Lecture 9 – Light-Matter Interaction Part 3 Structure Dependence

EECS 598-002 Winter 2006

Nanophotonics and Nano-scale Fabrication

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Schedule for the rest of the semester

- Introduction to light-matter interaction (1/26):
  - How to determine ε(r)?
  - The relationship to basic excitations.
- Basic excitations and measurement of ε(r). (1/31)
- **Structure dependence of ε(r) overview (2/2)**
- **Surface effects (2/7 & 2/9):**
  - **Surface EM wave**
  - **Surface polaritons**
  - Size dependence
- Case studies (2/14 – 2/21):
  - Quantum wells, wires, and dots
  - Nanophotonics in microscopy
  - Nanophotonics in plasmonics
- Dispersion engineering (2/23 – 3/9):
  - Material dispersion
  - Waveguide dispersion (photonic crystals)

Last time…

- Polaritons
  - Macroscopic Maxwell's equations describe the interaction of EM wave with a macroscopic medium (e.g. propagation, diffraction, scattering, etc.) through constitutive relations. In dielectric materials, constitutive relations are reduced to a dielectric function ε(r).
  - Polaritons are quasi-particles resulting from the quantization of an EM wave propagating through a macroscopic media.
  - In QM, "propagation" in a medium is described by the coupling of two SHO's.

$$\omega = \frac{ck}{\sqrt{\varepsilon_0}} \longrightarrow \omega = \frac{ck}{\sqrt{\varepsilon(\omega)}}$$

free-space — polariton

Overview

- Size
  - When the linear dimension of a structure is smaller than the wavelength of the light, the light sees the composite of the structure and its surroundings.
  - When the size of the structure is comparable to the wavelength of the basic excitations, dispersion relations for polaritons are greatly modified. (Discussed last time)

- Surface
  - When the structure is small enough that its surface area is comparable to its volume, effects that are pertaining to the surface start to dominate. Examples are electron scattering from the surface (which increases the decay rates of plasmons) and surface EM waves.

Plasmons in mesoscopic structures

Plasmon frequency: $\omega_p^2 = \dfrac{ne^2}{\varepsilon_0 m}$

If the structure consists of thin metal wires (r~1um) with air in the surroundings, we have (assuming wires are arranged into a cubic lattice of a period *a*):

$$n \rightarrow n\frac{\pi r^2}{a^2}$$

$$m \rightarrow m_{eff} \longleftarrow \text{Due to inductance}$$

$$\rightarrow \frac{\mu_0 \pi r^2 e^2 n}{2\pi} \ln\frac{a}{r}$$

$\Big\}$ Much lower ω_p
e.g. aluminum wires with r=1um, a=5mm:
ω_p 3622 THz->8.2 GHz.

Ref: J. B. Pendry et al., Phys. Rev. Lett. **76** (1996) 4773.

Negative refractive index

- Negative ε and negative μ → negative *n*

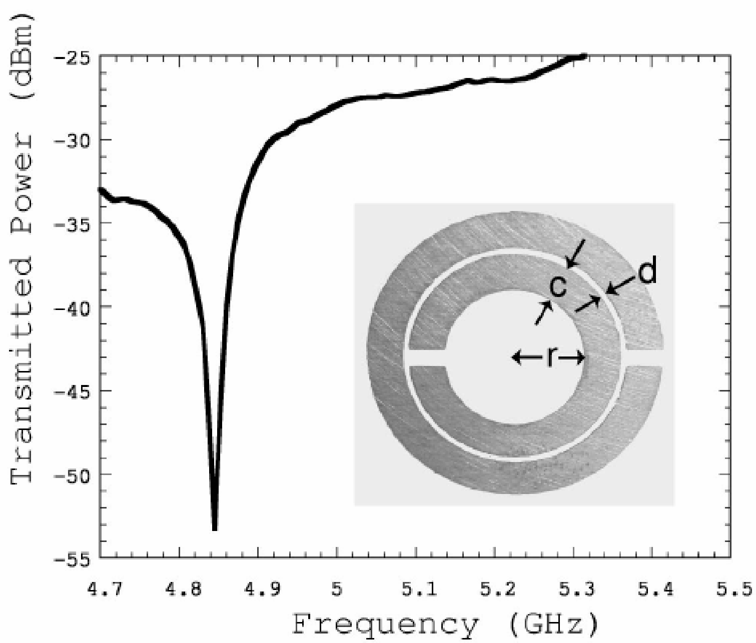


FIG. 1. Resonance curve of an actual copper split ring resonator (SRR). $c = 0.8$ mm, $d = 0.2$ mm, and $r = 1.5$ mm. The SRR has its resonance at about 4.845 GHz, and the quality factor has been measured to be $Q = f_0/\Delta f_{3dB} > 600$, consistent with numerical simulations.

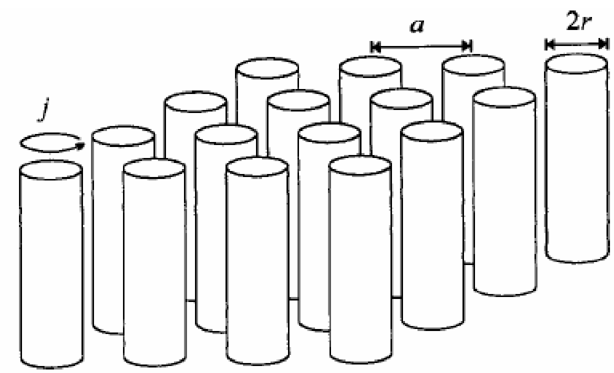
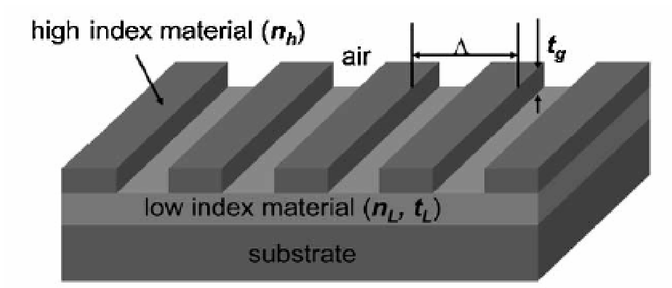


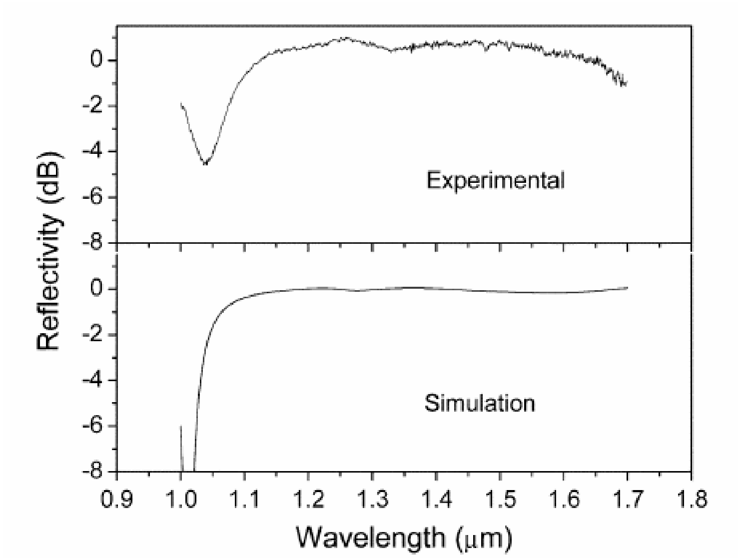
Fig. 2. Model A consists of a square array of metallic cylinders designed to have magnetic properties in the direction parallel to the axes of the cylinders.

$$\mu_{eff} = 1 - \frac{F\omega^2}{\omega^2 - \omega_0^2 + i\omega\Gamma}$$

Ref: D. R. Smith et al., Phys. Rev. Lett. **84** (2000) 4184.
J. B. Pendry et al., IEEE Trans. Micro. Theory & Tech., **47** (1999) 2075.

Subwavelength grating broadband mirror

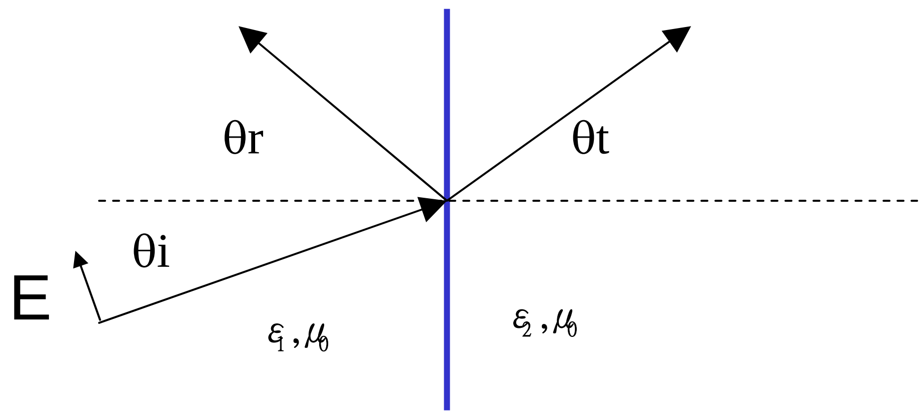


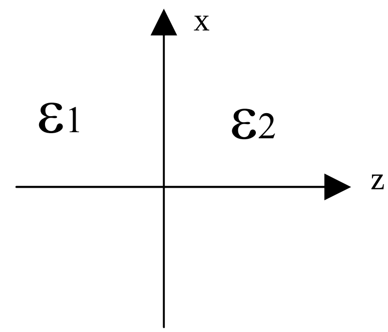


Ref: C. Mateus et al., Photon. Technol. Lett. **16** (2004) 1676.

Surface EM wave motivation

- Interface phenomena revisited:





$$k_{ix} = k_{tx} = k_{rx}$$

$$\frac{H_t}{H_i} = \frac{2k_{1z}/\varepsilon_1}{k_{1z}/\varepsilon_1 + k_{2z}/\varepsilon_2}$$

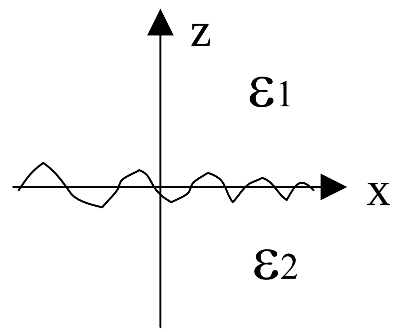
$$\frac{H_r}{H_i} = \frac{k_{1z}/\varepsilon_1 - k_{2z}/\varepsilon_2}{k_{1z}/\varepsilon_1 + k_{2z}/\varepsilon_2}$$

$\longrightarrow$ When $k_{1z}/\varepsilon_1 = -k_{2z}/\varepsilon_2$, reflection and refraction can be finite even without an incident wave.

Surface EM wave (TM polarized)

- Surface EM wave is an EM "mode" propagating along the surface (i.e. with its energy confined near the surface).

$$\begin{cases} z > 0: & \vec{E}_1 e^{ik_x x} e^{-\alpha_1 z} \\ z < 0: & \vec{E}_1 e^{ik_x x} e^{\alpha_2 z} \end{cases}$$



$$k_{1z} / \varepsilon_1 = -k_{2z} / \varepsilon_2$$

$$\Rightarrow i\alpha_1 / \varepsilon_1 = -i\alpha_2 / \varepsilon_2$$

$$\boxed{\Rightarrow \frac{\alpha_1}{\alpha_2} = -\frac{\varepsilon_1}{\varepsilon_2}}$$

TE polarized surface EM wave?

- If the electric field is TE polarized, we can show that the surface EM mode can not be supported.
  - Homework 2 (*Hint*: Consider the boundary condition at the interface and show that the boundary condition can not be satisfied if the TE wave decays exponentially on both sides of the interface.)

Electric field polarization

- The electric field has components both in parallel and in perpendicular to the x direction.

  ➔ Both transverse and longitudinal wave components exist in a surface EM wave.

Dispersion relation of surface EM waves

- Dispersion relation = ω versus kx

$$\begin{cases} k_x^2 + (i\alpha_1)^2 = \dfrac{\omega^2 \varepsilon_1}{c^2} \\ k_x^2 + (i\alpha_2)^2 = \dfrac{\omega^2 \varepsilon_2}{c^2} \end{cases}$$

$$\Rightarrow \omega = c k_x \sqrt{\frac{\varepsilon_1 + \varepsilon_2}{\varepsilon_1 \varepsilon_2}}$$

Surface polaritons

- Surface polaritons:

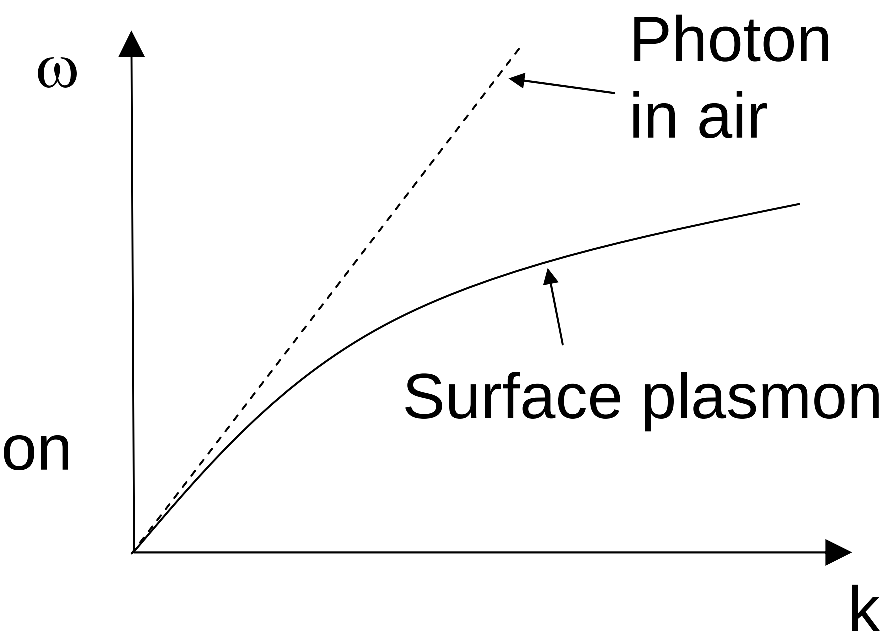
$$\omega = ck_x\sqrt{\frac{\varepsilon_1(\omega)+\varepsilon_2(\omega)}{\varepsilon_1(\omega)\varepsilon_2(\omega)}}$$

e.g. surface plasmon polaritons b/w air and metal

$$\varepsilon_1 = 1 \;\; ; \quad \varepsilon_2 = 1 - \frac{\omega_p^2}{\omega^2}$$

$$\Rightarrow \left(\frac{\omega}{c}\right)^2 = k^2 + \frac{\omega_p^2}{2} \pm \sqrt{k^4 + \left(\frac{\omega_p^2}{2}\right)^2}$$

We need to pick the “-” solution as $\varepsilon_2<0$.



Short wavelength limit

- As kx → infinity

$$\omega = ck_x \sqrt{\frac{\varepsilon_1(\omega)+\varepsilon_2(\omega)}{\varepsilon_1(\omega)\varepsilon_2(\omega)}} \to \infty$$

But since ω is finite, we need to demand: $\varepsilon_1 = -\varepsilon_2$

- Since in this limit, the wave sees only a very small region around the interface, the waves see a composite of these two different materials with a effective dielectric constant of $\varepsilon_1 + \varepsilon_2 = 0$. → The wave is a longitudinal wave in this limit.

Attenuation

- Attenuation length *L* = the propagation distance at which the field intensity drops to 1/e.

$$L = \mathrm{Re}(k_x)\left(\frac{c}{\omega}\right)^2 \frac{\left|\varepsilon_1+\varepsilon_2\right|^2}{|\varepsilon_1|^2\,\mathrm{Im}\,\varepsilon_2 + |\varepsilon_2|^2\,\mathrm{Im}\,\varepsilon_1}$$

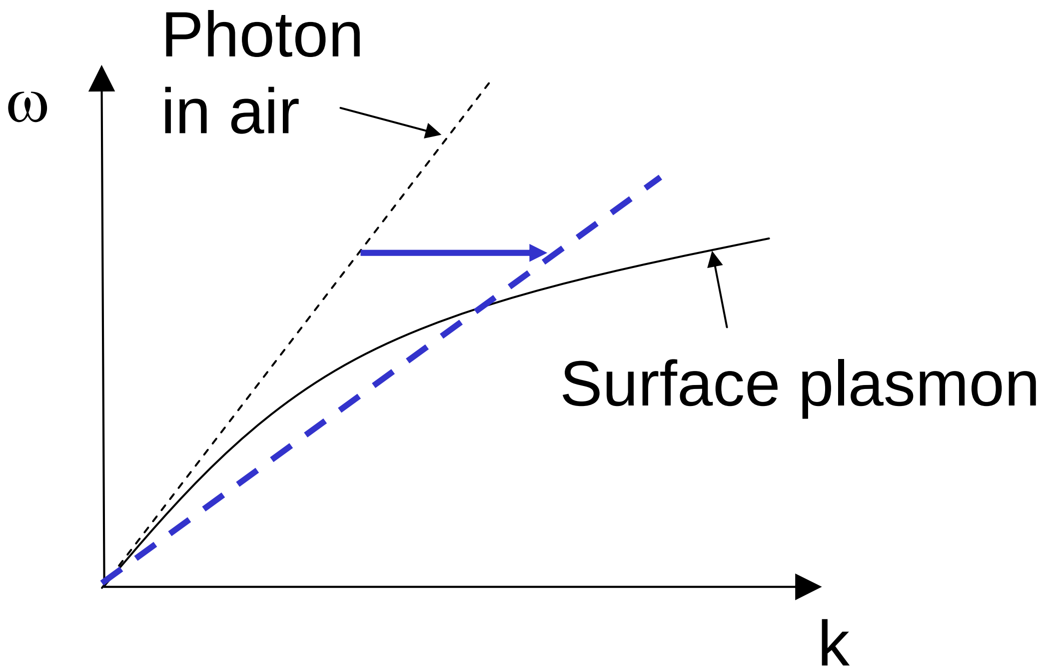
As a special case when $\varepsilon_1 = \text{real},\ \varepsilon_2 = \varepsilon_2' + \varepsilon_2''$ and $\varepsilon_2'' \gg \varepsilon_2'$ and ε_1

$$\Rightarrow \frac{c^2 k_x^2}{\omega^2} = \frac{\varepsilon_1(\varepsilon_2' + \varepsilon_2'')}{\varepsilon_1 + \varepsilon_2' + \varepsilon_2''} \approx \varepsilon_1\left(1 + i\frac{\varepsilon_1}{\varepsilon_2''}\right)$$

$$\Rightarrow k_x \approx \frac{\omega}{c}\sqrt{\varepsilon_1}\left(1 + i\frac{\varepsilon_1}{2\varepsilon_2''}\right)$$

$$\Rightarrow L = \frac{c}{\omega}\frac{\varepsilon_2''}{(\varepsilon_1)^{3/2}}$$ is very large

Optical excitation of surface polaritons



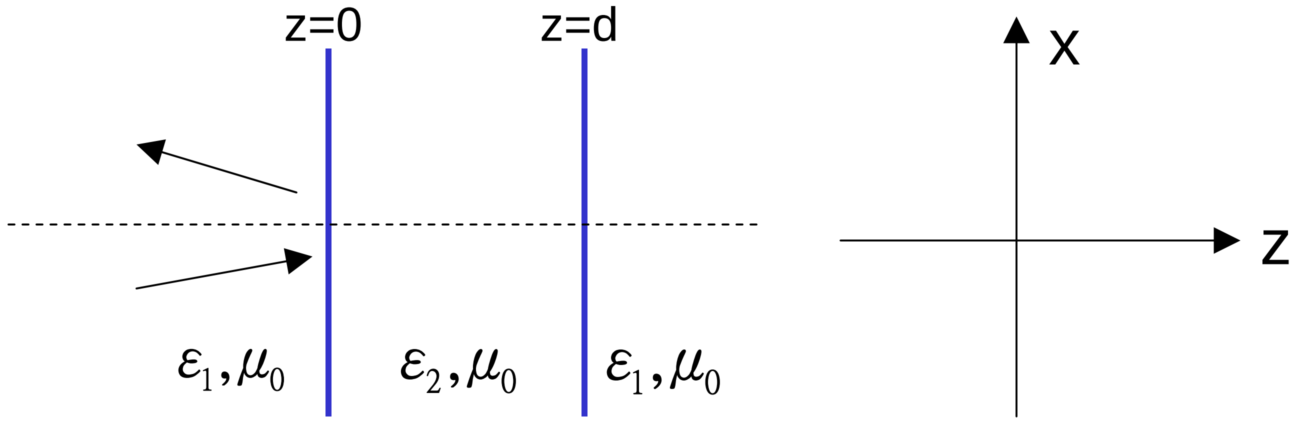
Need to increase the k vector in order to radiatively excite the surface polaritons.

Total internal reflection

- With light coming into the interface and experiencing total internal reflection, the kx component in the second media is larger than that in the free space.

$$k_2 \frac{\sin\theta_i}{\sin\theta_0}$$

Thin slab



$$\vec{E}_i = \hat{y}\left(E_i^+ e^{i(k_{ix}x+k_{iz}z)} + E_i^- e^{i(k_{ix}x-k_{iz}z)}\right) \qquad i = 1\cdots 3$$

$$\vec{H}_i = -ik_i\hat{x}\left(E_i^+ e^{i(k_{ix}x+k_{iz}z)} - E_i^- e^{i(k_{ix}x-k_{iz}z)}\right) \qquad ;\, E_3^- = 0$$

(TE polarized)

Applying boundary conditions at each interface, we have:

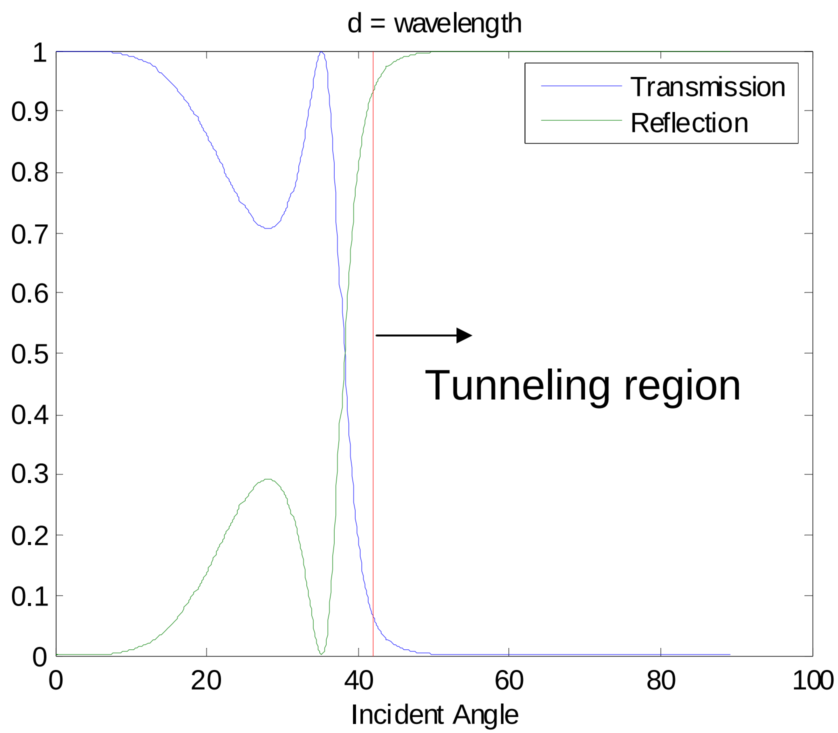
$$k_{1x} = k_{2x} = k_{3x}$$

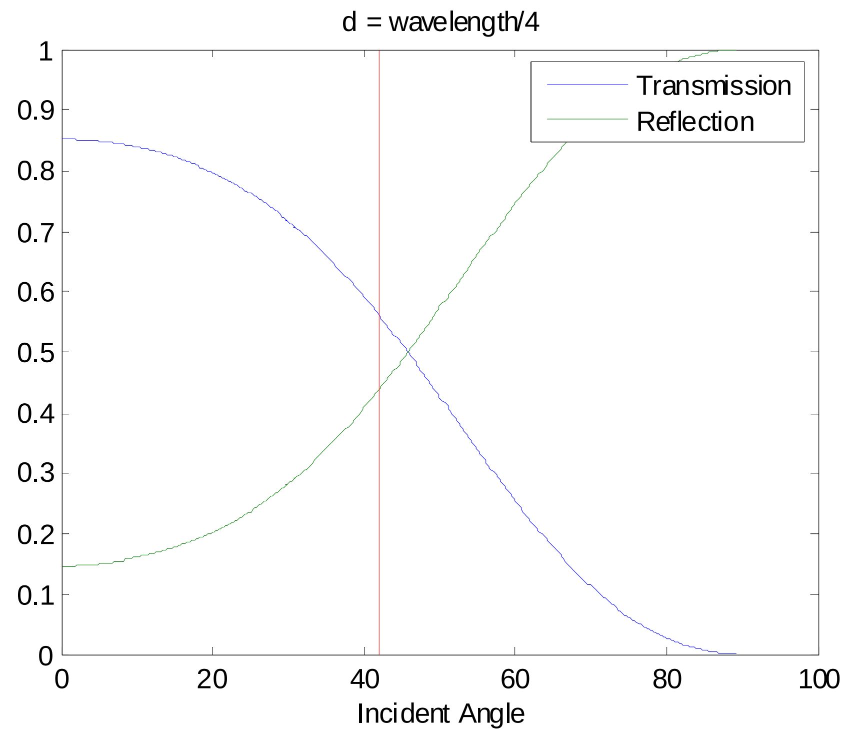
In the case $n_1 > n_2$ and when $k_{1x} > \frac{2\pi n_2}{\lambda}, k_{2z} = \sqrt{\left(\frac{2\pi n_2}{\lambda}\right)^2 - k_{2x}^2}$ is imaginary.

Thin slab (cont.)

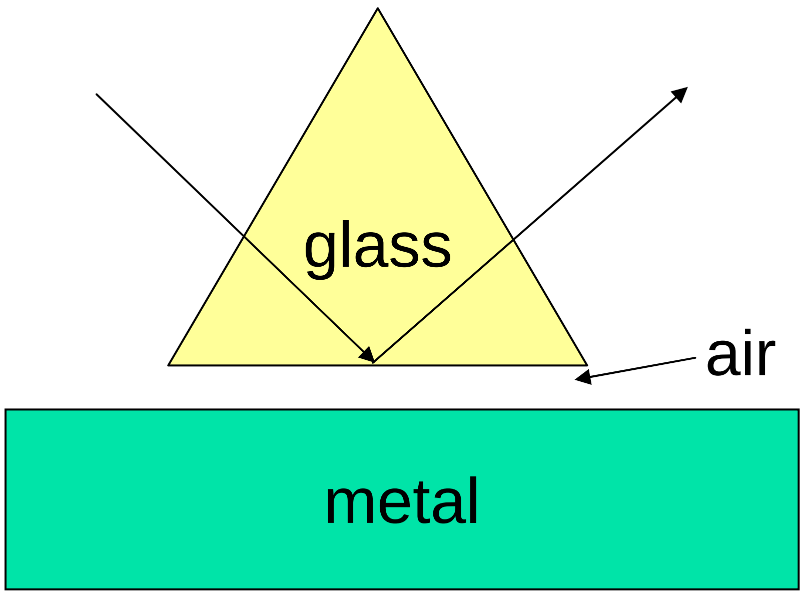
When the incident angle meets the total internal reflection condition at the 1-2 interface, there is no wave propagated in region 2 but there is still a finite transmission through the thin slab. The wave seems to "tunnel" through the slab!

For example, in the following plots, n_1=1.5 and n_2=1.





Prism coupling technique



Reading

- Prasad, chapters 2, 5 and 6.