# Lecture 14 – Dispersion engineering part 1 - Introduction

EECS 598-002 Winter 2006

Nanophotonics and Nano-scale Fabrication

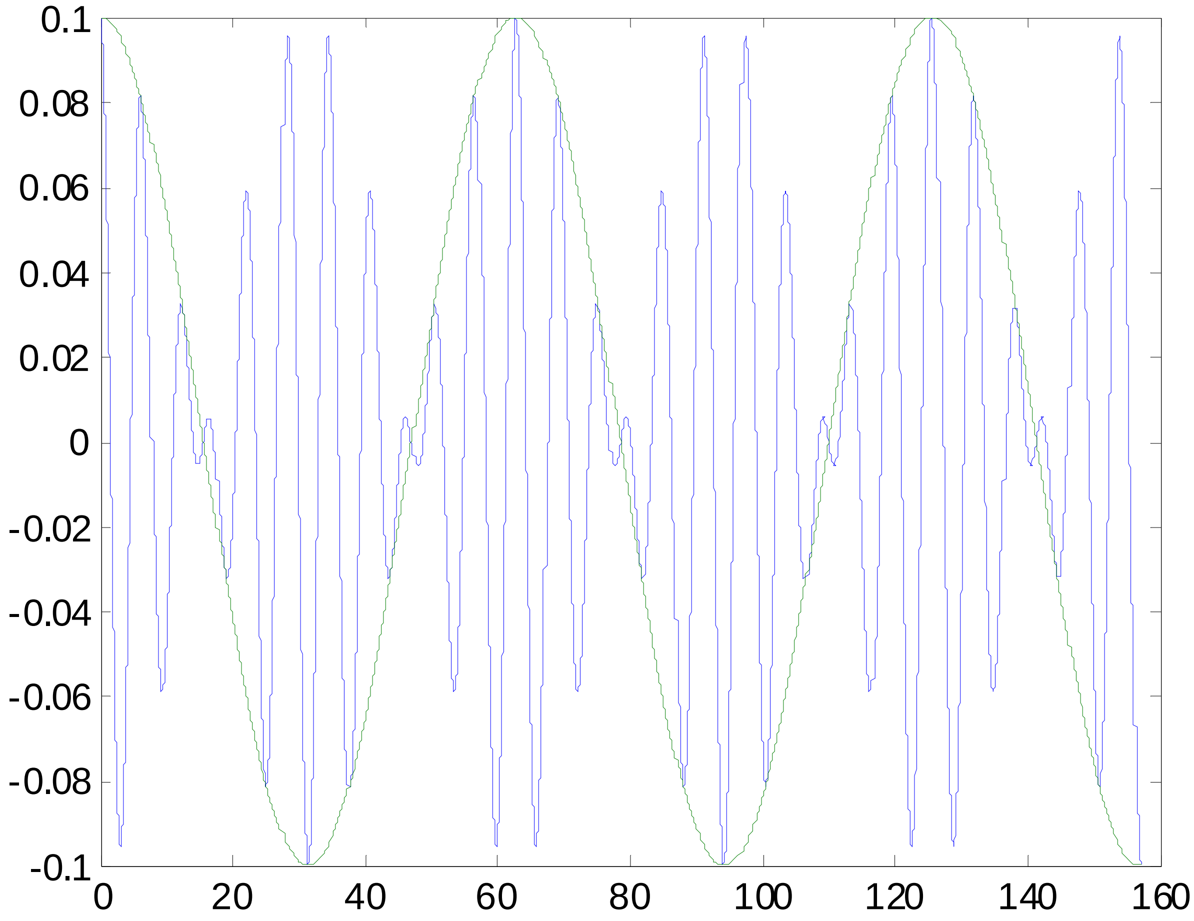
P.C.Ku

# Schedule for the rest of the semester

- Introduction to light-matter interaction (1/26):
  - How to determine $\varepsilon$(r)?
  - The relationship to basic excitations.
- Basic excitations and measurement of $\varepsilon$(r). (1/31)
- Structure dependence of $\varepsilon$(r) overview (2/2)
- Surface effects (2/7):
  - Surface EM wave
  - Surface polaritons
  - Size dependence
- Case studies (2/9 – 2/21):
  - Quantum wells, wires, and dots
  - Nanophotonics in microscopy
  - Nanophotonics in plasmonics
- **Dispersion engineering (2/23 – 3/7):**
  - **Material dispersion**
  - **Waveguide dispersion (photonic crystals)**

# Optical signal

$$\mathrm{Re}[Ee^{i(\beta z-\omega t+\phi)}]$$



# Phase, group, and signal velocities

- Phase velocity = velocity of the oscillation

$$v_p = \frac{1}{\sqrt{\mu_0 \varepsilon}}$$

- Group velocity = velocity of the envelope

$$v_g = \frac{\partial \omega(k)}{\partial k}$$ = slope of the dispersion curve

- Signal velocity

$v_s = v_g$ If the signal reaches the receiving end.

# Group velocity dispersion

- If $v_g$ depends on ω, the signal will experience distortion.

$$\beta(\omega) = \beta_0 + \frac{d\beta}{d\omega}(\Delta\omega) + \frac{d^2\beta}{d\omega^2}(\Delta\omega)^2 + \frac{d^3\beta}{d\omega^3}(\Delta\omega)^3 + \cdots$$

$$\equiv \beta_0 + \beta_1(\Delta\omega) + \beta_2(\Delta\omega)^2 + \beta_3(\Delta\omega)^3 + \cdots$$

If $\beta_3 = \beta_4 = \cdots = 0$ :

Input = gaussian pulse = $A(0,t) = A_0 \exp\left[-\frac{1}{2}\left(\frac{t}{T_0}\right)^2\right]$

Pulse width = $T_{FWHM} = 2\sqrt{\ln 2}\,T_0 \approx 1.665 T_0$

Output pulse width = $\frac{T_1}{T_0} = \sqrt{1 + \left(\frac{\beta_2 z}{T_0^2}\right)^2}$

# Slow and fast light

$$v_g = \frac{\partial \omega}{\partial k} \qquad k = \frac{\omega n(\omega,k)}{c} \longrightarrow \frac{n}{c} + \frac{\omega}{c}\left(\frac{\partial n}{\partial \omega} + \frac{\partial n}{\partial k}\frac{\partial k}{\partial \omega}\right) = \frac{\partial k}{\partial \omega} = \frac{1}{v_g}$$

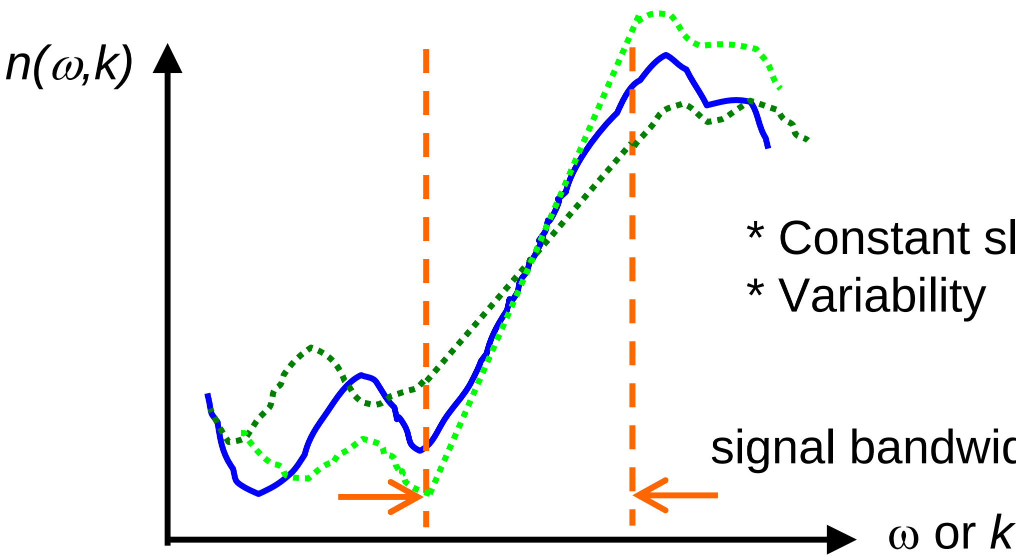
Dispersion engineering to vary group velocity

$$S = \frac{c}{v_g} = \frac{n + \omega \frac{\partial n}{\partial \omega}}{1 - \frac{\omega}{c}\frac{\partial n}{\partial k}}$$

Material dispersion

Spatial variation of *n*

## Control of signal velocity:




* Constant slope
* Variability

**Control by changing** $\frac{\partial n}{\partial \omega}$ **or** $\frac{\partial n}{\partial k}$

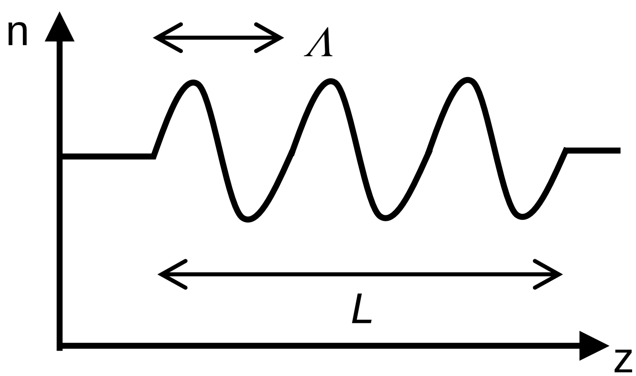
# Waveguide dispersion

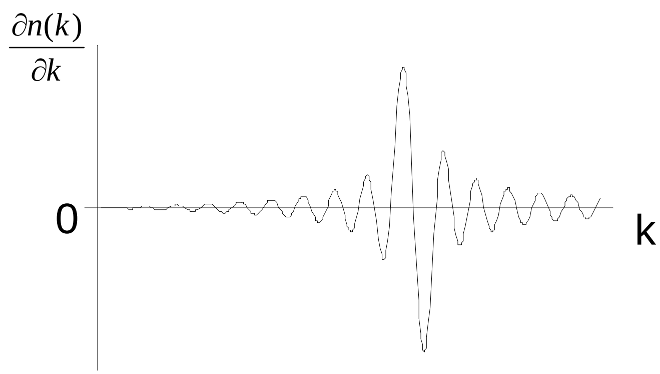
- Photonic crystals

$$n(z) = n + \Delta n \cos(2\pi z / \Lambda) \quad \text{for} \quad -L/2 < z < L/2$$

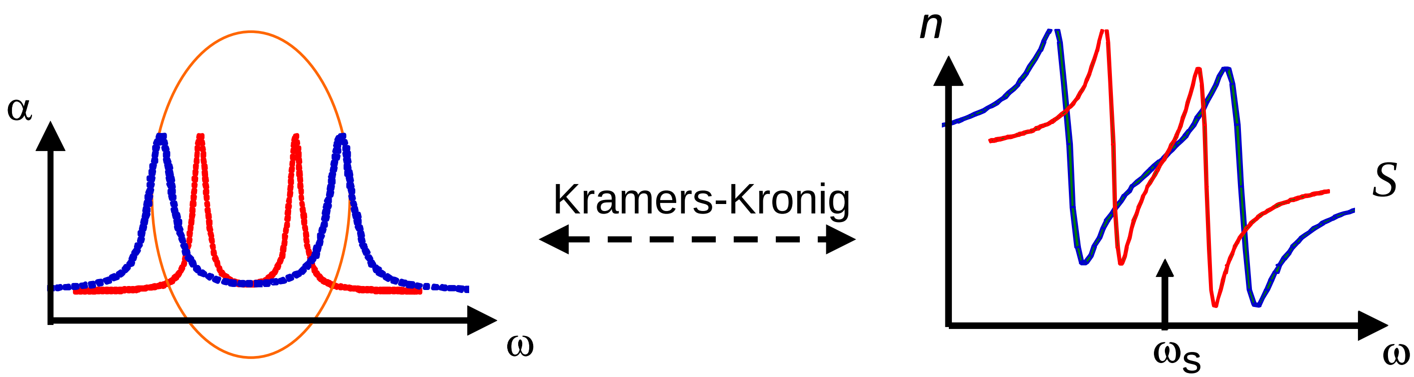
$$n(k) = \frac{\sqrt{2/\pi}\,\Delta n \Lambda \left( 2\pi \cos\frac{kL}{2} \sin\frac{\pi L}{\Lambda} - k\Lambda \sin\frac{kL}{2} \cos\frac{\pi L}{\Lambda} \right)}{4\pi^2 - k^2 \Lambda^2}$$

$$S = \frac{n}{1 - \frac{\omega}{c}\frac{\partial n}{\partial k}}$$



# Material dispersion and its control




$$S = \frac{c}{v_g} = \frac{n + \omega \frac{\partial n}{\partial \omega}}{1 - \frac{\omega}{c}\frac{\partial n}{\partial k}}$$

- Electromagnetically induced transparency (EIT)
- Population oscillation (coherence dip)

# EIT and slow light

## Light speed reduction to 17 metres per second in an ultracold atomic gas


**Lene Vestergaard Hau*†, S. E. Harris‡, Zachary Dutton*† & Cyrus H. Behroozi*§**

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† *Department of Physics, § Division of Engineering and Applied Sciences, Harvard University, Cambridge, Massachusetts 02138, USA*

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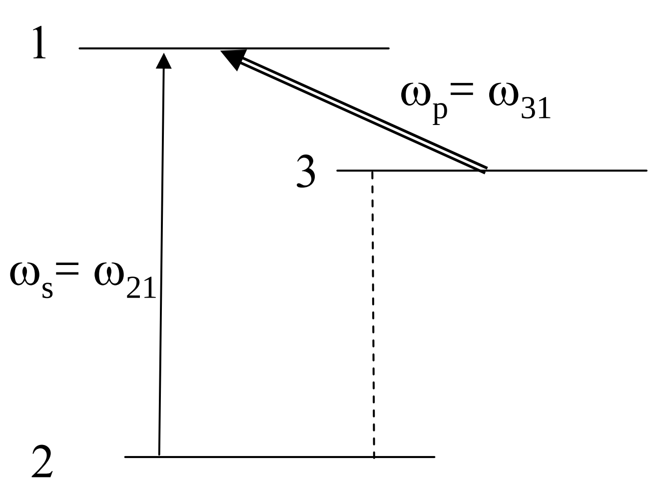
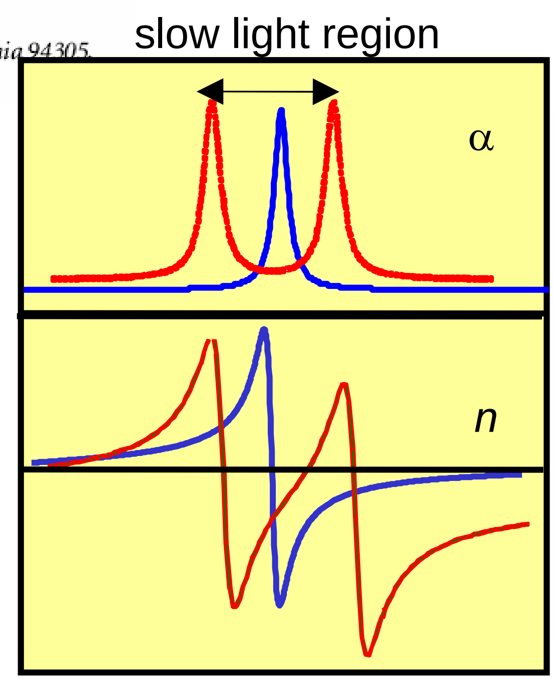
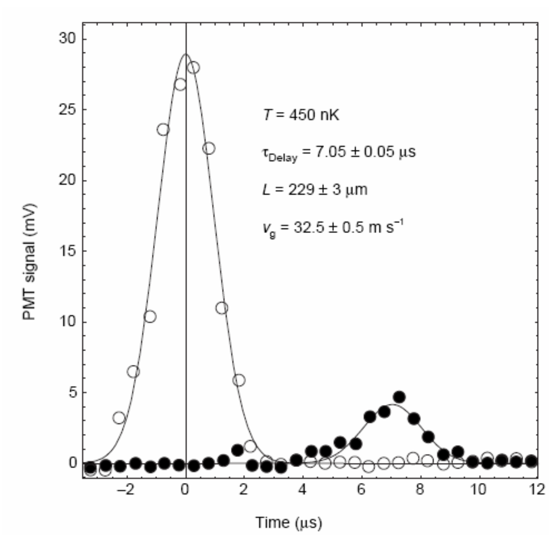
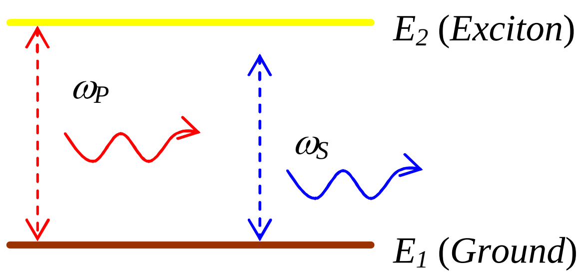


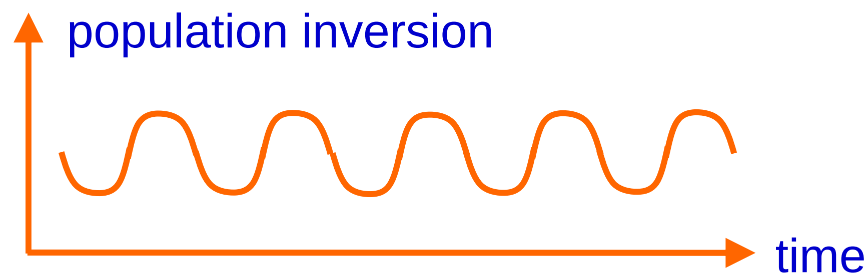



# Population oscillation (PO)

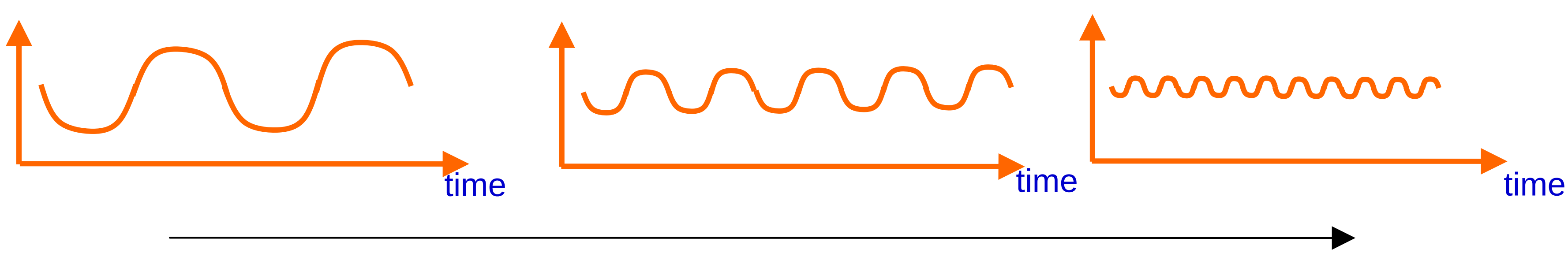
- Coherent interaction between intense pump and signal in a two-level system → population grating




**Population grating coherently scatters energy b/w pump and signal**

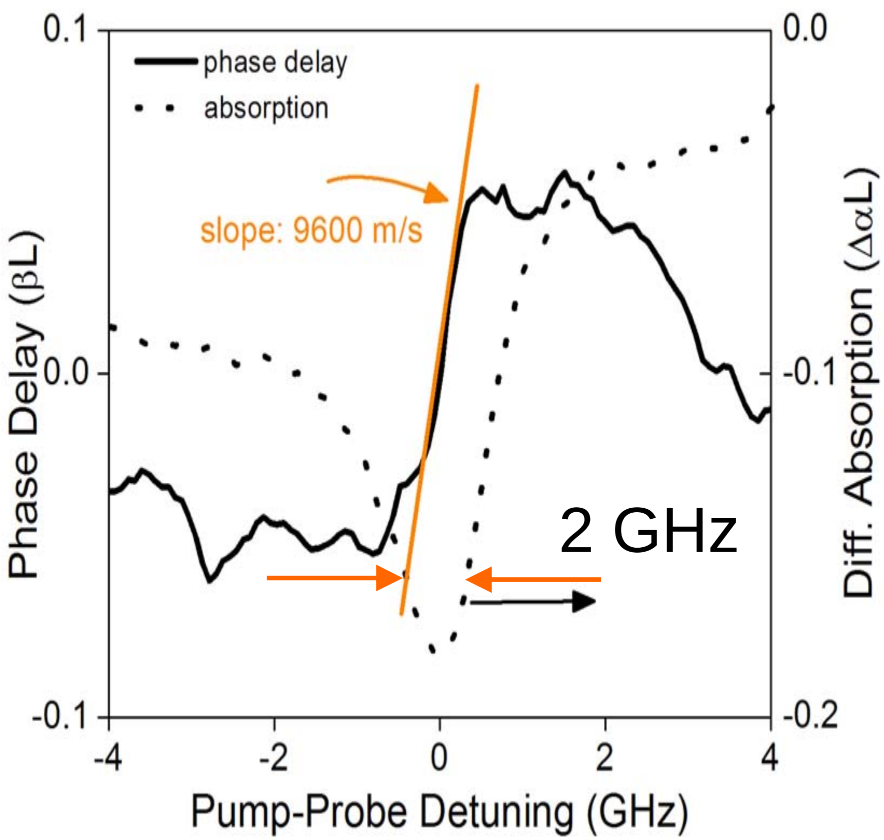
population inversion due to pump

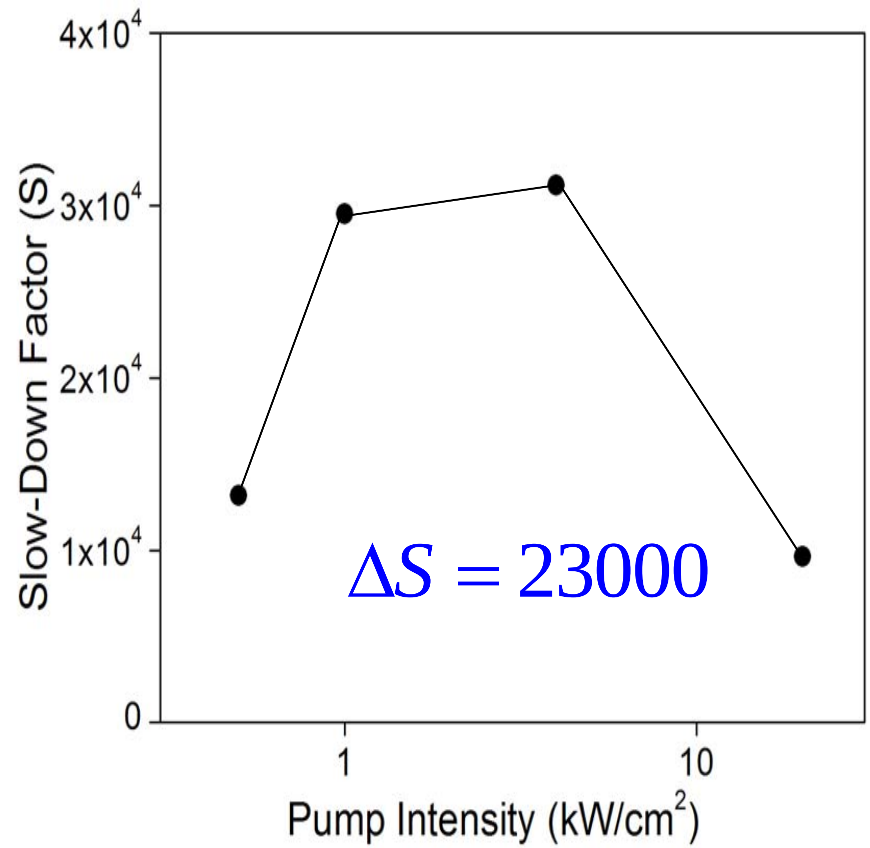



Increasing pump-signal detuning

# PO in semiconductor QWs

**First demonstration of light slowing down in semiconductor materials.**

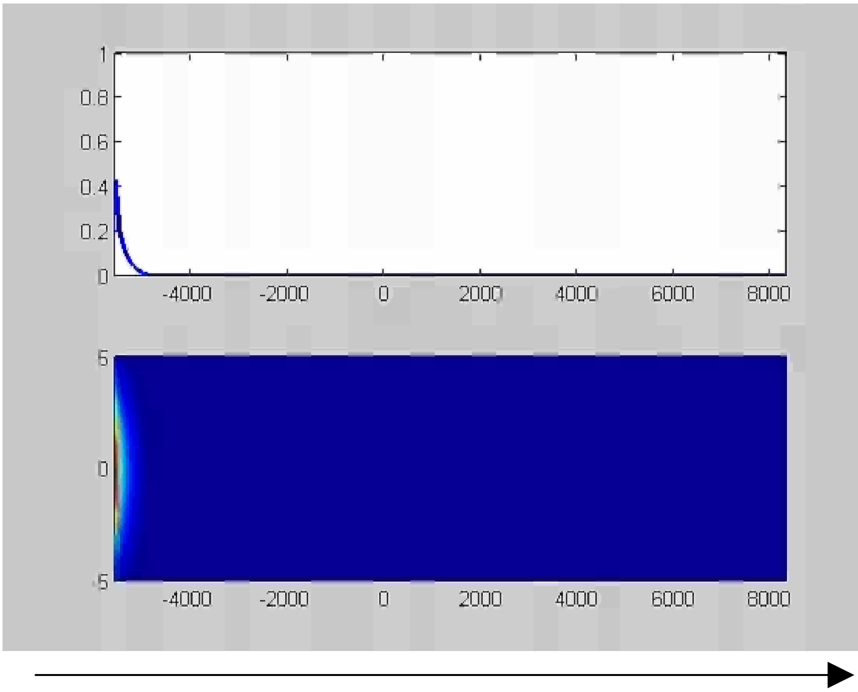


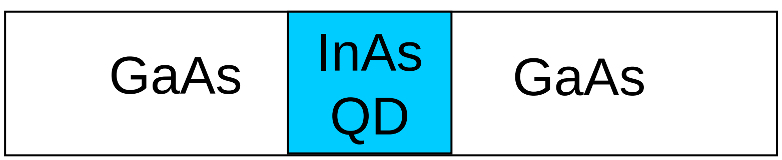

Ref: P. C. Ku et al, Opt. Lett., **29** (2004) 2291.

# Applications of slow/fast light

Computer simulation of 40 Gbs pseudorandom digitally modulated optical signal transmission through a uniform quantum dot waveguide




distance

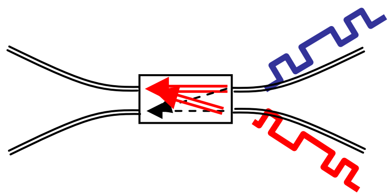



$\gamma_H$=1meV, $\gamma_{31}$=1$\mu$eV, uniform sample
Ku and Chang-Hasnain, UC Berkeley Press Release (2004)

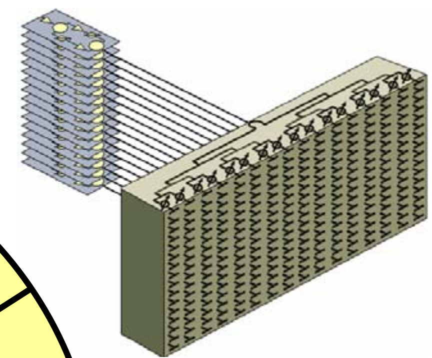
Slow light ↔ Optical signal
Brake ↔ Vehicle

# Scope of applications

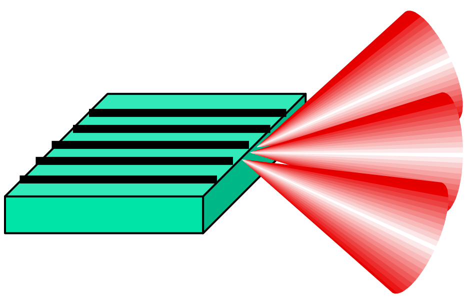
**optical switch**
**contention resolution**

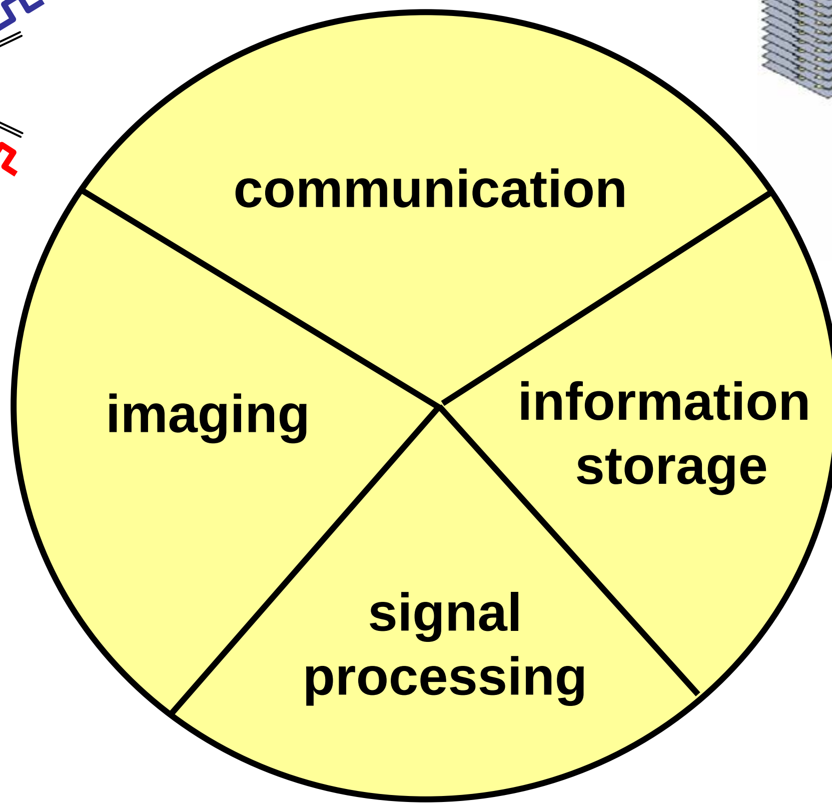


**phased array antenna**

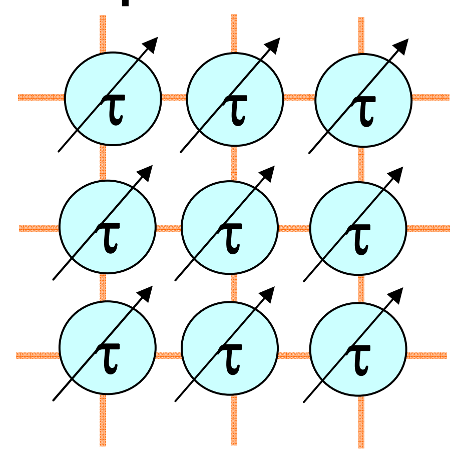


**beam steering**






**optical RAM**




**convolution**

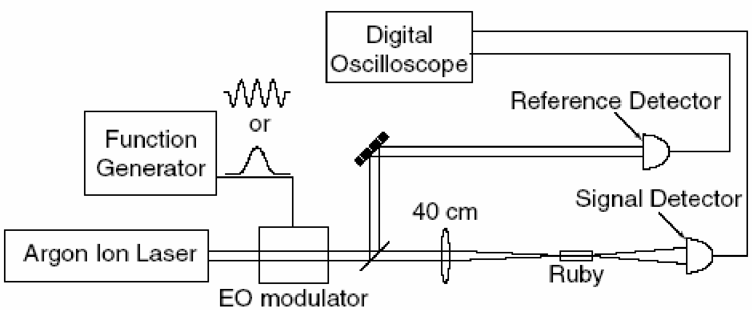
$$\int f_1(\tau) f_2(t-\tau)\,d\tau = F_1(\omega) F_2(\omega)$$

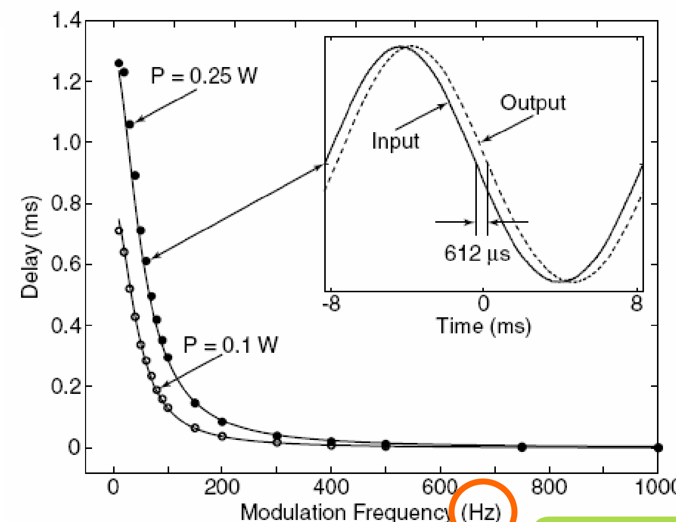
# Trade-off that matters in applications

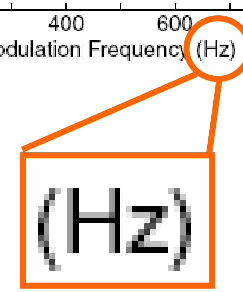
**Observation of Ultraslow Light Propagation in a Ruby Crystal at Room Temperature**

Matthew S. Bigelow, Nick N. Lepeshkin, and Robert W. Boyd
*The Institute of Optics, University of Rochester, Rochester, New York 14627*
(Received 31 October 2002; published 21 March 2003)

We have observed slow light propagation with a group velocity as low as 57.5 ± 0.5 m/s at room temperature in a ruby crystal. A quantum coherence effect, coherent population oscillations, produces a very narrow spectral "hole" in the homogeneously broadened absorption profile of ruby. The resulting rapid spectral variation of the refractive index leads to a large value of the group index. We observe slow light propagation both for Gaussian-shaped light pulses and for amplitude modulated optical beams in a system that is much simpler than those previously used for generating slow light.



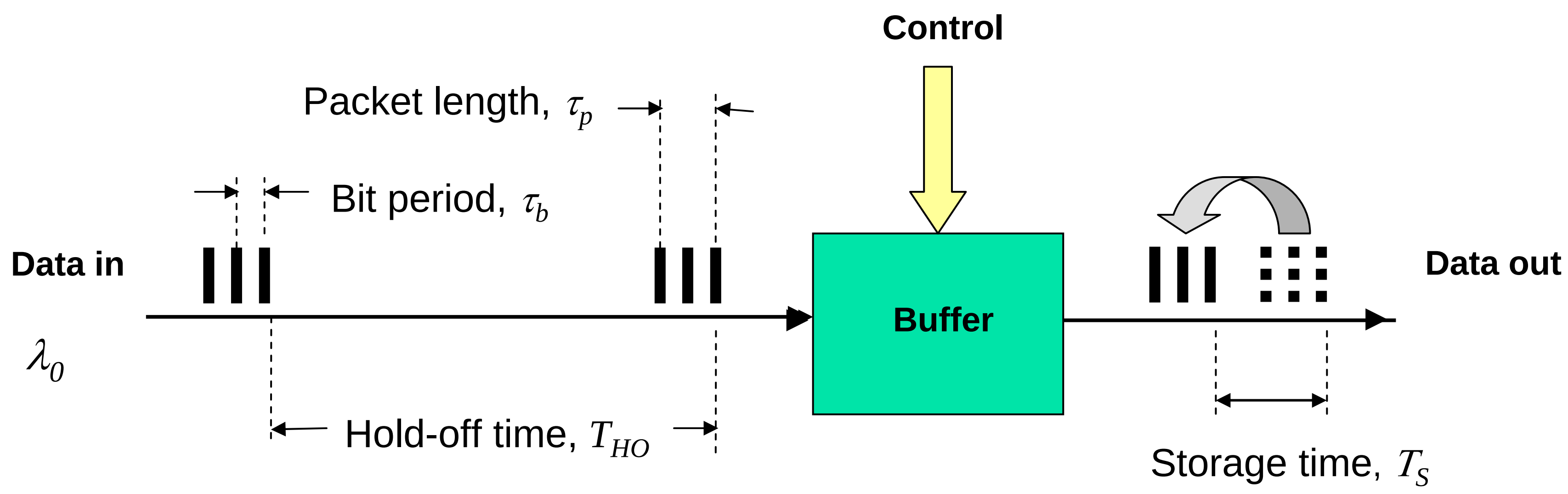




Many applications demand bandwidth

$$D_S = T_S B_{eff} / L = B_S / v_g \sim \text{const}$$

# Slow-light buffer




- Figure of merit : storage density $D_S$ and efficiency $E_S$

$$D_S = \frac{T_S\left(B_{packet}\tau_p / T_{HO}\right)}{L}$$ ~ bandwidth-delay product / $L$

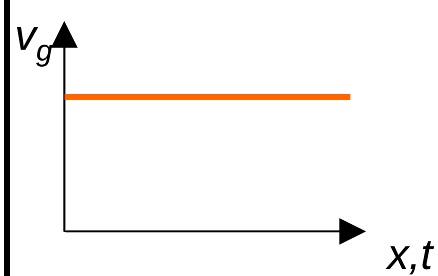
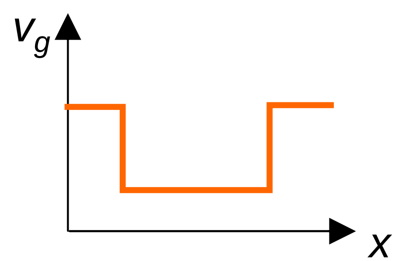
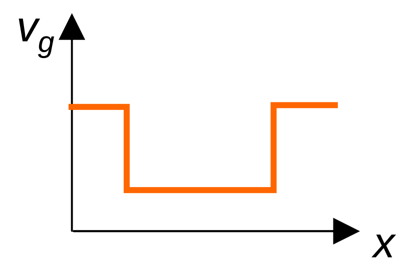
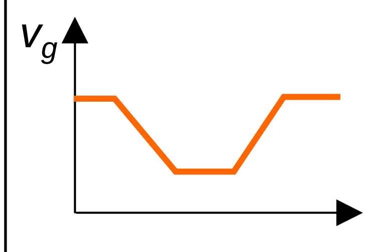
$$E_S = D_S T_S / L$$ ~ storage density x storage time

# Buffer classification and hold-off time

$$S = \frac{c}{v_g} = \frac{n + \omega \frac{\partial n}{\partial \omega}}{1 - \frac{\omega}{c}\frac{\partial n}{\partial k}}$$

Material dispersion

Spatial variation of $n$

| Class 1 | Class 2 | Class 3 | |
|---|---|---|---|
| | | A | B |
|  |  |  |  |
| Fiber delay line | EIT and PO | Photonic crystal based | Photonic crystal based with index tuning |
| $T_{HO}$ = Loop time | $T_{HO} = T_R$ | $T_{HO} = T_R$ | $T_{HO}$ = Loop time |

$T_R = \begin{cases} 0 \\ \text{Loop time if } v_g \text{ needs to be changed} \end{cases}$

# Scaling and fundamental limit

| Class | Single wavelength operation | | WDM | |
|---|---|---|---|---|
| 1 | $D_S = B_{packet} n / c \ll n_{avg} / \lambda_0$ | $E_S = B_{packet} (n/c)^2$ | $D_S = B_{fiber} n / c$ | $E_S = B_{fiber} (n/c)^2$ |
| 2 | $D_S = \Delta\alpha(0) \le n_{avg} / \lambda_0$ | $E_S = \alpha^2 / B_{DL}$<br>$B_{DL} = B_{packet} \tau_p / T_{HO}$ | $D_S = N \Delta\alpha(0)$ | $E_S = (N\alpha)^2 / B_{DL}$ |
| 3 | $D_S = n_{avg} / \lambda_0$ | $E_S = (n_{avg} / \lambda_0)^2 / B_{DL}$ | $D_S = N n_{avg} / \lambda_0$ | $E_S = (N n_{avg} / \lambda_0)^2 / B_{DL}$ |

R. S. Tucker, P. C. Ku, C. Chang-Hasnain, OFC 2005.

- Fundamental limit of storage density is inversely proportional to the optical wavelength in the device → 1/500 nm for 1.55 μm signal
- Storage efficiency rolls off for high bit rate signal.
- Slow-light buffer favors medium bit-rate WDM system.

# Can we overcome the trade-off?

- Adding gain to the system may overcome the bandwidth-delay product tradeoff. But distortion needs to be characterized and compensated.