Lecture 2 – EM Waves Part I

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Nanophotonics and Nano-scale Fabrication
P.C.Ku
Lecture 2-4 EM waves in nanoscale

- Interaction of EM waves with macroscopic media
  - Derivation of macroscopic Maxwell’s equations
  - Boundary conditions
  - Basic properties of EM waves
- Examples of interaction between EM waves and macroscopic media
  - Interface phenomena
  - Evanescent waves
- Generation of EM waves
  - Antenna basics
  - Dipole antenna
  - Near field and far field
- More on the interaction between EM waves and macroscopic media
  - Diffraction and scattering
  - Wave guiding
- Scaling and symmetry in EM waves
Illustrations of polarization and magnetization

Polarization \( P \)

\[
\vec{D} = \varepsilon_0 \vec{E} + \vec{P} = \varepsilon \vec{E}
\]

Magnetization \( M \)

\[
\vec{B} = \mu_0 \vec{H} + \vec{M} = \mu \vec{H}
\]

2\textsuperscript{nd} rank tensor
Examples of different macroscopic media

**Linear media**

**Isotropic media**
\[
\vec{D} = \varepsilon \vec{E} + \zeta \vec{H} \\
\vec{B} = \mu \vec{H} + \zeta \vec{E}
\]

**Dielectric media**
\[
\vec{D} = \varepsilon \vec{E} \\
\vec{B} = \mu_0 \vec{H}
\]

**Magnetic media**
\[
\vec{D} = \varepsilon_0 \vec{E} \\
\vec{B} = \mu \vec{H}
\]

**Nonlinear media**
\[
D_i = \varepsilon_0 E_i + 2 \chi^{(2)}_{ijk} E_j E_k + 4 \chi^{(3)}_{ijkl} E_j E_k E_l + \cdots \\
\vec{B} = \mu_0 \vec{H} + \mathbf{g}(H^2, H^3, \cdots)
\]

**Chiral media**
\[
\vec{D} = \varepsilon \vec{E} - \chi \frac{\partial \vec{H}}{\partial t} \\
\vec{B} = \mu \vec{H} + \chi \frac{\partial \vec{E}}{\partial t}
\]
Linear dielectric materials

- **Isotropic media**
  - Most of the materials can be regarded as isotropic dielectric materials. Another useful physical parameter refractive index $n$ is defined as $n = \sqrt{\varepsilon}$

- **Anisotropic media**
  - For example, in calcite:
    $$\varepsilon = \begin{pmatrix} \varepsilon & 0 & 0 \\ 0 & \varepsilon & 0 \\ 0 & 0 & \varepsilon_z \end{pmatrix}$$

Because of two different refractive indices in different propagation directions, they are called “birefringent.”
Magnetic materials

- Diamagnetic
  \[ \mu < \mu_0 \]
- Paramagnetic
  \[ \mu > \mu_0 \]
- Ferromagnetic

For examples, check [http://www.irm.umn.edu/hg2m/hg2m_b/hg2m_b.html](http://www.irm.umn.edu/hg2m/hg2m_b/hg2m_b.html)
Nonlinear optical (NLO) crystals

Frequency doubling through NLO.
Chiral materials

DNA

Amino acid

Sugar solutions

Phenylalanine (Phe / F)
Boundary conditions

C and S are infinitesimal contour and surface.

\[ \oint_C \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt} \oint_S \mathbf{B} \cdot d\mathbf{S} = 0 \Rightarrow E_{1t} = E_{2t} \]

\[ \oint_C \mathbf{H} \cdot d\mathbf{l} = \int_S \left( \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \right) \cdot d\mathbf{s} \Rightarrow \mathbf{n}_2 \times (\mathbf{H}_1 - \mathbf{H}_2) = \mathbf{J}_s \]

\[ \oint_S \mathbf{D} \cdot d\mathbf{s} = Q \Rightarrow D_{1n} - D_{2n} = \rho_S \]

\[ \oint_S \mathbf{B} \cdot d\mathbf{s} = 0 \Rightarrow B_{1n} - B_{2n} = 0 \]
Thin slab

\[ \bar{E}_i = \hat{y} \left( E_i^+ e^{i(k_{1x} x + k_{1z} z)} + E_i^- e^{i(k_{1x} x - k_{1z} z)} \right) \quad i = 1 \cdots 3 \]

\[ \bar{H}_i = -ik_i \hat{x} \left( E_i^+ e^{i(k_{1x} x + k_{1z} z)} - E_i^- e^{i(k_{1x} x - k_{1z} z)} \right) \quad ; \quad E_3^- = 0 \]

Applying boundary conditions at each interface, we have:

\[ k_{1x} = k_{2x} = k_{3x} \]

In the case \( n_1 > n_2 \) and when \( k_{1x} > \frac{2\pi n_2}{\lambda} \), \( k_{2z} = \sqrt{\left( \frac{2\pi n_2}{\lambda} \right)^2 - k_{2x}^2} \) is imaginary.
Thin slab (cont.)

When the incident angle meets the total internal reflection condition at the 1-2 interface, there is no wave propagated in region 2 but there is still a finite transmission through the thin slab. The wave seems to “tunnel” through the slab!

For example, in the following plots, $n_1=1.5$ and $n_2=1$. 

![Graph 1: D = wavelength, Tunneling region](image1)

![Graph 2: D = wavelength/4, Tunneling region](image2)
In a general EM problem, we have different length scales:

λ: wavelength
r: size of the scatterer
a: size of the diffracter
R1, R2, ...: distances b/w two objects

Near: $R << \lambda$
Intermediate: $R \sim \lambda$
Far: $R >> \lambda$