EECS 598-002 Nanophotonics and Nanofabrication
Winter 2006
Homework 1 (due Feb 2 Thursday by 5 pm)

Problem 1. Brewster's Angle

In the class, we derived the Snell's Law with the incident magnetic field polarized perpendicular to the plane (TM polarized). We noticed that there existed an angle at which the reflection vanished. We called it the Brewster's angle.

1. **Duality Principle:** Verify that the Maxwell's Equations remain the same with the following exchange of variables:

$$\begin{aligned} &\vec{E} \to \vec{H} \\ &\vec{H} \to -\vec{E} \\ &\mu \to \varepsilon \\ &\varepsilon \to \mu \\ &\vec{J} \to \vec{M} \\ &\vec{M} \to -\vec{J} \end{aligned}$$

This is the duality principle of the electromagnetics. Applying the duality principle to the TM-polarized results given in the class and argue that these are nothing but the TE case. Does the Brewster's angle exist in the TE case?

2. Please use the induced dipole moment picture we discussed in the class along with the radiation pattern of a dipole moment to explain qualitatively why Brewster's angle only exists in the TM case while not in the TE case.
3. List all applications you can think of from the Brewster's angle.

Problem 2. Left-Handed Materials

In a paper published in 1968 (Sov. Phys. Usp. **10** (1968) 509), Veselago investigated the propagation of plane waves in a medium that possesses both negative ε and μ.

1. From the Maxwell Equations, argue that the vectors **E**, **H**, and the wave vector **k** form a left-handed system, that is the direction of **ExH** is in opposite to the direction of **k**.
2. Explain why in a left-handed medium, the refractive index $n = \sqrt{\varepsilon\mu}$ must be taken the negative sign. (Ref: Veselago's paper)
3. Compare the refraction behavior between right-handed materials ($\varepsilon>0$ and $\mu>0$) and left-handed materials. You will need to re-derive the results in the class as $\mu_1 \neq \mu_2$ in this case.
4. What happens to the refracted wave if the total internal reflection condition is met? Describe the behaviors of the evanescent waves.

Problem 3. Photon tunneling and applications in optical microscopes:

In the class, we outlined the results of EM wave tunneling through a narrow layer under the total internal reflection condition. This has been applied to a photon scanning tunneling microscope (PSTM; J. Vac. Sci. Technol. B **9** (1991) 525.)

1. Calculate the reflection and transmission coefficients for the TE case. The reflection and transmission coefficients are defined as the fraction of intensity ($\propto |E|^2$) that is reflected or transmitted through the narrow layer, respectively.
2. Calculate the Poynting vector inside the narrow layer. Is it zero? If so, why? If not, why?
3. Explain briefly how this tunneling effect can be applied to a scanning tunneling microscope that uses light to detect the surface topography.
4. Do your best to explain a physical picture of this tunneling effect.