Size dependence (quantum confinement)

\[ g(E) = \text{Density of states} \]

- \( g(E) \) for bulk, sheet, wire, and dot as a function of energy.

- 3D, 2D, 1D, 0D representations of the density of states.
Self-assembled quantum dots growth

- Heteroepitaxial growth:
Growth modes

- Frank-van der Merwe (FvdM) = layer-by-layer growth (2D)
- Volmer-Weber (VW) = island growth (3D)
- Stranski-Krastanow (SK) = layer-by-layer + island growth
Minimization of total energy

Interface energy + Epilayer surface energy $\leq$ Substrate surface energy

If “$\leq$” $\rightarrow$ FvdM growth
Coherent island formation

- If interface energy + epilayer surface energy > substrate surface energy → coherent islands may form to lower the energy (SK mode).

\[
\begin{align*}
\Delta E_V &< 0 & \text{Relaxation of elastic energy} \\
\Delta E_{SURF} &> 0 & \text{Relaxation of elastic energy} \\
\Delta E_{DIS} &> 0 & \text{Energy of the dislocation interface}
\end{align*}
\]

\[
\Gamma = \frac{\Delta E_{DIS}}{\Delta E_{SURF}}
\]

\(Q = \text{amount of deposited materials}\)
SK example: InAs on GaAs

FIG. 3. Density of self-assembled dots vs InAs coverage. Treating these data as a first-order phase transition gives a critical thickness of 1.50 monolayers.

Evolution of growth

FIG. 1. A series of $1 \times 1 \mu m^2$ atomic force microscopy images of a range of coverages of InAs on (100) GaAs, from one monolayer in (a) up to four monolayers in (f), produced by use of a Ta shadow mask. The best size uniformity ($\pm 10\%$ in height, $\pm 7\%$ in diameter) of self-assembled islands of InAs is found only at the initial stages of their formation.

Critical island size

- The total energy of the coherent island is:

\[
E_{TOTAL} = E_{edge} + \Delta E_{SURF} + \Delta E_V
\]

\[
\frac{\partial E_{TOTAL}}{\partial L} = 0 \Rightarrow L = L_{critical}
\]

If \( L > L_{crit} \rightarrow \) Oswald ripening to reduce the \( E_{surf} \)
Stacked QD’s

FIG. 1. Plan-view (a) and cross-section (b) TEM images of triple InAs/GaAs dot stacks with 4 nm separation layer thickness.

QD’s on patterned substrate

MBE grown

MBE grown

FIG. 2. The AFM images of the self-assembled InGaAs QDs formed on (a) the nonpatterned GaAs (100) substrate as a reference, and the Ga_2O_3 patterned substrate with (b) the spacing of 0.3 μm (denoted by S_1), (c) 0.6 μm (denoted by S_2), and (d) 0.9 μm (denoted by S_3), all the AFM scan size is 1×1 μm.

FIG. 4. The AFM image of the two-dimensionally arrayed self-assembled InGaAs QDs formed on the mesh-like patterned Ga_2O_3 substrate. The size of each window area is 0.2×0.15 μm. The white rectangles appearing horizontally in between the patterned regions coming from the proximity effect should be neglected. AFM scan size is 3×3 μm.

“Patterned”?

- The simplest version of patterned substrates is substrates coated with oxide or nitride masks.

- But it can also be the following:
  - Strained induced pattern
  - Etched substrates
Selective-area on nonplanar substrates

MBE grown

FIG. 3. AFM images of ≲100 nm wide stripe mesas from the 1.45 ML InAs deposition sample. (a) Low magnification AFM image of ≳100 nm wide stripe mesa. Note that InAs islands are present only on the mesa top. Panels (b), (c), and (d) are high-magnification AFM images showing the arrangement of InAs islands in the form of chains on the mesa top for mesas of 100, 60, and 30 nm widths, respectively.

Strained selective-area growth

InAs dot

<table>
<thead>
<tr>
<th>InGa1-xAs base layer (x=0.07~0.21)</th>
<th>43~1200 nm</th>
</tr>
</thead>
<tbody>
<tr>
<td>GaAs buffer layer</td>
<td>200 nm</td>
</tr>
<tr>
<td>GaAs(001) sub.</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 1. A schematic diagram of the sample structure.

![InAs dot diagram](image)

![AFM images](image)

Fig. 3. The AFM images of selectively formed InAs dots on the In$_{0.15}$Ga$_{0.85}$As base layer. The thickness of InGaAs base layer is (a) 162 nm (NT = 12) and (b) 325 nm (NT = 27).

MOCVD grown

(a) Mask pattern for SAG of InAs quantum dots

(b) AFM images

Fig. 3. (a) Schematic diagram of the mask pattern used in selective area growth of quantum dots and (b) AFM images at several positions. The mask width is 20, 50, 100 and 145 μm for \( W_1 \) to \( W_4 \), respectively, and the window width \( w \) is 5 μm.

Single and double QD’s

FIG. 3. SEM images of the InAs SQQD(s) formed on top of the SA-grown GaAs, when the width of the top (001) facet was (a) 35 (growth saturated), (b) 77, (c) 115, and (d) 220 nm.

Uniformity

- Homogeneously broadening linewidth

**FIG. 5.** Full width at half maximum homogeneous linewidths estimated from the measured dephasing. Below 100 K, the open squares are the linewidths of the broadband, while the closed squares are the linewidths of the zero-phonon line. The dotted line is the thermal energy $k_B T$. The stars are the ratio between the area of the zero-phonon line and the total area of the line.

Linewidth for QD array

**FIG. 1.** Variation of PL FWHM as a function of InAs growth rate, obtained under an excitation density of about 0.5 W/cm$^2$ at 7 K.

**FIG. 2.** FWHM dependence of QDs with the growth rate of 0.011 ML/s on excitation density. A linear variation of the FWHM with excitation density is distinctly visible. The inset is a 7 K PL spectrum of the QDs, showing the narrowest FWHM of 16.5 meV, obtained with an excitation density of approximately 0.05 mW/cm$^2$.

Quest for uniform QD array

- The linewidth from a QD array is > 2000 times more than its intrinsic linewidth!

There is belief that there is no physical limit on why we can not get uniform dots. But it is a challenge!