# Lecture 7 – Light-Matter Interaction Part 1
# Basic excitation and coupling

EECS 598-002 Winter 2006

Nanophotonics and Nano-scale Fabrication

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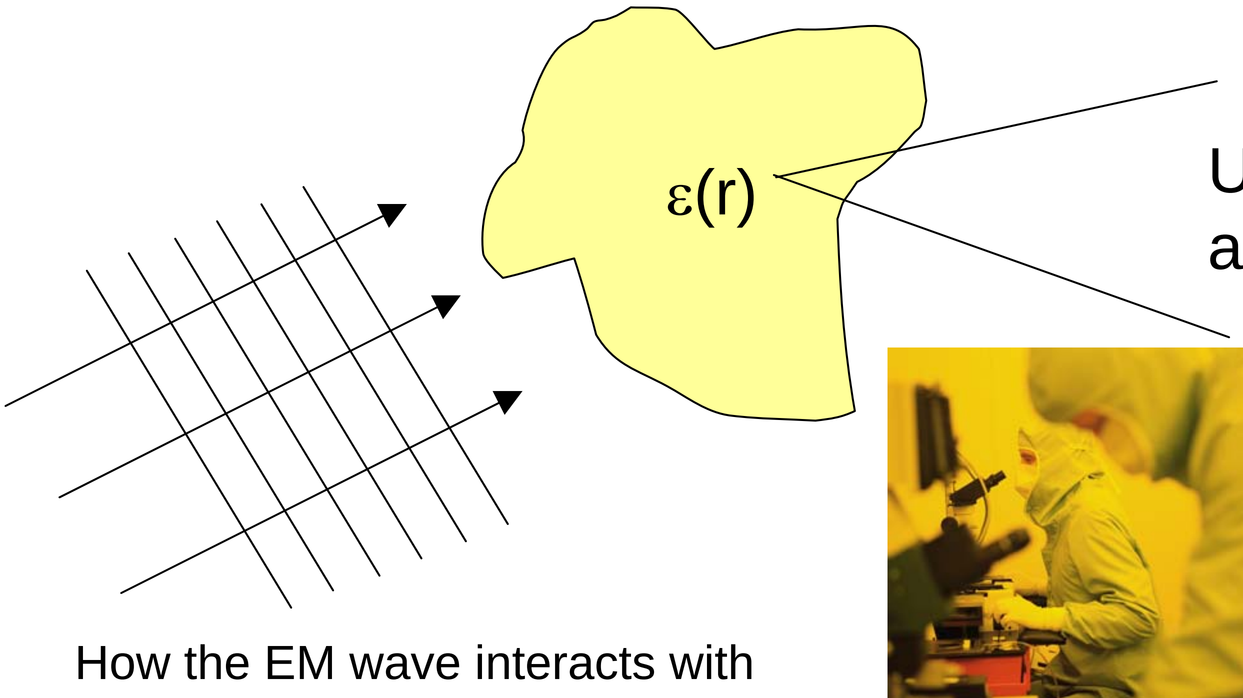
# What we have learned?

- Nanophotonics studies the interaction of photons and matters (including electrons, nuclei, phonons, plasmons, excitons, and etc.)




Uncertainty principle
and quantization

How the EM wave interacts with
a medium with known ε(r)?

# Schedule for the rest of the semester

- Introduction to light-matter interaction (today):
  - How to determine $\varepsilon$(r)?
  - The relationship to basic excitations.
- Basic excitations and measurement of $\varepsilon$(r). (1/31)
- Structure dependence of $\varepsilon$(r) overview (2/2)
- Surface effects (2/7 & 2/9):
  - Surface EM wave
  - Surface polaritons
  - Size dependence
- Case studies (2/14 – 2/21):
  - Quantum wells, wires, and dots
  - Nanophotonics in microscopy
  - Nanophotonics in plasmonics
- Dispersion engineering (2/23 – 3/9):
  - Material dispersion
  - Waveguide dispersion (photonic crystals)

# How to determine ε(r)?

- As a simplest example for metals, "if" we could treat the electron and the ion classically using Newton's Laws and the movement of electrons and ions follows the incoming electric field linearly:

$$m\ddot{x} + m\gamma\dot{x} = -qE_i e^{-i\omega t}$$

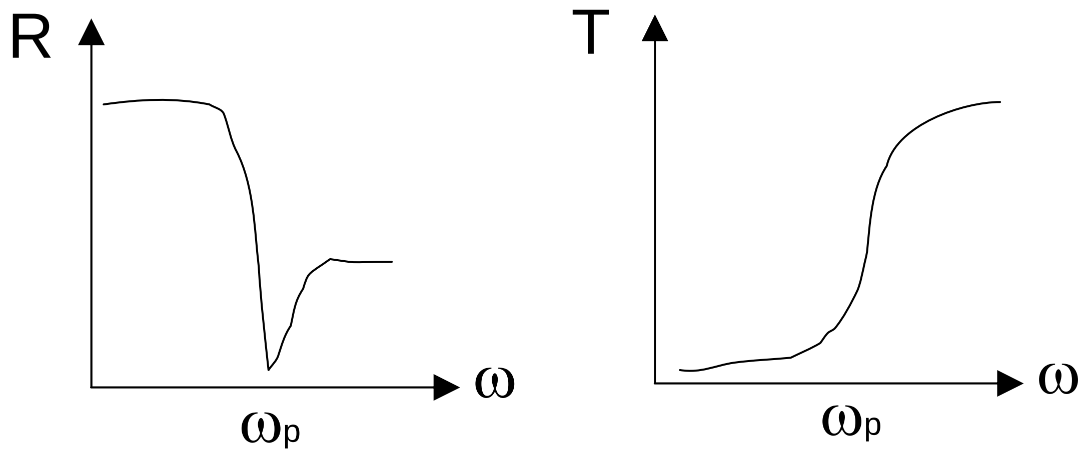
$$\Rightarrow x = \frac{qE_i e^{-i\omega t}}{m\left(\omega^2 + i\gamma\omega\right)}$$

$$\varepsilon = \varepsilon_0 + P/E = \varepsilon_0 - nqx/E = \varepsilon_0 - \frac{nq^2(\omega - i\gamma)}{m\omega\left(\omega^2+\gamma^2\right)} \equiv \varepsilon_0\left[1 - \frac{\omega_p^2(\omega - i\gamma)}{\omega\left(\omega^2+\gamma^2\right)}\right]$$

If $\omega \ll \gamma,\ \ \varepsilon \approx i\dfrac{\omega_p^2}{\omega\gamma\left(\omega^2+\gamma^2\right)}$ ⟶ Highly absorptive

If $\omega \gg \gamma,\ \ \varepsilon \approx \varepsilon_0\left[1 - \dfrac{\omega_p^2}{\omega^2}\right]$ ⟶ Real & positive if $\omega > \omega_p$

# What happens at $\omega_p$?




## Without interaction with external EM field:

If the electron density changes slightly from its equilibrium: $n = n_0 + n_1$

There will be an induced electric field: $\nabla \cdot \vec{E} = -e(n - n_0)$

The continuity equation: $\nabla \cdot (-en\vec{v}) + \dfrac{\partial(-en)}{\partial t} = 0$

Newton's law: $-e\vec{E} = m\dot{\vec{v}}$

If there is no field when $n_1 = 0$, we get $\boxed{\dfrac{\partial^2 n_1}{\partial t^2} + \omega_p^2 n_1 = 0}$

SHO for electron density fluctuation at $\omega_p$

# Plasmons

PHYSICAL REVIEW VOLUME 126, NUMBER 1 APRIL 1, 1962

**Plural Scattering of 20-kev Electrons in Aluminum**

L. Marton, J. Arol Simpson, H. A. Fowler, and N. Swanson
*National Bureau of Standards, Washington, D. C.*

(Received October 20, 1961)

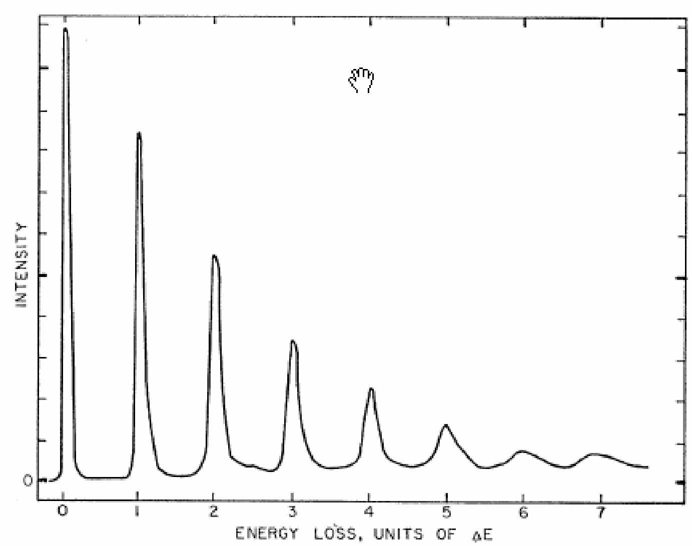



Fig. 5. Energy-loss spectrogram at $\theta=0$ for aluminum foil of thickness 2580±100 A. Angular distribution is shown in Fig. 9. Primary energy 20 kev.

$\omega_p$ or $\lambda_p$:

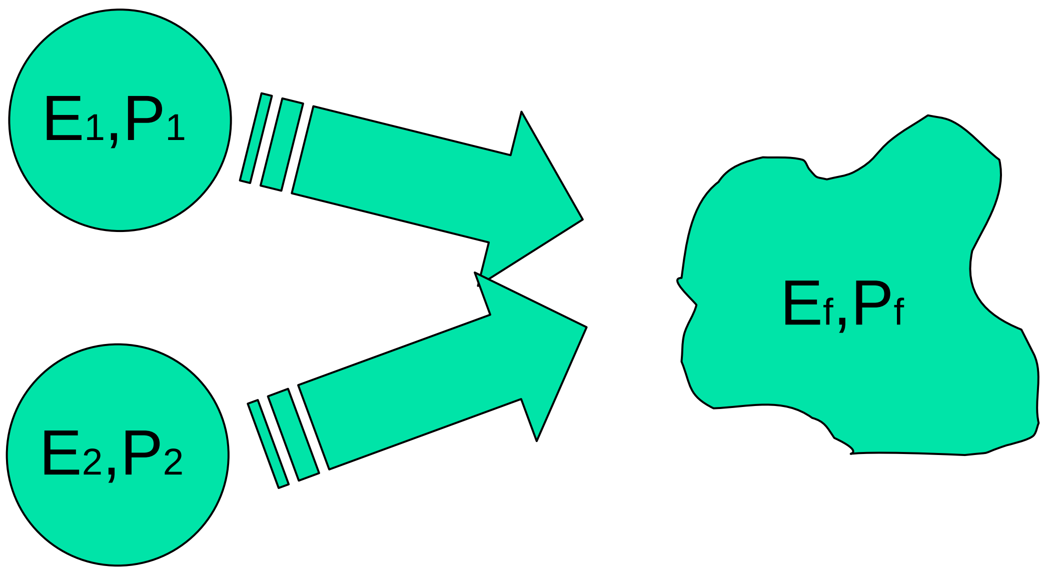
In metals: ~100nm

In semiconductors: ~1μm

# Interaction between light and matter

- In reality, however, we cannot treat microscopic physics with Newton's Laws.
- But without going into details, we can make a few observations just by simple physical intuition.
- Despite of the need for QM, from the example in metals, we can still treat the interaction between light and matter as the coupling between two harmonic oscillators.

# Conservation of energy and momentum




$$E_1 + E_2 = E_f$$

$$\vec{P}_1 + \vec{P}_2 = \vec{P}_f$$

$E = \dfrac{\hbar^2 k^2}{2m}$ for massive particles (electrons, etc.)

or $\hbar\omega$ for massless particles (photons, phonons, etc.)

$\vec{P} = \hbar\vec{k}$

# Conservation conditions

- Conservation of energy and momentum b/w the initial and final states.
- The final state must exist!  (The interaction, however, can go through one or more intermediate states which can be virtual.)
- In addition, if the interaction has rotational symmetry (i.e. it depends only on *r* e.g. the Coulomb interaction), the angular momentum must also be conserved → selection rule.
- We have also learned the electric charge is a conserved quantity.

# Fermi's golden rule

- The discussions above can be described quantitatively by the Fermi's golden rule which results from the lowest-order contribution from the time-dependent perturbation theory.

$$w = \frac{2\pi}{\hbar} \left| \langle f | H_I | i \rangle \right|^2 g(E_f) \delta(E - (E_f - E_i))$$

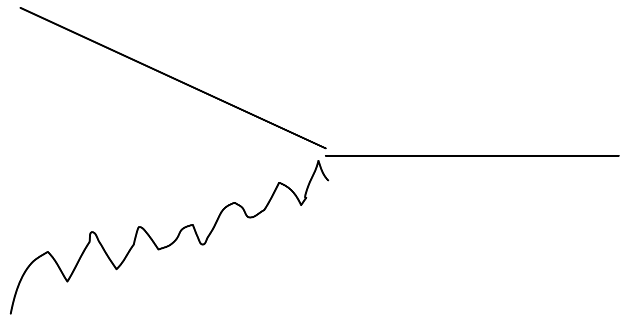
Energy conservation

Final state must exist

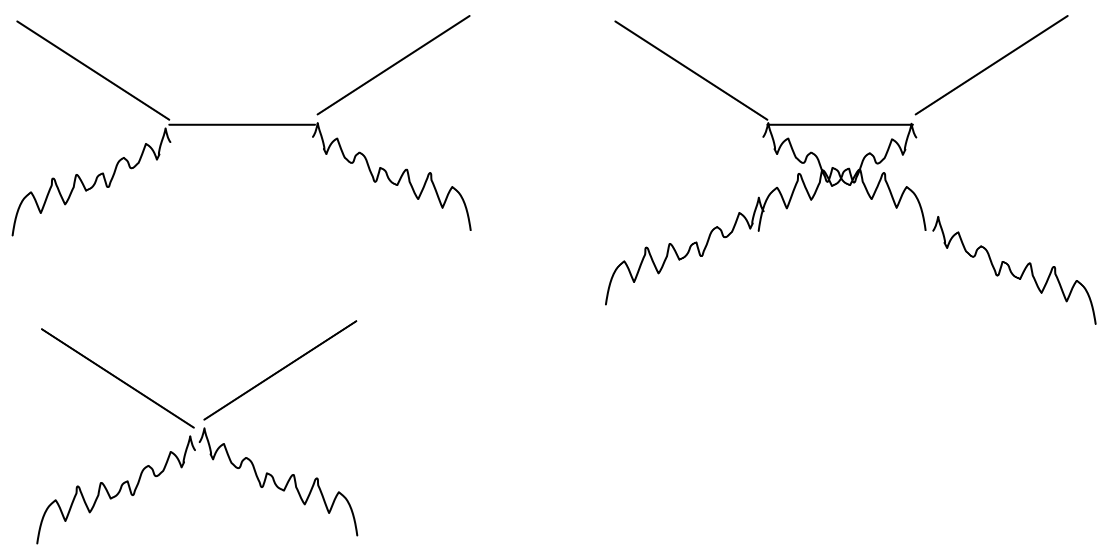
Momentum conservation
and selection rule

# Time-dependent perturbation theory

First order



Second order



# Framework of the calculation for $\varepsilon(r)$

- Near resonance, the first order dominates.
- But if far away from the resonance, we need to include the second order effect (i.e. one intermediate state). This makes the QM calculation of the entire dispersion curve rather tedious.
- In most of the cases, the determination of only the imaginary part of $\varepsilon(r)$ is much simpler. It relates to only a few resonances. We will prove in the following simply by knowing $\mathrm{Im}(\varepsilon(r))$ allows us to determine the entire $\mathrm{Re}(\varepsilon(r))$ without going through any lengthy QM calculations.

# Kramers-Kronig relation

- Because the dielectric function represents a response of the matter to the incoming EM field and any physical response should be causal (nothing happens before the cause), $\varepsilon(\omega)$ satisfies the Karmers-Kronig relation as follows:

$$\mathrm{Re}\left[\varepsilon(\omega)\right] = \varepsilon_0 + \frac{1}{\pi}\mathrm{Pr}\int_{-\infty}^{\infty}\frac{\mathrm{Im}\left[\varepsilon(\omega')\right]}{\omega' - \omega}d\omega' = \varepsilon_0 + \frac{2}{\pi}\mathrm{Pr}\int_{0}^{\infty}\frac{\omega'\,\mathrm{Im}\left[\varepsilon(\omega')\right]}{\omega'^2 - \omega^2}d\omega'$$

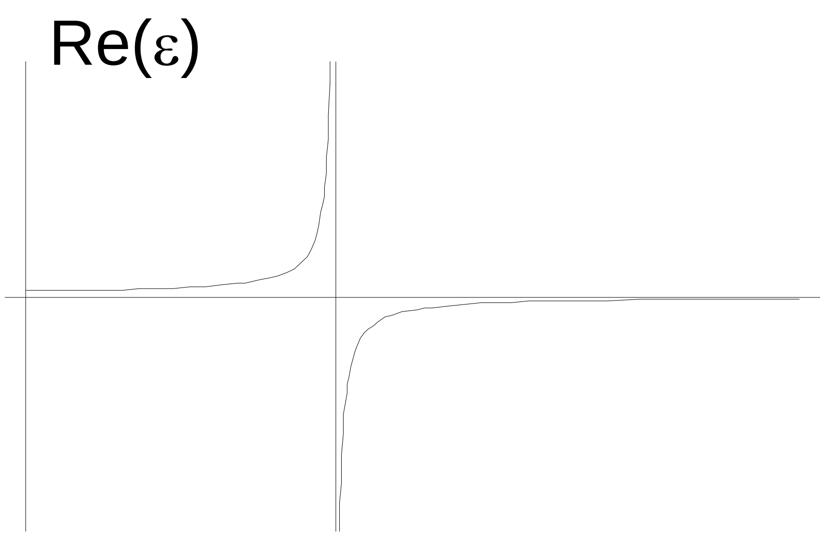
$$\mathrm{Im}\left[\varepsilon(\omega)\right] = -\frac{2\omega}{\pi}\mathrm{Pr}\int_{0}^{\infty}\frac{\mathrm{Re}\left[\varepsilon(\omega')\right] - \varepsilon_0}{\omega'^2 - \omega^2}d\omega'$$

For the derivation of the Kramers-Kronig relation, please refer to J. D. Jackson, *Classical Electrodynamics 2nd ed.*, section 7.10 (c).

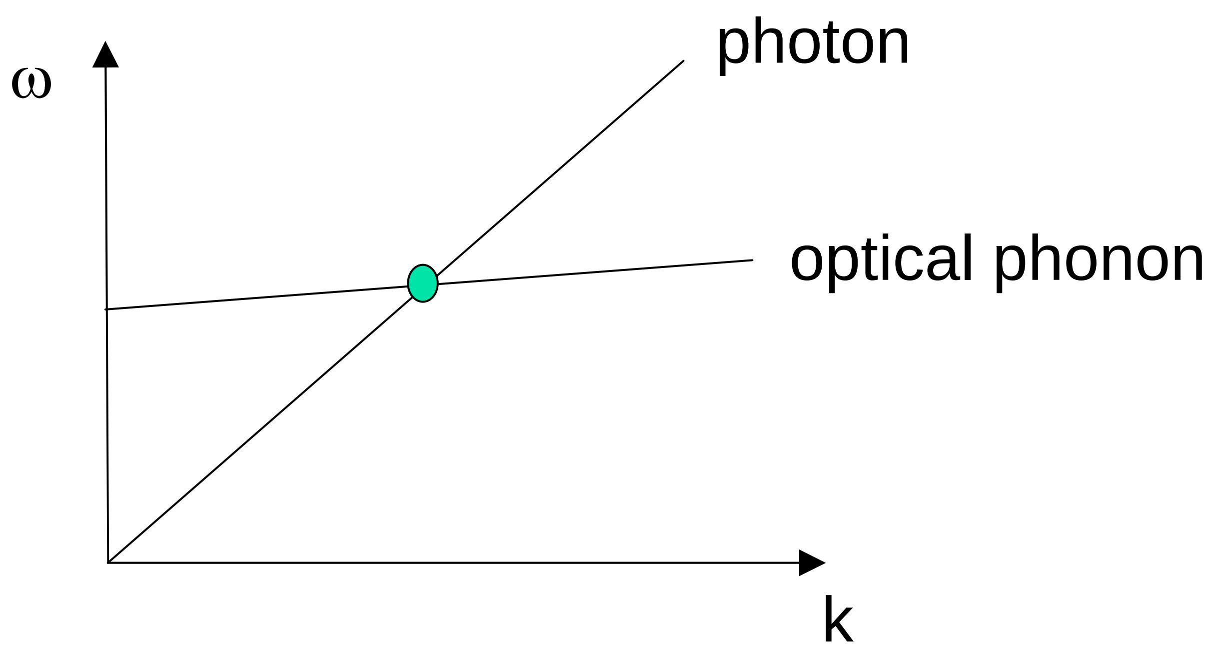
# Example of a sharp resonance

$$\mathrm{Im}\left[\varepsilon(\omega)\right] = \kappa\delta(\omega - \omega_0)$$

$$\begin{aligned}\mathrm{Re}\left[\varepsilon(\omega)\right] &= \varepsilon_0 + \frac{2}{\pi}\mathrm{Pr}\int_0^\infty \frac{\omega'\,\mathrm{Im}\left[\varepsilon(\omega')\right]}{\omega'^2 - \omega^2}d\omega' \\ &= \varepsilon_0 + \frac{2\kappa}{\pi}\mathrm{Pr}\int_{-\infty}^{\infty} \frac{\omega'\,\delta(\omega' - \omega_0)}{\omega'^2 - \omega^2}d\omega' \\ &= \varepsilon_0 + \frac{2\kappa}{\pi}\frac{\omega_0}{\omega_0^2 - \omega^2}\end{aligned}$$



# Conservation of E & P revisited




For resonance (excitation) to happen, two dispersion curves must intersect one another.

# Coupling of two classical harmonic oscillators

Please refer to S. L. Chuang, *Physics of Optoelectronic Devices*, section 8.2.

# ε(r) for an ensemble of harmonic oscillators

$$m\ddot{x} + m\gamma\dot{x} = -kx - qE_i e^{-i\omega t} \equiv -m\omega_0^2 x - qE_i e^{-i\omega t}$$

$$\Rightarrow x = \frac{qE_i e^{-i\omega t}}{m\left[(\omega^2 - \omega_0^2) + i\gamma\omega\right]}$$

$$\varepsilon = \varepsilon_0 + P/E = \varepsilon_0 - nqx/E = \varepsilon_0 - \frac{nq^2}{m\left[(\omega^2 - \omega_0^2) + i\gamma\omega\right]} = \varepsilon_0\left[1 - \frac{\omega_p^2}{(\omega^2 - \omega_0^2) + i\gamma\omega}\right]$$

$$\text{If } \omega \approx \omega_0,\ \ \mathrm{Im}\,\varepsilon \approx \frac{\omega_p^2}{\omega_0}\frac{\gamma/2}{(\omega-\omega_0)^2 + (\gamma/2)^2}$$

$$\mathrm{Re}\,\varepsilon \approx \varepsilon_0\left[1 - \frac{\omega_p^2}{2\omega_0}\frac{\omega-\omega_0}{(\omega-\omega_0)^2 + (\gamma/2)^2}\right]$$

# Polariton – dispersion relation

- Polaritons are mixtures (coupling) of photons and harmonic oscillators in the media.

$$\varepsilon = \varepsilon_0 \left[ 1 - \frac{\omega_p^2}{(\omega^2 - \omega_0^2) + i\gamma\omega} \right] = \frac{k^2}{\omega^2 \mu_0}$$

$$\text{when } \gamma = 0$$

$$k^2 = \frac{\omega^2}{c^2} \left[ \frac{\omega^2 - (\omega_0^2 + \omega_p^2)}{\omega^2 - \omega_0^2} \right]$$