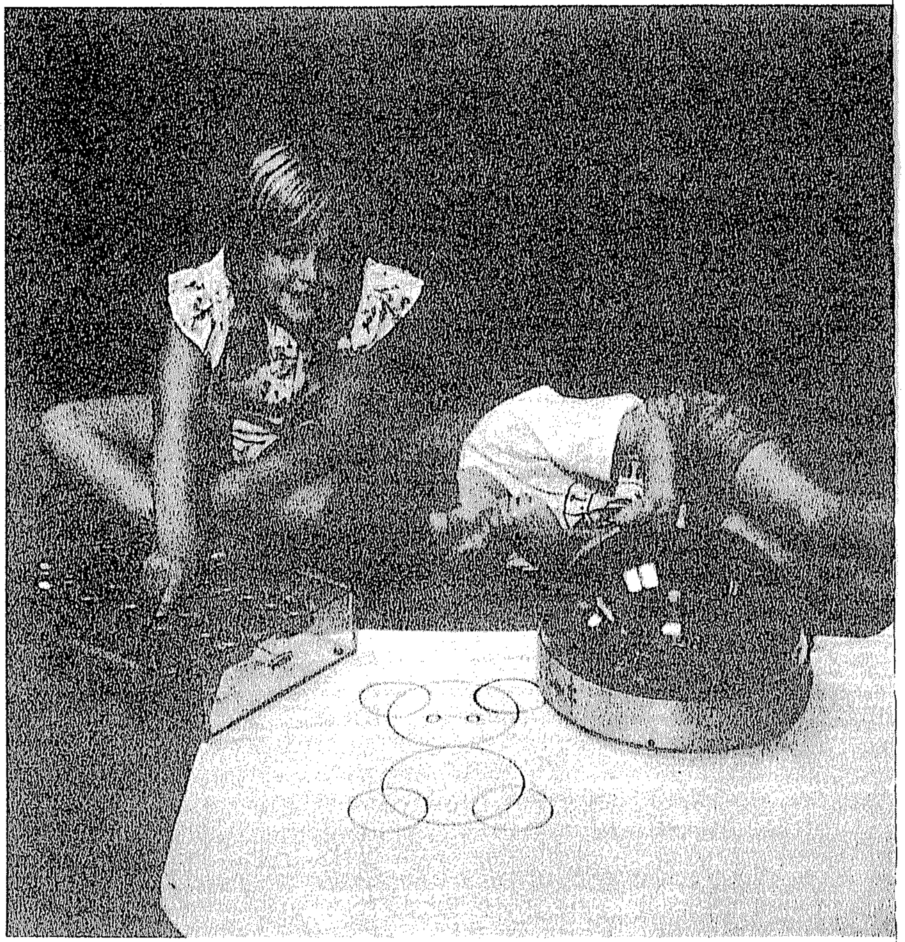

Frontispiece: LOGO Turtle.

MINDSTORMS

Children, Computers, and Powerful Ideas

SECOND EDITION

SEYMOUR PAPERT

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Chapter 2

Mathophobia: The Fear of Learning

PLATO WROTE over his door, "Let only geometers enter." Times have changed. Most of those who now seek to enter Plato's intellectual world neither know mathematics nor sense the least contradiction in their disregard for his injunction. Our culture's schizophrenic split between "humanities" and "science" supports their sense of security. Plato was a philosopher, and philosophy belongs to the humanities as surely as mathematics belongs to the sciences.

This great divide is thoroughly built into our language, our worldview, our social organization, our educational system, and, most recently, even our theories of neurophysiology. It is self-perpetuating: The more the culture is divided, the more each side builds separation into its new growth.

I have already suggested that the computer may serve as a force to break down the line between the "two cultures." I know that the humanist may find it questionable that a "technology" could change his assumptions about what kind of knowledge is relevant to his or her perspective of understanding people. And to the scientist dilution of rigor by the encroachment of "wishy-washy" humanistic thinking can be no less threatening. Yet the computer

presence might, I think, plant seeds that could grow into a less dissociated cultural epistemology.

The status of mathematics in contemporary culture is one of the most acute symptoms of its dissociation. The emergence of a "humanistic" mathematics, one that is not perceived as separated from the study of man and "the humanities," might well be the sign that a change is in sight. So in this book I try to show how the computer presence can bring children into a more humanistic as well as a more humane relationship with mathematics. In doing so I shall have to go beyond discussion of mathematics. I shall have to develop a new perspective on the process of learning itself.

It is not uncommon for intelligent adults to turn into passive observers of their own incompetence in anything but the most rudimentary mathematics. Individuals may see the direct consequences of this intellectual paralysis in terms of limiting job possibilities. But the indirect, secondary consequences are even more serious. One of the main lessons learned by most people in math class is a sense of having rigid limitations. They learn a balkanized image of human knowledge which they come to see as a patchwork of territories separated by impassable iron curtains. My challenge is not to the sovereignty of the intellectual territories but to the restrictions imposed on easy movement among them. I do not wish to reduce mathematics to literature or literature to mathematics. But I do want to argue that their respective ways of thinking are not as separate as is usually supposed. And so, I use the image of a Mathland—where mathematics would become a natural vocabulary—to develop my idea that the computer presence could bring the humanistic and mathematical/scientific cultures together. In this book, Mathland is the first step in a larger argument about how the computer presence can change not only the way we teach children mathematics, but, much more fundamentally, the way in which our culture as a whole thinks about knowledge and learning.

To my ear the word "mathophobia" has two associations. One of these is a widespread fear of mathematics, which often has the intensity of a real phobia. The other comes from the meaning of the stem "math." In Greek it means "learning" in a general sense.* In

*The original meaning is present in the word "polymath," a person of many learnings. A less well-known word with the same stem which I shall use in later chapters is "mathetic," having to do with learning.

our culture, fear of learning is no less endemic (although more frequently disguised) than fear of mathematics. Children begin their lives as eager and competent learners. They have to *learn* to have trouble with learning in general and mathematics in particular. In both senses of "math" there is a shift from mathophile to mathophobe, from lover of mathematics and of learning to a person fearful of both. We shall look at how this shift occurs and develop some idea of how the computer presence could serve to counteract it. Let me begin with some reflections on what it is like to learn as a child.

That children learn a great deal seems so obvious to most people that they believe it is scarcely worth documenting. One area in which a high rate of learning is very plain is the acquisition of a spoken vocabulary. At age two very few children have more than a few hundred words. By the time they enter first grade, four years later, they know thousands of words. They are evidently learning many new words every day.

While we can "see" that children learn words, it is not quite as easy to see that they are learning mathematics at a similar or greater rate. But this is precisely what has been shown by Piaget's life-long study of the genesis of knowledge in children. One of the more subtle consequences of his discoveries is the revelation that adults fail to appreciate the extent and the nature of what children are learning, because knowledge structures we take for granted have rendered much of that learning invisible. We see this most clearly in what have come to be known as Piagetian "conservations" (see Figure 2).

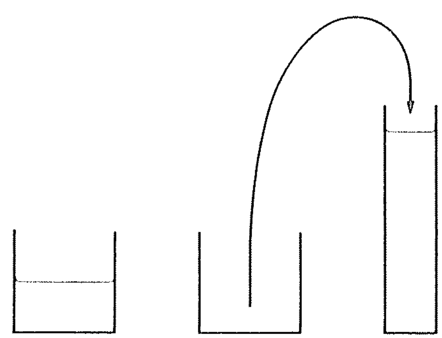

Figure 2 The Conservation of Liquids

For an adult it is obvious that pouring liquid from one glass to another does not change the volume (ignoring such little effects as drops that spilled or remained behind). The conservation of volume is so obvious that it seems not to have occurred to anyone before Piaget that children of four might not find it obvious at all.* A substantial intellectual growth is needed before children develop the "conservationist" view of the world. The conservation of volume is only one of many conservations they all learn. Another is the conservation of numbers. Again, it does not occur to most adults that a child must learn that counting a collection of objects in a different order should yield the same result. For adults counting is simply a method of determining how many objects "there are." The result of the operation is an "objective fact" independent of the act of counting. But the separation of number from counting (of product from process) rests on epistemological presuppositions not only unknown to preconservationist children, but alien to their worldview. These conservations are only part of a vast structure of "hidden" mathematical knowledge that children learn by themselves. In the intuitive geometry of the child of four or five, a straight line is not necessarily the shortest distance between two points, and walking slowly between two points does not necessarily take more time than walking fast. Here, too, it is not merely the "item" of knowledge that is missing, but the epistemological presupposition underlying the idea of "shortest" as a property of the path rather than of the action of traversing it.

None of this should be understood as mere *lack* of knowledge on the part of the children. Piaget has demonstrated how young children hold theories of the world that, in their own terms, are perfectly coherent. These theories, spontaneously "learned" by all children, have well-developed components that are not less "mathematical," though expressing a different mathematics, than the one generally accepted in our (adult) culture. The hidden learning process has at least two phases: Already in the preschool years every child first constructs one or more preadult theorizations of the

*People have lived with children for a long time. The fact that we had to wait for Piaget to tell us how children think and *what we all forget about our thinking as children* is so remarkable that it suggests a Freudian model of "cognitive repression."

world and then moves toward more adultlike views. And all this is done through what I have called Piagetian learning, a learning process that has many features the schools should envy: It is effective (all the children get there), it is inexpensive (it seems to require neither teacher nor curriculum development), and it is humane (the children seem to do it in a carefree spirit without explicit external rewards and punishments).

The extent to which adults in our society have lost the child's positive stance toward learning varies from individual to individual. An unknown but certainly significant proportion of the population has almost completely given up on learning. These people seldom, if ever, engage in deliberate learning and see themselves as neither competent at it nor likely to enjoy it. The social and personal cost is enormous: Mathophobia can, culturally and materially, limit people's lives. Many more people have not completely given up on learning but are still severely hampered by entrenched negative beliefs about their capacities. Deficiency becomes identity: "I can't learn French, I don't have an ear for languages;" "I could never be a businessman, I don't have a head for figures;" "I can't get the hang of parallel skiing, I never was coordinated." These beliefs are often repeated ritualistically, like superstitions. And, like superstitions, they create a world of taboos; in this case, taboos on learning. In this chapter and chapter 3, we discuss experiments that demonstrate that these self-images often correspond to a very limited reality—usually to a person's "school reality." In a learning environment with the proper emotional and intellectual support, the "uncoordinated" can learn circus arts like juggling and those with "no head for figures" learn not only that they can do mathematics but that they can enjoy it as well.

Although these negative self-images can be overcome, in the life of an individual they are extremely robust and powerfully self-reinforcing. If people believe firmly enough that they cannot do math, they will usually succeed in preventing themselves from doing whatever they recognize as math. The consequences of such self-sabotage is personal failure, and each failure reinforces the original belief. And such beliefs may be most insidious when held not only by individuals, but by our entire culture.

Our children grow up in a culture permeated with the idea that there are "smart people" and "dumb people." The social construction of the individual is as a bundle of aptitudes. There are people who are "good at math" and people who "can't do math." Everything is set up for children to attribute their first unsuccessful or unpleasant learning experiences to their own disabilities. As a result, children perceive failure as relegating them either to the group of "dumb people" or, more often, to a group of people "dumb at x" (where, as we have pointed out, x often equals mathematics). Within this framework children will define themselves in terms of their limitations, and this definition will be consolidated and reinforced throughout their lives. Only rarely does some exceptional event lead people to reorganize their intellectual self-image in such a way as to open up new perspectives on what is learnable.

This belief about the structure of human abilities is not easy to undermine. It is never easy to uproot popular beliefs. But here the difficulty is compounded by several other factors. First, popular theories about human aptitudes seem to be supported by "scientific" ones. After all, psychologists talk in terms of measuring aptitudes. But the significance of what is measured is seriously questioned by our simple thought experiment of imagining Mathland.

Although the thought experiment of imagining a Mathland leaves open the question of how a Mathland can actually be created, it is completely rigorous as a demonstration that the accepted beliefs about mathematical aptitude do not follow from the available evidence.[1] But since truly mathophobic readers might have trouble making this experiment their own, I shall reinforce the argument by casting it in another form. Imagine that children were forced to spend an hour a day drawing dance steps on squared paper and had to pass tests in these "dance facts" before they were allowed to dance physically. Would we not expect the world to be full of "dancophobes"? Would we say that those who made it to the dance floor and music had the greatest "aptitude for dance"? In my view, it is no more appropriate to draw conclusions about mathematical aptitude from children's unwillingness to spend many hundreds of hours doing sums.

One might hope that if we pass from parables to the more rigorous methods of psychology we could get some "harder" data on the problem of the true ceilings of competence attainable by individuals. But this is not so: The paradigm in use by contemporary educational psychology is focused on investigations of how children learn or (more usually) don't learn mathematics in the "anti-Mathland" in which we all live. The direction of such research has an analogy in the following parable:

> Imagine someone living in the nineteenth century who felt the need to improve methods of transportation. He was persuaded that the route to new methods started with a deep understanding of the existing problems. So he began a careful study of the differences among horse-drawn carriages. He carefully documented by the most refined methods how speed varied with the form and substance of various kinds of axles, bearings, and harnessing techniques.

In retrospect, we know that the road that led from nineteenth-century transportation was quite different. The invention of the automobile and the airplane did not come from a detailed study of how their predecessors, such as horse-drawn carriages, worked or did not work. Yet, this is the model for contemporary educational research. The standard paradigms for education research take the existing classroom or extracurricular culture as the primary object of study. There are many studies concerning the poor notions of math or science students acquire from today's schooling. There is even a very prevalent "humanistic" argument that "good" pedagogy should take these poor ways of thinking as its starting point. It is easy to sympathize with the humane intent. Nevertheless I think that the strategy implies a commitment to preserving the traditional system. It is analogous to improving the axle of the horse-drawn cart. But the real question, one might say, is whether we can invent the "educational automobile." Since this question (the central theme of this book) has not been addressed by educational psychology, we must conclude that the "scientific" basis for beliefs about aptitudes is really very shaky. But these beliefs are institutionalized in schools, in testing systems, and in college admissions criteria and consequently, their social basis is as firm as their scientific basis is weak.

From kindergarten on, children are tested for verbal and quanti-

tative aptitudes, conceived of as "real" and separable entities. The results of these tests enter into the social construction of each child as a bundle of aptitudes. Once Johnny and his teacher have a shared perception of Johnny as a person who is "good at" art and "poor at" math, this perception has a strong tendency to dig itself in. This much is widely accepted in contemporary educational psychology. But there are deeper aspects to how school constructs aptitudes. Consider the case of a child I observed through his eighth and ninth years. Jim was a highly verbal and mathophobic child from a professional family. His love for words and for talking showed itself very early, long before he went to school. The mathophobia developed at school. My theory is that it came as a direct result of his verbal precocity. I learned from his parents that Jim had developed an early habit of describing in words, often aloud, whatever he was doing as he did it. This habit caused him minor difficulties with parents and preschool teachers. The real trouble came when he hit the arithmetic class. By this time he had learned to keep "talking aloud" under control, but I believe that he still maintained his inner running commentary on his activities. In his math class he was stymied: He simply did not know how to talk about doing sums. He lacked a vocabulary (as most of us do) and a sense of purpose. Out of this frustration of his verbal habits grew a hatred of math, and out of the hatred grew what the tests later confirmed as poor aptitude.

For me the story is poignant. I am convinced that what shows up as intellectual weakness very often grows, as Jim's did, out of intellectual strengths. And it is not only verbal strengths that undermine others. Every careful observer of children must have seen similar processes working in different directions: For example, a child who has become enamored of logical order is set up to be turned off by English spelling and to go on from there to develop a global dislike for writing.

The Mathland concept shows how to use computers as vehicles to escape from the situation of Jim and his dyslexic counterpart. Both children are victims of our culture's hard-edged separation between the verbal and the mathematical. In the Mathland we shall describe in this chapter, Jim's love and skill for language could be mobilized to serve his formal mathematical development instead of opposing it, and the other child's love for logic could be recruited to serve the development of interest in linguistics.

The concept of mobilizing a child's multiple strengths to serve all domains of intellectual activity is an answer to the suggestion that differing aptitudes may reflect actual differences in brain development. It has become commonplace to talk as if there are separate brains, or separate "organs" in the brain, for mathematics and for language. According to this way of thinking, children split into the verbally and the mathematically apt depending on which brain organs are strongest. But the argument from anatomy to intellect reflects a set of epistemological assumptions. It assumes, for example, that there is only one route to mathematics and that if this route is "anatomically blocked," the child cannot get to the destination. Now, in fact, for most children in contemporary societies there may indeed be only one route into "advanced" mathematics, the route via school math. But even if further research in brain biology confirms that this route depends on anatomical brain organs that might be missing in some children, it would not follow that mathematics itself is dependent on these brain organs. Rather, it would follow that we should seek out other routes. Since this book is an argument that alternate routes do exist, it can be read as showing how the dependency of function on brain is itself a social construct.

Let us grant, for the sake of argument, that there is a special part of the brain especially good at performing the mental manipulations of numbers we teach children in school, and let's call it the MAD, or "math acquisition device."[2] On this assumption it would make sense that in the course of history humankind would have evolved methods of doing and of teaching arithmetic that take full advantage of the MAD. But while these methods would work for most of us, and so for society as a whole, reliance on them would be catastrophic for an individual whose MAD happened to be damaged or inaccessible for some other (perhaps "neurotic") reason. Such a person would fail at school and be diagnosed as a victim of "dyscalculia." And as long as we insist on making children learn arithmetic by the standard route, we will continue to "prove" by

objective tests that these children really cannot "do arithmetic." But this is like proving that the deaf children cannot have language because they don't hear. Just as sign languages use hands and eyes to bypass the more usual speaking organs so, too, alternative ways of doing mathematics that bypass the MAD may be as good as, even if different from, the usual ones.

But we do not have to appeal to neurology to explain why some children do not become fluent in mathematics. The analogy of the dance class without music or dance floor is a serious one. Our education culture gives mathematics learners scarce resources for making sense of what they are learning. As a result our children are forced to follow the very worst model for learning mathematics. This is the model of rote learning, where material is treated as meaningless; it is a *dissociated* model. Some of our difficulties in teaching a more culturally integrable mathematics have been due to an objective problem: Before we had computers there were very few good points of contact between what is most fundamental and engaging in mathematics and anything firmly planted in everyday life. But the computer—a mathematics-speaking being in the midst of the everyday life of the home, school, and workplace—is able to provide such links. The challenge to education is to find ways to exploit them.

Mathematics is certainly not the only example of dissociated learning. But it is a very good example for precisely the reason that many readers are probably now wishing that I would talk about something else. Our culture is so mathophobic, so math-fearing, that if I could demonstrate how the computer can bring us into a new relationship to mathematics, I would have a strong foundation for claiming that the computer has the ability to change our relation to other kinds of learning we might fear. Experiences in Mathland, such as entering into a "mathematical conversation," give the individual a liberating sense of the possibilities of doing a variety of things that may have previously seemed "too hard." In this sense, contact with the computer can open access to knowledge for people, not instrumentally by providing them with processed information, but by challenging some constraining assumptions they make about themselves.

The computer-based Mathland I propose extends the kind of natural, Piagetian learning that accounts for children's learning a first language to learning mathematics. Piagetian learning is typically deeply embedded in other activities. For example, the infant does not have periods set aside for "learning talking." This model of learning stands in opposition to dissociated learning, learning that takes place in relative separation from other kinds of activities, mental and physical. In our culture, the teaching of mathematics in schools is paradigmatic of dissociated learning. For most people, mathematics is taught and taken as medicine. In its dissociation of mathematics, our culture comes closest to caricaturing its own worst habits of epistemological alienation. In LOGO environments we have done some blurring of boundaries: No particular computer activities are set aside as "learning mathematics."

The problem of making mathematics "make sense" to the learner touches on the more general problem of making a language of "formal description" make sense. So before turning to examples of how the computer helps give meaning to mathematics, we shall look at several examples where the computer helped give meaning to a language of formal description in domains of knowledge that people do not usually count as mathematics. In our first example the domain is grammar, for many people a subject only a little less threatening than math.

Well into a year-long study that put powerful computers in the classrooms of a group of "average" seventh graders, the students were at work on what they called "computer poetry." They were using computer programs to generate sentences. They gave the computer a syntactic structure within which to make random choices from given lists of words. The result is the kind of concrete poetry we see in the illustration that follows. One of the students, a thirteen-year-old named Jenny, had deeply touched the project's staff by asking on the first day of her computer work, "Why were we chosen for this? We're not the brains." The study had deliberately chosen children of "average" school performance. One day Jenny came in very excited. She had made a discovery. "Now I know why we have nouns and verbs," she said. For many years in school Jenny had been drilled in grammatical categories. She had

never understood the differences between nouns and verbs and adverbs. But now it was apparent that her difficulty with grammar was not due to an inability to work with logical categories. It was something else. She had simply seen no purpose in the enterprise. She had not been able to make any sense of what grammar was about in the sense of what it might be *for*. And when she had asked what it was for, the explanations that her teachers gave seemed manifestly dishonest. She said she had been told that "grammar helps you talk better."

INSANE RETARD MAKES BECAUSE SWEET SNOOPY SCREAMS
SEXY WOLF LOVES THATS WHY THE SEXY LADY HATES
UGLY MAN LOVES BECAUSE UGLY DOG HATES
MAD WOLF HATES BECAUSE INSANE WOLF SKIPS
SEXY RETARD SCREAMS THATS WHY THE SEXY RETARD HATES
THIN SNOOPY RUNS BECAUSE FAT WOLF HOPS
SWEET FOGINY SKIPS A FAT LADY RUNS

Jenny's Concrete Poetry

In fact, tracing the connection between learning grammar and improving speech requires a more distanced view of the complex process of learning language than Jenny could have been given at the age she first encountered grammar. She certainly didn't see any way in which grammar could help talking, nor did she think her talking needed any help. Therefore she learned to approach grammar with resentment. And, as is the case for most of us, resentment guaranteed failure. But now, as she tried to get the computer to generate poetry, something remarkable happened. She found herself classifying words into categories, not because she had been told she had to but because she needed to. In order to "teach" her computer to make strings of words that would look like English, she had to "teach" it to choose words of an appropriate class. What she learned about grammar from this experience with a machine was anything but mechanical or routine. Her learning was deep and meaningful. Jenny did more than learn definitions for particular grammatical classes. She understood the general idea that words (like things) can be placed in different groups or sets, and that doing so could work for her. She not only "understood" grammar, she changed her relationship to it. It was "hers," and during her year with the computer, incidents like this helped Jenny change her image of herself. Her performance changed too; her previously low to average grades became "straight A's" for her remaining years of school. She learned that she could be "a brain" after all.

It is easy to understand why math and grammar fail to make sense to children when they fail to make sense to everyone around them and why helping children to make sense of them requires more than a teacher making the right speech or putting the right diagram on the board. I have asked many teachers and parents what they thought mathematics to be and why it was important to learn it. Few held a view of mathematics that was sufficiently coherent to justify devoting several thousand hours of a child's life to learning it, and children sense this. When a teacher tells a student that the reason for those many hours of arithmetic is to be able to check the change at the supermarket, the teacher is simply not believed. Children see such "reasons" as one more example of adult double talk. The same effect is produced when children are told school math is "fun" when they are pretty sure that teachers who say so spend their leisure hours on anything except this allegedly fun-filled activity. Nor does it help to tell them that they need math to become scientists—most children don't have such a plan. The children can see perfectly well that the teacher does not like math any more than they do and that the reason for doing it is simply that it has been inscribed into the curriculum. All of this erodes children's confidence in the adult world and the process of education. *And I think it introduces a deep element of dishonesty into the educational relationship.*

Children perceive the school's rhetoric about mathematics as double talk. In order to remedy the situation we must first acknowledge that the child's perception is fundamentally correct. The *kind of mathematics* foisted on children in schools is not meaningful, fun, or even very useful. This does not mean that an individual child cannot turn it into a valuable and enjoyable personal game.

For some the game is scoring grades; for others it is outwitting the teacher and the system. For many, school math is enjoyable in its repetitiveness, precisely because it is so mindless and dissociated that it provides a shelter from having to think about what is going on in the classroom. But all this proves is the ingenuity of children. It is not a justification for school math to say that *despite* its intrinsic dullness, inventive children can find excitement and meaning in it.

It is important to remember the distinction between *mathematics*—a vast domain of inquiry whose beauty is rarely suspected by most nonmathematicians—and something else which I shall call *math* or *school math.*

I see "school math" as a social construction, a kind of QWERTY. A set of historical accidents (which shall be discussed in a moment) determined the choice of certain mathematical topics as *the* mathematical baggage that citizens should carry. Like the QWERTY arrangement of typewriter keys, school math did make some sense in a certain historical context. But, like QWERTY, it has dug itself in so well that people take it for granted and invent rationalizations for it long after the demise of the historical conditions that made sense of it. Indeed, for most people in our culture it is inconceivable that school math could be very much different: This is the only mathematics they know. In order to break this vicious circle I shall lead the reader into a new area of mathematics, Turtle geometry, that my colleagues and I have created as a better, more meaningful first area of formal mathematics for children. The design criteria of Turtle geometry are best understood by looking a little more closely at the historical conditions responsible for the shape of school math.

Some of these historical conditions were pragmatic. Before electronic calculators existed it was a practical social necessity that many people be "programmed" to perform such operations as long division quickly and accurately. But now that we can purchase calculators cheaply we should reconsider the need to expend several hundred hours of every child's life on learning such arithmetic functions. I do not mean to deny the intellectual value of some knowledge, indeed, of a lot of knowledge, about numbers. Far from it. But we can now select this knowledge on coherent, rational grounds. We can free ourselves from the tyranny of the superficial, pragmatic considerations that dictated past choices about what knowledge should be learned and at what age.

But utility was only one of the historical reasons for school math. Others were of a *mathetic* nature. Mathetics is the set of guiding principles that govern learning. Some of the historical reasons for school math had to do with what was learnable and teachable in the precomputer epoch. As I see it, a major factor that determined what mathematics went into school math was what could be done in the setting of school classrooms with the primitive technology of pencil and paper. For example, children can draw graphs with pencil and paper. So it was decided to let children draw many graphs. The same considerations influenced the emphasis on certain kinds of geometry. For example, in school math "analytic geometry" has become synonymous with the representation of curves by equations. As a result every educated person vaguely remembers that $y = x^2$ is the equation of a parabola. And although most parents have very little idea of why anyone should know this, they become indignant when their children do not. They assume that there must be a profound and objective reason known to those who better understand these things. Ironically, their mathophobia keeps most people from trying to examine those reasons more deeply and thus places them at the mercy of the self-appointed math specialists. Very few people ever suspect that the reason for what is included and what is not included in school math might be as crudely technological as the ease of production of parabolas with pencils! This is what could change most profoundly in a computer-rich world: The range of easily produced mathematical constructs will be vastly expanded.

Another mathetic factor in the social construction of school math is the technology of grading. A living language is learned by speaking and does not need a teacher to verify and grade each sentence. A dead language requires constant "feedback" from a teacher. The activity known as "sums" performs this feedback function in school math. These absurd little repetitive exercises have only one merit: They are easy to grade. But this merit has bought them a firm place at the center of school math. In brief, I maintain that

construction of school math is strongly influenced by what seemed to be teachable when math was taught as a "dead" subject, using the primitive, passive technologies of sticks and sand, chalk and blackboard, pencil and paper. The result was an intellectually incoherent set of topics that violates the most elementary mathetic principles of what makes certain material easy to learn and some almost impossible.

Faced with the heritage of school, math education can take two approaches. The traditional approach accepts school math as a given entity and struggles to find ways to teach it. Some educators use computers for this purpose. Thus, paradoxically, the most common use of the computer in education has become force-feeding indigestible material left over from the precomputer epoch. In Turtle geometry the computer has a totally different use. There the computer is used as a mathematically expressive medium, one that frees us to design personally meaningful and intellectually coherent and easily learnable mathematical topics for children. Instead of posing the educational problem as "how to teach the existing school math," we pose it as "reconstructing mathematics," or more generally, as reconstructing knowledge in such a way that no great effort is needed to teach it.

All "curriculum development" could be described as "reconstructing knowledge." For example, the New Math curriculum reform of the sixties made some attempt to change the content of school math. But it could not go very far. It was stuck with having to do sums, albeit different sums. The fact that the new sums dealt with sets instead of numbers, or arithmetic in base two instead of base ten made little difference. Moreover, the math reform did not provide a challenge to the inventiveness of creative mathematicians and so never acquired the sparkle of excitement that marks the product of new thought. The name itself—"New Math"—was a misnomer. There was very little new about its mathematical content: It did not come from a process of invention of children's mathematics but from a process of trivialization of mathematician's mathematics. Children need and deserve something better than selecting out pieces of old mathematics. Like clothing passed down to the younger siblings, it never fits comfortably.

Turtle geometry started with the goal of fitting children. Its primary design criterion was to be *appropriable*. Of course it had to have serious mathematical content, but we shall see that appropriability and serious mathematic thinking are not at all incompatible. On the contrary: We shall end up understanding that some of the most personal knowledge is also the most profoundly mathematical. In many ways mathematics—for example the mathematics of space and movement and repetitive patterns of action—is what comes most naturally to children. It is into this mathematics that we sink the tap-root of Turtle geometry. As my colleagues and I have worked through these ideas, a number of principles have given more structure to the concept of an appropriable mathematics. First, there was the *continuity principle*: The mathematics must be continuous with well-established personal knowledge from which it can inherit a sense of warmth and value as well as "cognitive" competence. Then there was the *power principle*: It must empower the learner to perform personally meaningful projects that could not be done without it. Finally there was a *principle of cultural resonance*: The topic must make sense in terms of a larger social context. I have spoken of Turtle geometry making sense to children. But it will not truly make sense to children unless it is accepted by adults too. A dignified mathematics for children cannot be something we permit ourselves to inflict on children, like unpleasant medicine, although we see no reason to take it ourselves.