FoCM'11 FOUNDATIONS of COMPUTATIONAL MATHEMATICS

Random Matrices, Computations and Applications
July 12 – 14, 2011

Venue

The conference is being held at the Budapest University of Technology and Economics (BME), Building K Address: H-1111 Budapest, Muegyetem rkp. 3-9.
For additional details consult the conference website:
`http://www.damtp.cam.ac.uk/user/na/FoCM11/venue.html`

SCHEDULE

Tuesday, July 12

14:00 –14:20	Christ Richmond *Asymptotic Mean Squared Error Performance of Maximum-Likelihood DOA Estimation with Estimated Noise Covariance*
14:25 – 14:45	Gregory Schehr *Extreme statistics of non-intersecting Brownian motions and random matrix theory*
14:50 – 15:35	Thomas Antonsen *Statistics of Wave Fields in Complex Enclosures in the Frequency and Time Domain*
15:40 – 16:25	Jared Tanner *The surprising structure of Gaussian point clouds and its implications for signal processing*
16:25 – 17:00	Coffee Break
17:00 – 17:45	Balint Virag *Matrices with less randomness: the critical Anderson model in 1D*
17:50 – 18:35	Sheehan Olver *Calculating finite dimensional GUE random matrix distributions with general weights*

Wednesday, July 13

14:00 –14:45	Per-Gunnar Martinsson *Randomized Methods for Very Large-Scale Linear Algebra*
14:50 – 15:35	Allard Mosk (Semi-Plenary) *Opaque lenses: Random matrices at work in optics*
15:40 – 16:25	Joel Tropp *User-Friendly Tail Bounds for Sums of Random Matrices*
16:25 – 17:00	Coffee Break
17:00 – 17:45	Ioana Dumitriu *Bipartite biregular graphs and the Marcenko-Pastur Law*
17:50 – 18:35	Jamal Najim *Two statistical applications of Large Random Matrix Theory to wireless communication*
20:00 – 22:00	Group Dinner *Venue TBA* (suggestions welcome!)

Thursday, July 14

14:00 –14:45	Folkmar Bornemann *Accurate Numerical Computations for Orthogonal and Symplectic Ensembles*
14:50 – 15:35	David Gleich *Spectral methods for linear systems with random inputs: A parameterized matrix view*
15:40 – 16:25	Ralf Muller *On large covariance matrices of band-limited random signals*
16:25 – 17:00	Coffee Break
17:00 – 17:45	Yang Chen (Semi-Plenary) *Painleve Transcendents and the Information Theory of MIMO Communication Systems*
17:50 – 18:10	Alan Edelman *What are the Eigenvalues of a Sum of Non-Commuting Random Symmetric Matrices? : A "Quantum Information" inspired Answer.*
18:15 – 18:35	Raj Rao Nadakuditi *Phase transitions in the eigen-analysis of signal-plus-noise random matrices*

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ABSTRACTS
(in alphabetic order by speaker surname)

Speaker: **Thomas Antonsen** (University of Maryland)
Title: *Statistics of Wave Fields in Complex Enclosures in the Frequency and Time Domain*
Abstract: The statistical properties of the scattering of electromagnetic waves into and out of enclosures is of interest in the typical case in which precise knowledge of the configuration of the enclosure and its contents is not available. A general approach is to separate the process of coupling energy through the ports of the enclosure, which can be analyzed using first principles, from the process of excitation of the modes of the enclosure, which is treated statistically using random matrix theory, RMT. In our past work we have shown this approach to be quite useful in describing the results of both computations and experiments so long as one considers statistics over a large enough range of frequencies. Over a narrow range of frequencies deviations between the predictions of the model for elements of the scattering matrix, and calculated or measured values were observed. We show here that these deviations are the result of internal reflections of wave energy in the cavity that occur along relatively short ray paths starting and ending at a port. Further, we show that with knowledge of the shortest ray paths the statistical model can be improved substantially. Our theory is then tested by comparison with direct numerical simulations and measurements.

In our approach the response of the cavity is described by the matrix impedance that gives the voltage at each port in terms of linear combinations of the currents at all ports. In the simplest treatment the only relevant parameters are the radiation impedances of the port-antennas, and the volume and quality factor of the cavity. We address the issue of short ray paths by expressing the cavity impedance in terms of a geometric optics propagator. We approximate the propagator as a finite unitary matrix. By assuming that it is drawn from one of Dyson's circular ensembles, we recover our previous statistics for the impedance. We then find ensembles for the propagator, which have the correct symmetries and short-trajectory corrections, but are otherwise unconstrained. From these, we calculate the impedance statistics of our cavities with short ray paths and find that they are equivalent to treating the walls and the antenna combined as a single port within a larger cavity.

A second topic to be discussed involves the information contained in the time domain response of a complicated wave enclosure to a pulse of energy. In a transmission problem a short pulse of energy injected into a cavity in one port emerges from a second port as a stretched out waveform known as a coda. This waveform contains information about the contents of the cavity. In particular, if it is time reversed and sent in the second port, a reproduction of the original short pulse will appear at the first port. The fidelity of this reproduction will be degraded if the contents of the enclosure are altered. Applications of this effect to sensing and attack will be discussed.

Speaker: **Folkmar Bornemann** (Technische Universität München)
Title: *Accurate Numerical Computations for Orthogonal and Symplectic Ensembles*
Abstract: The highly accurate numerical evaluation of higher order gap probabilities and k-th largest eigenvalue distributions can be based, for the unitary ensembles (UE), on their representation by derivatives

of Fredholm determinants. Though corresponding formulae of matrix-operator determinants are available for orthogonal (OE) and symplectic (SE) ensembles, they suffer from regularity and complexity issues. We show how the Forrester-Rains interrelations of OE, SE, and UE can be used to circumvent these problems. Special attention is given to the case of finite ensembles.

Speaker: **Yang Chen** (Imperial College)
Title: *Painleve Transcendents and the Information Theory of MIMO Communication Systems*
Abstract: In this talk I will show that the moment generating function of the Shannon capacity–essentially an entropy–which characterizes the ultimate limits on communications achievable by any transmission scheme, is described by a particular Painleve V transcendant, in the single user case. (Briefly, in the multiuser situation one finds a PV I transcendent.) For large number of antennas (transmitting and receiving), I will demonstrate how the potential theory approach (involves singular integral equations) maybe usefully employed together with this PV to systematically compute higher moments of the capacity, in a large n expansion. This shows deviation from Gaussian as the signal to noise ratio P increases. [Joint work with Matthew McKay]

Speaker: **Ioana Dumitriu** (University of Washington)
Title: *Bipartite biregular graphs and the Marcenko-Pastur Law*
Abstract: Inspired by a series of breakthrough advances in establishing universality laws for Wigner (random symmetric) matrices, random graph theory is experiencing a similar surge of universality results. At the base of the "universality pyramid" is the local semicircle law (giving a "rate of convergence" for the empirical spectral distribution to the semicircle law, and representing–often–the first step toward further results, like delocalization of eigenvectors). The two models of random graphs widely studied so far are Erdos-Renyi and random regular–and both have been shown to obey the semicircle laws (both global and local). The next natural model is the bipartite biregular random graph, and, sure enough, the methods extend. What may seem surprising at first glance is that this time the limiting law is not semicircular, but essentially derived from Marcenko-Pastur. At second glance, this parallel to the Wishart-like (random positive-definite) matrices can be intuitively explained in terms of linear algebra. The actual results takes a few more technical steps to compute. This is joint work with Tobias Johnson.

Speaker: **Alan Edelman** (Massachusetts Institute of Technology)
Title: What are the Eigenvalues of a Sum of Non-Commuting Random Symmetric Matrices? : A "Quantum Information" inspired Answer
Abstract: We show how interpolating a classical probability sum with a free probability sum gives a sliding scale that allows the matching of four moments. While this method was inspired by a problem from quantum information, we anticipate that it is more widely applicable. Numerical simulations show distributions that match much more closely than standard truncated moment matching methods such as Pearson, Gram- Charlier, and others.

Speaker: **David Gleich** (Sandia National Labs (Livermore))
Title: *Spectral methods for linear systems with random inputs: A parameterized matrix view*
Abstract: There are a variety of real-world problems that correspond to approximating the expectation and variance of the solution of a linear system where the matrix and right hand side depend on a random variable. Such problems arise (i) when studying the effect of uncertain parameters on the outcome of a computational simulation and (ii) when studying more realistic models of ranking the nodes of a web-graph, among other situations. We phrase both problems as parameterized matrix equations and study the behavior of methods to approximate the desired statistics with polynomials, that is, spectral methods. Our results shed important insight on how to utilize these methods in real-world engineering situations. (In collaboration with Paul G. Constantine.)

Speaker: **Per-Gunnar Martinsson (University of Colorado, Boulder)**
Title: Randomized Methods for Very Large-Scale Linear Algebra

Abstract: Very large data sets arise in medical imaging, in analyzing large networks such as the World Wide Web, in image and video processing, and in many other applications. Existing algorithms for extracting information from such data sets are accurate and robust, but are often designed for a single processor computer with the data available in fast Random Access Memory (RAM). Future software must be able to fully exploit modern hardware dominated by multiple processors, capacious but slow memory, streaming data, etc. (Not coincidentally, such hardware is exceptionally well suited for _generating_ huge data sets.)

This talk will describe a set of recently developed techniques for standard linear algebraic computations (such as computing a partial singular value decomposition of a matrix) that are very well suited for implementation on parallel architectures and for processing data stored outside of RAM, or streamed. These techniques use randomized sampling to reduce the effective dimensionality of the data. Remarkably, randomized sampling does not only loosen communication constraints, but does so while maintaining, or even improving, the accuracy and robustness of existing deterministic techniques.

Speaker: **Allard Mosk** (Universiteit Twente, The Netherlands)
Title: *Opaque lenses: Random matrices at work in optics*
Materials such as white paint, skin or bone are opaque because they have a microscopic disordered structure that scatters light. Laser light impinging on such a material becomes diffuse and only a fraction of it is transmitted. This diffusion process preserves phase coherence and is described by a random scattering matrix. By controlling the shape of the incoming wavefront we can cause coherent light to interfere constructively at a target point behind the sample, thereby effectively inverting the transmission matrix. We find that at the target point the light forms a tight focus that is up to 1000 times brighter than the diffuse background [1] and can achieve a higher resolution than is possible without scattering [2]. Optical manipulation of the incident waves can also be used to perform an experimental verification of predictions of random matrix theory. Theorists have predicted that in any non-absorbing disordered sample there exist open eigenchannels: specific linear combinations of incoming waves which experience a transmittance of nearly one. Such eigenchannels have not yet been directly observed in condensed matter systems, however, they are the cause of physical phenomena such as universal conductance fluctuations. By carefully constructing a suitable wavefront we find we can selectively couple light to the open eigenchannels in a disordered optical material. As a result, the total diffuse transmittance increases. The magnitude of the increase is exactly as predicted by random matrix theory.[2]

(*) I will present work performed in cooperation with: D. Akbulut, F. van Beijnum, E.G. van Putten, I.M. Vellekoop, J. Bertolotti, W.L. Vos, and A. Lagendijk

[1] I.M. Vellekoop and A.P. Mosk, Focusing coherent light through opaque strongly scattering media, Opt. Lett. 32, 2309 (2007). [2] E. G. van Putten, D. Akbulut, J. Bertolotti, W. L. Vos, A. Lagendijk, and A. P. Mosk, Scattering Lens Resolves sub-100 nm Structures with Visible Light, Phys. Rev. Lett. (In press, 2011). [3] I.M. Vellekoop and A.P. Mosk, Universal optimal transmission of light through disordered materials, Phys. Rev. Lett. 101, 120601 (2008).

Speaker: **Ralf Muller (NTNU)**
Title: *On large covariance matrices of band-limited random signals*
Abstract: We consider the asymptotic eigenvalue distributions of certain covariance matrices. We create random signals by passing sequences of independent identically distributed random variables through continuous-time linear time-invariant filters and analyze the distribution of eigenvalues of their covariance matrices for an infinite number of signals. We find that under certain conditions on the Fourier transforms of the filters' impulse responses, random time-shifts of the signals against each may or may nor affect the asymptotic eigenvalue distributions. The problems addressed here have applications in asynchronous code-division multiple-access.

Speaker: **Raj Rao Nadakuditi (University of Michigan)**
Title: *Phase transitions in the eigen-analysis of signal-plus-noise random matrices*
Abstract: Motivated by signal-plus-noise type models in high dimensional statistical signal processing and

machine learning, we consider the eigenvalues and eigenvectors of finite, low rank perturbations of large random matrices.

Applications in mind are as diverse as radar, sonar, wireless communications, bio-informatics and machine learning. We provide an application-independent approach that brings into sharp focus a phase transition that is connected to the informational limit of high-dimensional eigen-analysis. We highlight the random matrix origin of this informational limit, the connection with "free" harmonic analysis and discuss implications for high-dimensional statistical signal processing and learning.

Speaker: **Jamal Najim** (Ecole Nationale Suprieure des Tlcommunications)
Title: *Two statistical applications of Large Random Matrix Theory to wireless communication*
Abstract: In this talk, we will present two recent applications of Large Random Matrix Theory to estimation in high dimension. Both results are motivated by wireless communication issues. Building upon X. Mestre's work, we will present an estimator for the population covariance matrix based on the sample covariance matrix and study its fluctuations in a context where the population covariance matrix is structured. If time permits, we will also present another application from wireless communication, namely the estimation of the ergodic capacity under colored interference. From a mathematical point of view, the aim is to estimate a functional of the population covariance matrix based on the observations in a less structured model than before.

[1] J-F. Yao, R. Couillet, J. Najim, M. Debbah, E. Moulines. "Fluctuations of the population covariance matrix estimators in sample covariance matrix models", soon to be posted on the ArXiv.

[2] A. Kammoun, R. Couillet, J. Najim and M. Debbah. "Performance of capacity inference methods under colored interference", available on the ArXiv.

Speaker: **Sheehan Olver** (Oxford University)
Title: *Calculating finite dimensional GUE random matrix distributions with general weights*
Abstract: The Gaussian unitary ensembles (GUE) play a very important role in describing a vast range of phenomena. Gap statistics for GUE can be expressed in terms of Fredholm determinants whose kernel depends on orthogonal polynomials with respect to general weights. We can efficiently and accurately evaluate these orthogonal polynomialsand thus the associated Fredholm determinantsby expressing them as RiemannHilbert problems. This formulation depends on the equilibrium measure, which can also be readily calculated, using a straightforward Newton iteration.

Speaker: **Christ Richmond** (MIT Lincoln Laboratory)
Title: *Asymptotic Mean Squared Error Performance of Maximum-Likelihood DOA Estimation with Estimated Noise Covariance*
Abstract: The mean squared error (MSE) performance prediction of Maximum-Likelihood (ML) Direction-Of-Arrival (DOA) angle estimation has been studied extensively by several authors (see refs in [1]). Most recently ML DOA performance prediction of both the threshold and asymptotic regions in the presence of a general form of deterministic array response mismatch was considered assuming the noise-plus-interference covariance matrix (NICM) was known [1,2]. The error probabilities required for predicting the threshold region MSE of the general case of deterministic mismatch are derived in [3] including the unknown NICM case. Approximations of the asymptotic MSE performance were explored in [2] based on a stochastic representation of the ML filter weight vector, yielding only moderate success due to the approximate representation of the functional dependence of some parameters. Herein an exact general expression for the asymptotic DOA MSE performance for the unknown NICM case is derived based on a Taylor Series expansion that accounts for the exact functional dependence of all parameters. It is shown herein that the MSE expression derived in [1] is augmented by an additive term that accounts for the loss due exclusively to NICM estimation.

[1] C. D. Richmond, "On the Threshold Region Mean-Squared Error Performance of Maximum-Likelihood Direction-Of-Arrival Estimation in the Presence of Signal Model Mismatch," Proceedings of the Fourth IEEE SAM Processing Workshop, pp. 268-272, Waltham, MA, July 2006.

[2] C. D. Richmond, "Signal Model Mismatch and Maximum-Likelihood Mean-Squared Error Performance," Proceedings of the Adaptive Sensor Array Processing Workshop, MIT Lincoln Laboratory, June 2006.

[3] C. D. Richmond, "Mean Squared Error and Threshold SNR Prediction of Maximum-Likelihood Signal Parameter Estimation with Estimated Colored Noise Covariances," IEEE Transactions on Information Theory, Vol. 52, No. 5, pp. 2146-2164, May 2006.

Speaker: **Gregory Schehr** (University of Paris)
Title: *Extreme statistics of non-intersecting Brownian motions and random matrix theory*
Abstract: Since the seminal papers of P. G. de Gennes and M. E Fisher, vicious, i.e. non-intersecting, random walkers have been studied in various physical situations, ranging from wetting and melting to stochastic growth processes. I will first briefly review the connections between certain constrained vicious walkers models, called "watermelons", and random matrix theory. I will then show that the distribution of the maximal height of N vicious walkers in "watermelons" configurations is given, in the large N limit, by the Tracy-Widom distribution associated to the Gaussian Orthogonal Ensemble (GOE).

Speaker: **Jared Tanner** (University of Edinburgh)
Title: *The surprising structure of Gaussian point clouds and its implications for signal processing*
Abstract: We will explore connections between the structure of high-dimensional convex polytopes, random matrices, and information acquisition for compressible signals. A classical result in the field of convex polytopes is that if N points are distributed Gaussian i.i.d. at random in dimension $n << N$, then only order $(logN)^n$ of the points are vertices of their convex hull. However, provided n grows slowly with N, then with high probability all of the points are vertices of its convex hull. More surprisingly, a rich "neighborliness" structure emerges in the faces of the convex hull. One implication of this phenomenon is that an N-vector with k non-zeros can be recovered computationally efficiently from only n random projections with $n = 2eklog(N/n)$. Alternatively, the best k-term approximation of a signal in any basis can be recovered from $2eklog(N/n)$ non-adaptive measurements, which is within a log factor of the optimal rate achievable for adaptive sampling. Analogous results are available using standard spectral analysis in random matrix theory; these results will be compared.

This work is joint with Bah, Blanchard, Cartis, and Donoho,

Speaker: **Joel Tropp** (Caltech)
Title: *User-Friendly Tail Bounds for Sums of Random Matrices*
Abstract:We introduce a new methodology for studying the maximum eigenvalue of a sum of independent, symmetric random matrices. This approach results in a complete set of extensions to the classical tail bounds associated with the names Azuma, Bennett, Bernstein, Chernoff, Freedman, Hoeffding, and McDiarmid. Results for rectangular random matrices follow as a corollary. This research is inspired by the work of Ahlswede–Winter and Rudelson–Vershynin, but the new methods yield essential improvements over earlier results. We believe that these techniques have the potential to simplify the study a large class of random matrices.

Speaker: **Balint Virag** (University of Toronto)
Title: *Matrices with less randomness: the critical Anderson model in 1D*
Abstract: Consider the adjacency matrix of a long path and add independent noise on the diagonal. Given the right variance, this model has a scaling limit. The limit is related to random matrices, but the eigenvalue repulsion is much stronger. If we increase the dimension by epsilon, we can get usual random matrix behavior.