson and Montgomery the critical zeros (after a certain rescaling) look like the bulk eigenvalue limit point process of the Gaussian Unitary Ensemble. It is natural to ask if this point process can be described as the spectrum of a random self adjoint operator. We show that this is indeed the case: in fact for any $\beta > 0$ the bulk limit of the Gaussian beta ensemble can be obtained as the spectrum of a self adjoint random differential operator. (This is joint work with Balint Virag.)

Speaker: **Dong Wang** (National University of Singapore)

Title: *Uniform slow decorrelation and TASEP with general initial conditions*

Abstract: The slow decorrelation phenomenon is first observed in the polynuclear growth model by Ferrari, and then proved in many models in KPZ universality class by Corwin, Ferrari and Peche. It is believed to be a feature of KPZ universality class. In this talk, I will show that the pointwise slow decorrelation can be generalized to a uniform slow decorrelation, at least for last passage percolation with geometric weight distribution. As an application, It enables us to write down the asymptotics of the current fluctuation of the discrete TASEP with general initial conditions. This is joint work with Ivan Corwin and Zhipeng Liu.