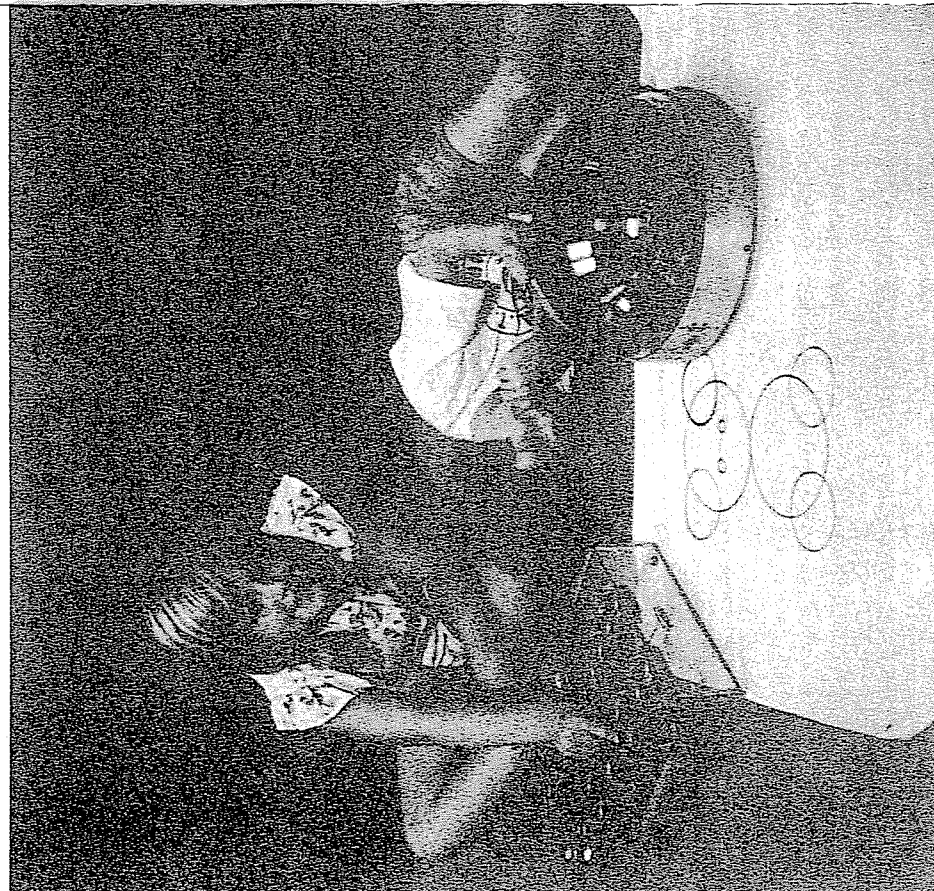


# MINDSTORMS

Children, Computers,  
and Powerful Ideas

SECOND EDITION

SEYMOUR PAPERT



Frontpiece: LEGO Turtle.



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# The Gears of My Childhood

BEFORE I WAS two years old I had developed an intense involvement with automobiles. The names of car parts made up a very substantial portion of my vocabulary: I was particularly proud of knowing about the parts of the transmission system, the gearbox, and most especially the differential. It was, of course, many years later before I understood how gears work; but once I did, playing with gears became a favorite pastime. I loved rotating circular objects against one another in gearlike motions and, naturally, my first "erector set" project was a crude gear system.

I became adept at turning wheels in my head and at making chains of cause and effect: "This one turns this way so that must turn that way so . . ." I found particular pleasure in such systems as the differential gear, which does not follow a simple linear chain of causality since the motion in the transmission shaft can be distributed in many different ways to the two wheels depending on what resistance they encounter. I remember quite vividly my excitement at discovering that a system could be lawful and completely comprehensible without being rigidly deterministic.

I believe that working with differentials did more for my mathematical development than anything I was taught in elementary school. Gears, serving as models, carried many otherwise abstract

ideas into my head. I clearly remember two examples from school math. I saw multiplication tables as gears, and my first brush with equations in two variables (e.g.,  $3x + 4y = 10$ ) immediately evoked the differential. By the time I had made a mental gear model of the relation between  $x$  and  $y$ , figuring how many teeth each gear needed, the equation had become a comfortable friend.

Many years later when I read Piaget this incident served me as a model for his notion of assimilation, except I was immediately struck by the fact that his discussion does not do full justice to his own idea. He talks almost entirely about cognitive aspects of assimilation. But there is also an affective component. Assimilating equations to gears certainly is a powerful way to bring old knowledge to bear on a new object. But it does more as well. I am sure that such assimilations helped to endow mathematics, for me, with a positive affective tone that can be traced back to my infantile experiences with cars. I believe Piaget really agrees. As I came to know him personally I understood that his neglect of the affective comes more from a modest sense that little is known about it than from an arrogant sense of its irrelevance. But let me return to my childhood.

One day I was surprised to discover that some adults—even most adults—did not understand or even care about the magic of the gears. I no longer think much about gears, but I have never turned away from the questions that started with that discovery: How could what was so simple for me be incomprehensible to other people? My proud father suggested "being clever" as an explanation. But I was painfully aware that some people who could not understand the differential could easily do things I found much more difficult. Slowly I began to formulate what I still consider the fundamental fact about learning: Anything is easy if you can assimilate it to your collection of models. If you can't, anything can be painfully difficult. Here too I was developing a way of thinking that would be resonant with Piaget's. *The understanding of learning must be genetic*. It must refer to the genesis of knowledge. What an individual can learn, and how he learns it, depends on what models he has available. This raises, recursively, the question of how he learned these models. Thus the "laws of learning" must be about how intellectual structures grow

out of one another and about how, in the process, they acquire both logical and emotional form.

This book is an exercise in an applied genetic epistemology expanded beyond Piaget's cognitive emphasis to include a concern with the affective. It develops a new perspective for education research focused on creating the conditions under which intellectual models will take root. For the last two decades this is what I have been trying to do. And in doing so I find myself frequently reminded of several aspects of my encounter with the differential gear. First, I remember that no one told me to learn about differential gears. Second, I remember that there was *feeling/love*, as well as understanding in my relationship with gears. Third, I remember that my first encounter with them was in my second year. If any "scientific" educational psychologist had tried to "measure" the effects of this encounter, he would probably have failed. It had profound consequences but, I conjecture, only very many years later. A "pre- and post-" test at age two would have missed them.

Piaget's work gave me a new framework for looking at the gears of my childhood. The gear can be used to illustrate many powerful "advanced" mathematical ideas, such as groups or relative motion. But it does more than this. As well as connecting with the formal knowledge of mathematics, it also connects with the "body knowledge," the sensorimotor schemata of a child. You can *be* the gear, you can understand how it turns by projecting yourself into its place and turning with it. It is this double relationship—both abstract and sensory—that gives the gear the power to carry powerful mathematics into the mind. In a terminology I shall develop in later chapters, the gear acts here as a *transitional object*.

A modern-day Montessori might propose, if convinced by my story, to create a gear set for children. Thus every child might have the experience I had. But to hope for this would be to miss the essence of the story. *I fell in love with the gears*. This is something that cannot be reduced to purely "cognitive" terms. Something very personal happened, and one cannot assume that it would be repeated for other children in exactly the same form.

My thesis could be summarized as: What the gears cannot do

the computer might. The computer is the Proteus of machines. Its essence is its universality, its power to simulate. Because it can take on a thousand forms and can serve a thousand functions, it can appeal to a thousand tastes. This book is the result of my own attempts over the past decade to turn computers into instruments flexible enough so that many children can each create for themselves something like what the gears were for me.

**MINDSTORMS**

## Introduction

# Computers for Children

JUST A FEW YEARS AGO people thought of computers as expensive and exotic devices. Their commercial and industrial uses affected ordinary people, but hardly anyone expected computers to become part of day-to-day life. This view has changed dramatically and rapidly as the public has come to accept the reality of the personal computer, small and inexpensive enough to take its place in every living room or even in every breast pocket. The appearance of the first rather primitive machines in this class was enough to catch the imagination of journalists and produce a rash of speculative articles about life in the computer-rich world to come. The main subject of these articles was what people will be able to do with their computers. Most writers emphasized using computers for games, entertainment, income tax, electronic mail, shopping, and banking. A few talked about the computer as a teaching machine.

This book too poses the question of what will be done with personal computers, but in a very different way. I shall be talking about how computers may affect the way people think and learn. I begin to characterize my perspective by noting a distinction between two ways computers might enhance thinking and change patterns of access to knowledge.

Instrumental uses of the computer to help people think have

been dramatized in science fiction. For example, as millions of "Star Trek" fans know, the starship *Enterprise* has a computer that gives rapid and accurate answers to complex questions posed to it. But no attempt is made in "Star Trek" to suggest that the human characters aboard think in ways very different from the manner in which people in the twentieth century think. Contact with the computer has not, as far as we are allowed to see in these episodes, changed how these people think about themselves or how they approach problems. In this book I discuss ways in which the computer presence could contribute to mental processes not only instrumentally but in more essential, conceptual ways, influencing how people think even when they are far removed from physical contact with a computer (just as the gears shaped my understanding of algebra although they were not physically present in the math class). It is about an end to the culture that makes science and technology alien to the vast majority of people. Many cultural barriers impede children from making scientific knowledge their own. Among these barriers the most visible are the physically brutal effects of deprivation and isolation. Other barriers are more political. Many children who grow up in our cities are surrounded by the artifacts of science but have good reason to see them as belonging to "the others"; in many cases they are perceived as belonging to the social enemy. Still other obstacles are more abstract, though ultimately of the same nature. Most branches of the most sophisticated modern culture of Europe and the United States are so deeply "mathophobic" that many privileged children are as effectively (if more gently) kept from appropriating science as their own. In my vision, space-age objects, in the form of small computers, will cross these cultural barriers to enter the private worlds of children everywhere. They will do so not as mere physical objects. This book is about how computers can be carriers of powerful ideas and of the seeds of cultural change, how they can help people form new relationships with knowledge that cut across the traditional lines separating humanities from sciences and knowledge of the self from both of these. It is about using computers to challenge current beliefs about who can understand what and at what age. It is about using computers to question standard assumptions in developmen-

tal psychology and in the psychology of aptitudes and attitudes. It is about whether personal computers and the cultures in which they are used will continue to be the creatures of "engineers" alone or whether we can construct intellectual environments in which people who today think of themselves as "humanists" will feel part of, not alienated from, the process of constructing computational cultures.

But there is a world of difference between what computers can do and what society will choose to do with them. Society has many ways to resist fundamental and threatening change. Thus, this book is about facing choices that are ultimately political. It looks at some of the forces of change and of reaction to those forces that are called into play as the computer presence begins to enter the politically charged world of education.

Much of the book is devoted to building up images of the role of the computer very different from current stereotypes. All of us, professionals as well as laymen, must consciously break the habits we bring to thinking about the computer. Computation is in its infancy. It is hard to think about computers of the future without projecting onto them the properties and the limitations of those we think we know today. And nowhere is this more true than in imagining how computers can enter the world of education. It is not true to say that the image of a child's relationship with a computer I shall develop here goes far beyond what is common in today's schools. My image does not go beyond: It goes in the opposite direction.

In many schools today, the phrase "computer-aided instruction" means making the computer teach the child. One might say the computer is being used to program the child. In my vision, the child programs the computer and, in doing so, both acquires a sense of mastery over a piece of the most modern and powerful technology and establishes an intimate contact with some of the deepest ideas from science, from mathematics, and from the art of intellectual model building.

I shall describe learning paths that have led hundreds of children to becoming quite sophisticated programmers. Once programming is seen in the proper perspective, there is nothing very surprising about the fact that this should happen. Programming a computer

means nothing more or less than communicating to it in a language that it and the human user can both "understand." And learning languages is one of the things children do best. Every normal child learns to talk. Why then should a child not learn to "talk" to a computer?

There are many reasons why someone might expect it to be difficult. For example, although babies learn to speak their native language with spectacular ease, most children have great difficulty learning foreign languages in schools and, indeed, often learn the written version of their own language none too successfully. Isn't learning a computer language more like the difficult process of learning a foreign written language than the easy one of learning to speak one's own language? And isn't the problem further compounded by all the difficulties most people encounter learning mathematics?

Two fundamental ideas run through this book. The first is that it is possible to design computers so that learning to communicate with them can be a natural process, more like learning French by living in France than like trying to learn it through the unnatural process of American foreign-language instruction in classrooms. Second, learning to communicate with a computer may change the way other learning takes place. The computer can be a mathematics-speaking and an alphabetic-speaking entity. We are learning how to make computers with which children love to communicate. When this communication occurs, children learn mathematics as a living language. Moreover, mathematical communication and alphabetic communication are thereby both transformed from the alien and therefore difficult things they are for most children into natural and therefore easy ones. The idea of "talking mathematics" to a computer can be generalized to a view of learning mathematics in "Mathland"; that is to say, in a context which is to learning mathematics what living in France is to learning French.

In this book the Mathland metaphor will be used to question deeply engrained assumptions about human abilities. It is generally assumed that children cannot learn formal geometry until well into their school years and that most cannot learn it too well even then. But we can quickly see that these assumptions are based on ex-

tremely weak evidence by asking analogous questions about the ability of children to learn French. If we had to base our opinions on observation of how poorly children learned French in American schools, we would have to conclude that most people were incapable of mastering it. But we know that all normal children would learn it very easily if they lived in France. My conjecture is that much of what we now see as too "formal" or "too mathematical" will be learned just as easily when children grow up in the computer-rich world of the very near future.

I use the examination of our relationship with mathematics as a thematic example of how technological and social processes interact in the construction of ideas about human capacities. And mathematical examples will also help to describe a theory of how learning works and of how it goes wrong.

I take from Jean Piaget<sup>1</sup> a model of children as builders of their own intellectual structures. Children seem to be innately gifted learners, acquiring long before they go to school a vast quantity of knowledge by a process I call "Piagetian learning," or "learning without being taught." For example, children learn to speak, learn the intuitive geometry needed to get around in space, and learn enough of logic and rhetorics to get around parents—all this without being "taught." We must ask why some learning takes place so early and spontaneously while some is delayed many years or does not happen at all without deliberately imposed formal instruction.

If we really look at the "child as builder" we are on our way to an answer. All builders need materials to build with. Where I am at variance with Piaget is in the role I attribute to the surrounding cultures as a source of these materials. In some cases the culture supplies them in abundance, thus facilitating constructive Piagetian learning. For example, the fact that so many important things (knives and forks, mothers and fathers, shoes and socks) come in pairs is a "material" for the construction of an intuitive sense of number. But in many cases where Piaget would explain the slower development of a particular concept by its greater complexity or formality, I see the critical factor as the relative poverty of the culture in those materials that would make the concept simple and concrete. In yet other cases the culture may provide materials but



block their use. In the case of formal mathematics, there is both a shortage of formal materials and a cultural block as well. The mathophobia endemic in contemporary culture blocks many people from learning anything they recognize as "math," although they may have no trouble with mathematical knowledge they do not perceive as such.

We shall see again and again that the consequences of mathophobia go far beyond obstructing the learning of mathematics and science. They interact with other endemic "cultural toxins," for example, with popular theories of aptitudes, to contaminate peoples' images of themselves as learners. Difficulty with school math is often the first step of an invasive intellectual process that leads us all to define ourselves as bundles of aptitudes and ineptitudes, as being "mathematical" or "not mathematical," "artistic" or "not artistic," "musical" or "not musical," "profound" or "superficial," "intelligent" or "dumb." Thus deficiency becomes identity and learning is transformed from the early child's free exploration of the world to a chore beset by insecurities and self-imposed restrictions.

Two major themes—that children can learn to use computers in a masterful way, and that learning to use computers can change the way they learn everything else—have shaped my research agenda on computers and education. Over the past ten years I have had the good fortune to work with a group of colleagues and students at MIT (the LOGO<sup>2</sup> group in the Artificial Intelligence Laboratory) to create environments in which children can learn to communicate with computers. The metaphor of imitating the way the child learns to talk has been constantly with us in this work and has led to a vision of education and of education research very different from the traditional ones. For people in the teaching professions, the word "education" tends to evoke "teaching," particularly classroom teaching. The goal of education research tends therefore to be focused on how to improve classroom teaching. But if, as I have stressed here, the model of successful learning is the way a child learns to talk, a process that takes place without deliberate and organized teaching, the goal set is very different. I see the classroom as an artificial and inefficient learning environment that society has been forced to invent because its informal environments fail in certain essential learning domains, such as writing or gram-

mar or school math. I believe that the computer presence will enable us to so modify the learning environment outside the classrooms that much if not all the knowledge schools presently try to teach with such pain and expense and such limited success will be learned, as the child learns to talk, painlessly, successfully, and without organized instruction. This obviously implies that schools as we know them today will have no place in the future. But it is an open question whether they will adapt by transforming themselves into something new or wither away and be replaced.

Although technology will play an essential role in the realization of my vision of the future of education, my central focus is not on the machine but on the mind, and particularly on the way in which intellectual movements and cultures define themselves and grow. Indeed, the role I give to the computer is that of a *carrier* of cultural "germs" or "seeds" whose intellectual products will not need technological support once they take root in an actively growing mind. Many if not all the children who grow up with a love and aptitude for mathematics owe this feeling, at least in part, to the fact that they happened to acquire "germs" of the "math culture" from adults, who, one might say, knew how to speak mathematics, even if only in the way that Moliere had M. Jourdain speak prose without knowing it. These "math-speaking" adults do not necessarily know how to solve equations; rather, they are marked by a turn of mind that shows up in the logic of their arguments and in the fact that for them to play is often to play with such things as puzzles, puns, and paradoxes. Those children who prove recalcitrant to math and science education include many whose environments happened to be relatively poor in math-speaking adults. Such children come to school lacking elements necessary for the easy learning of school math. School has been unable to supply these missing elements, and, by forcing the children into learning situations doomed in advance, it generates powerful negative feelings about mathematics and perhaps about learning in general. Thus is set up a vicious self-perpetuating cycle. For these same children will one day be parents and will not only fail to pass on mathematical germs but will almost certainly infect their children with the opposing and intellectually destructive germs of mathophobia.

Fortunately it is sufficient to break the self-perpetuating cycle at

one point for it to remain broken forever. I shall show how computers might enable us to do this, thereby breaking the vicious cycle without creating a dependence on machines. My discussion differs from most arguments about "nature versus nurture" in two ways. I shall be much more specific both about what kinds of nurturance are needed for intellectual growth and about what can be done to create such nurturance in the home as well as in the wider social context.

Thus this book is really about how a culture, a way of thinking, an idea comes to inhabit a young mind. I am suspicious of thinking about such problems too abstractly, and I shall write here with particular restricted focus. I shall in fact concentrate on those ways of thinking that I know best. I begin by looking at what I know about my own development. I do this in all humility, without any implication that what I have become is what everyone should become. But I think that the best way to understand learning is first to understand specific, well-chosen cases and then to worry afterward about how to generalize from this understanding. You can't think seriously about thinking without thinking about thinking about something. And the something I know best how to think about is mathematics. When in this book I write of mathematics, I do not think of myself as writing for an audience of mathematicians interested in mathematical thinking for its own sake. My interest is in universal issues of how people think and how they learn to think.

When I trace how I came to be a mathematician, I see much that was idiosyncratic, much that could not be duplicated as part of a generalized vision of education reform. And I certainly don't think that we would want everyone to become a mathematician. But I think that the kind of pleasure I take in mathematics should be part of a general vision of what education should be about. If we can grasp the essence of one person's experiences, we may be able to replicate its consequences in other ways, and in particular this consequence of finding beauty in abstract things. And so I shall be writing quite a bit about mathematics. I give my apologies to readers who hate mathematics, but I couple that apology with an offer to help them learn to like it a little better—or at least to change their image of what "speaking mathematics" can be all about.

In the Foreword of this book I described how gears helped mathematical ideas to enter my life. Several qualities contributed to their effectiveness. First, they were part of my natural "landscape," embedded in the culture around me. This made it possible for me to find them myself and relate to them in my own fashion. Second, gears were part of the world of adults around me and through them I could relate to these people. Third, I could use my body to think about the gears. I could feel how gears turn by imagining my body turning. This made it possible for me to draw on my "body knowledge" to think about gear systems. And finally, because, in a very real sense, the relationship between gears contains a great deal of mathematical information, I could use the gears to think about formal systems. I have described the way in which the gears served as an "object-to-think-with." I made them that for myself in my own development as a mathematician. The gears have also served me as an object-to-think-with in my work as an educational researcher. My goal has been the design of other objects that children can make theirs for themselves and in their own ways. Much of this book will describe my path through this kind of research. I begin by describing one example of a constructed computational "object-to-think-with." This is the "Turtle."<sup>3</sup>

The central role of the Turtle in this book should not be taken to mean that I propose it as a panacea for all educational problems. I see it as a valuable educational object, but its principal role here is to serve as a model for other objects, yet to be invented. My interest is in the process of invention of "objects-to-think-with," objects in which there is an intersection of cultural presence, embedded knowledge, and the possibility for personal identification.

The Turtle is a computer-controlled cybernetic animal. It exists within the cognitive minicultures of the "LOGO environment," LOGO being the computer language in which communication with the Turtle takes place. The Turtle serves no other purpose than of being good to program and good to think with. Some Turtles are abstract objects that live on computer screens. Others, like the floor Turtles shown in the frontispiece are physical objects that can be picked up like any mechanical toy. A first encounter often begins by showing the child how a Turtle can be made to move by

typing commands at a keyboard. FORWARD 100 makes the Turtle move in a straight line a distance of 100 Turtle steps of about a millimeter each. Typing RIGHT 90 causes the Turtle to pivot in place through 90 degrees. Typing PENDOWN causes the Turtle to lower a pen so as to leave a visible trace of its path while PENUPI instructs it to raise the pen. Of course the child needs to explore a great deal before gaining mastery of what the numbers mean. But the task is engaging enough to carry most children through this learning process.

The idea of programming is introduced through the metaphor of teaching the Turtle a new word. This is simply done, and children often begin their programming experience by programming the Turtle to respond to new commands invented by the child such as SQUARE or TRIANGLE or SQ or TRI or whatever the child wishes, by drawing the appropriate shapes. New commands once defined can be used to define others. For example just as the house in Figure 1 is built out of a triangle and a square, the program for drawing it is built out of the commands for drawing a square and a triangle. Figure 1 shows four steps in the evolution of this program. From these simple drawings the young programmer can go on in many different directions. Some work on more complex drawings, either figural or abstract. Some abandon the use of the Turtle as a drawing instrument and learn to use its touch sensors to program it to seek out or avoid objects.<sup>4</sup> Later children learn that the computer can be programmed to make music as well as move Turtles and combine the two activities by programming Turtles to dance. Or they can move on from floor Turtles to "screen Turtles," which they program to draw moving pictures in bright colors. The examples are infinitely varied, but in each the child is learning how to exercise control over an exceptionally rich and sophisticated "micro-world."

Readers who have never seen an interactive computer display might find it hard to imagine where this can lead. As a mental exercise they might like to imagine an electronic sketchpad, a computer graphics display of the not-too-distant future. This is a television screen that can display moving pictures in color. You can also "draw" on it, giving it instructions, perhaps by typing, perhaps by

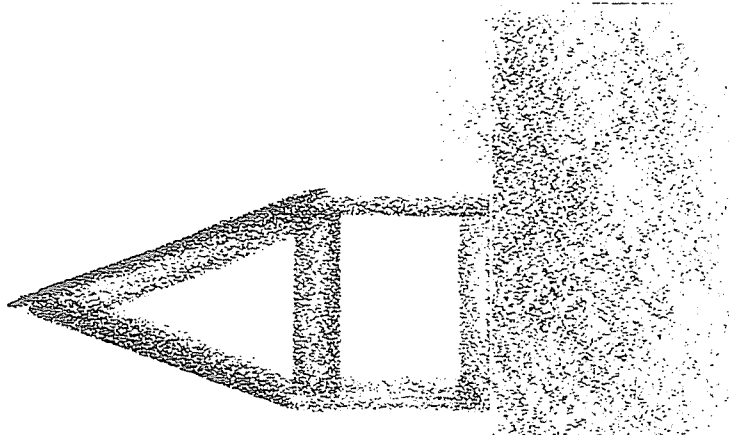
speaking, or perhaps by pointing with a wand. On request, a palette of colors could appear on the screen. You can choose a color by pointing at it with the wand. Until you change your choice, the wand draws in that color. Up to this point the distinction from traditional art materials may seem slight, but the distinction becomes very real when you begin to think about editing the drawing. You can "talk to your drawing" in computer language. You can "tell" it to replace this color with that. Or set a drawing in motion. Or make two copies and set them in counterrotating motion. Or replace the color palette with a sound palette and "draw" a piece of music. You can file your work in computer memory and retrieve it at your pleasure, or have it delivered into the memory of any of the many millions of other computers linked to the central communication network for the pleasure of your friends.

That all this would be fun needs no argument. But it is more than fun. Very powerful kinds of learning are taking place. Children working with an electronic sketchpad are learning a language for talking about shapes and fluxes of shapes, about velocities and rates of change, about processes and procedures. They are learning to speak mathematics, and acquiring a new image of themselves as mathematicians.

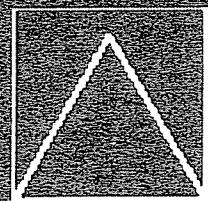
In my description of children working with Turtles, I implied that children can learn to program. For some readers this might be tantamount to the suspension of disbelief required when we enter a theater to watch a play. For them programming is a complex and marketable skill acquired by some mathematically gifted adults. But my experience is very different. I have seen hundreds of elementary school children learn very easily to program, and evidence is accumulating to indicate that much younger children could do so as well. The children in these studies are not exceptional, or rather, they are exceptional in every conceivable way. Some of the children were highly successful in school, some were diagnosed as emotionally or cognitively disabled. Some of the children were so severely afflicted by cerebral palsy that they had never purposefully manipulated physical objects. Some of them had expressed their talents in "mathematical" forms, some in "verbal" forms, and some in artistically "visual" or in "musical" forms.

Figure 1

A Plan

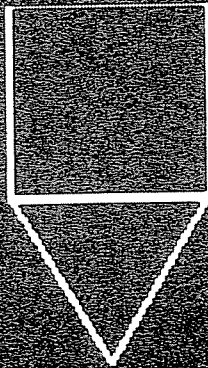


A Bug

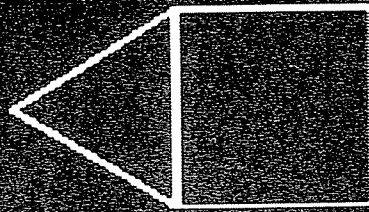


TO HOUSE  
SO  
TRI

And a Compromise



TO HOUSE  
SO  
RIGHT 30  
TRI  
END



TO HOUSE  
RIGHT 90  
SO  
RIGHT 30  
TRI  
END



Of course these children did not achieve a fluency in programming that came close to matching their use of spoken language. If we take the Mathland metaphor seriously, their computer experience was more like learning French by spending a week or two on vacation in France than like living there. But like children who have spent a vacation with foreign-speaking cousins, they were clearly on their way to "speaking computer."

When I have thought about what these studies mean I am left with two clear impressions. First, that all children will, under the right conditions, acquire a proficiency with programming that will make it one of their more advanced intellectual accomplishments. Second, that the "right conditions" are very different from the kind of access to computers that is now becoming established as the norm in schools. The conditions necessary for the kind of relationships with a computer that I will be writing about in this book require more and freer access to the computer than educational planners currently anticipate. And they require a kind of computer language and a learning environment around that language very different from those the schools are now providing. They even require a kind of computer rather different from those that the schools are currently buying.

It will take most of this book for me to convey some sense of the choices among computers, computer languages, and more generally, among computer cultures, that influence how well children will learn from working with computation and what benefits they will get from doing so. But the question of the *economic* feasibility of free access to computers for every child can be dealt with immediately. In doing so I hope to remove any doubts readers may have about the "economic realism" of the "vision of education" I have been talking about.

My vision of a new kind of learning environment demands free contact between children and computers. This could happen because the child's family buys one or a child's friends have one. For purposes of discussion here (and to extend our discussion to all social groups) let us assume that it happens because schools give every one of their students his or her own powerful personal computer. Most "practical" people (including parents, teachers, school

principals, and foundation administrators) react to this idea in much the same way: "Even if computers could have all the effects you talk about, it would still be impossible to put your ideas into action. Where would the money come from?"

What these people are saying needs to be faced squarely. They are wrong. Let's consider the cohort of children who will enter kindergarten in the year 1987, the "Class of 2000," and let's do some arithmetic. The direct public cost of schooling a child for thirteen years, from kindergarten through twelfth grade is over \$20,000 today (and for the class of 2000, it may be closer to \$30,000). A conservatively high estimate of the cost of supplying each of these children with a personal computer with enough power for it to serve the kinds of educational ends described in this book, and of upgrading, repairing, and replacing it when necessary would be about \$1,000 per student, distributed over thirteen years in school. Thus, "computer costs" for the class of 2,000 would represent only about 5 percent of the total public expenditure on education, and this would be the case even if nothing else in the structure of educational costs changed because of the computer presence. But in fact computers in education stand a good chance of making other aspects of education cheaper. Schools might be able to reduce their cycle from thirteen years to twelve years; they might be able to take advantage of the greater autonomy the computer gives students and increase the size of classes by one or two students without decreasing the personal attention each student is given. Either of these two moves would "recuperate" the computer cost.

My goal is not educational economics: It is not to use computation to shave a year off the time a child spends in an otherwise unchanged school or to push an extra child into an elementary school classroom. The point of this little exercise in educational "budget balancing" is to do something to the state of mind of my readers as they turn to the first chapter of this book. I have described myself as an educational utopian—not because I have projected a future of education in which children are surrounded by high technology, but because I believe that certain uses of very powerful computational technology and computational ideas can provide children with new possibilities for learning, thinking, and growing emotion-

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ally as well as cognitively. In the chapters that follow I shall try to give you some idea of these possibilities, many of which are dependent on a computer-rich future, a future where a computer will be a significant part of every child's life. But I want my readers to be very clear that what is "utopian" in my vision and in this book is a particular way of using computers, of forging new relationships between computers and people—that the computer will be there to be used is simply a conservative premise.

# Chapter 1 Computers and Computer Cultures

IN MOST contemporary educational situations where children come into contact with computers the computer is used to put children through their paces, to provide exercises of an appropriate level of difficulty, to provide feedback, and to dispense information. The computer programming the child. In the LOGO environment the relationship is reversed: The child, even at preschool ages, is in control: The child programs the computer. And in teaching the computer how to think, children embark on an exploration about how they themselves think. The experience can be heady: Thinking about thinking turns the child into an epistemologist, an experience not even shared by most adults.

This powerful image of child as epistemologist caught my imagination while I was working with Piaget. In 1964, after five years at Piaget's Center for Genetic Epistemology in Geneva, I came away impressed by his way of looking at children as the active builders of their own intellectual structures. But to say that intellectual structures are built by the learner rather than taught by a teacher does not mean that they are built from nothing. On the contrary: Like other builders, children appropriate to their own use materials they find about them, most saliently the models and metaphors suggested by the surrounding culture.

when a child learns to program, the process of learning is transformed. It becomes more active and self-directed. In particular, the knowledge is acquired for a recognizable personal purpose. The child does something with it. The new knowledge is a source of power and is experienced as such from the moment it begins to form in the child's mind.

I have spoken of mathematics being learned in a new way. But much more is affected than mathematics. One can get an idea of the extent of what is changed by examining another of Piaget's ideas. Piaget distinguishes between "concrete" thinking and "formal" thinking. Concrete thinking is already well on its way by the time the child enters the first grade at age 6 and is consolidated in the following several years. Formal thinking does not develop until the child is almost twice as old, that is to say at age 12, give or take a year or two, and some researchers have even suggested that many people never achieve fully formal thinking. I do not fully accept Piaget's distinction, but I am sure that it is close enough to reality to help us make sense of the idea that the consequences for intellectual development of one innovation could be qualitatively greater than the cumulative quantitative effects of a thousand others. Stated most simply, my conjecture is that the computer can concretize (and personalize) the formal. Seen in this light, it is not just another powerful educational tool. It is unique in providing us with the means for addressing what Piaget and many others see as the obstacle which is overcome in the passage from child to adult thinking. I believe that it can allow us to shift the boundary separating concrete and formal. Knowledge that was accessible only through formal processes can now be approached concretely. And the real magic comes from the fact that this knowledge includes those elements one needs to become a formal thinker.

This description of the role of the computer is rather abstract. I shall concretize it, anticipating discussions which occur in later chapters of this book, by looking at the effect of working with computers on two kinds of thinking Piaget associates with the formal stage of intellectual development: combinatorial thinking, where one has to reason in terms of the set of all possible states of a system, and self-referential thinking about thinking itself.

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Piaget writes about the order in which the child develops different intellectual abilities. I give more weight than he does to the influence of the materials a particular culture provides in determining that order. For example, our culture is very rich in materials useful for the child's construction of certain components of numerical and logical thinking. Children learn to count; they learn that the result of counting is independent of order and special arrangement; they extend this "conservation" to thinking about the properties of liquids as they are poured and of solids which change their shape. Children develop these components of thinking pre-consciously and "spontaneously," that is to say without deliberate teaching. Other components of knowledge, such as the skills involved in doing permutations and combinations, develop more slowly, or do not develop at all without formal schooling. Taken as a whole this book is an argument that in many important cases this developmental difference can be attributed to our culture's relative poverty in materials from which the apparently "more advanced" intellectual structures can be built. This argument will be very different from cultural interpretations of Piaget that look for differences between city children in Europe or the United States and tribal children in African jungles. When I speak here of "our" culture I mean something less parochial. I am not trying to contrast New York with Chad. I am interested in the difference between pre-computer cultures (whether in American cities or African tribes) and the "computer cultures" that may develop everywhere in the next decades.

I have already indicated one reason for my belief that the computer presence might have more fundamental effects on intellectual development than did other new technologies, including television and even printing. The metaphor of computer as mathematics-speaking entity puts the learner in a qualitatively new kind of relationship to an important domain of knowledge. Even the best of educational television is limited to offering quantitative improvements in the kinds of learning that existed without it. "Sesame Street" might offer better and more engaging explanations than a child can get from some parents or nursery school teachers, but the child is still in the position of listening to explanations. By contrast,

In a typical experiment in combinatorial thinking, children are asked to form all the possible combinations (or "families") of beads of assorted colors. It really is quite remarkable that most children are unable to do this systematically and accurately until they are in the fifth or sixth grades. Why should this be? Why does this task seem to be so much more difficult than the intellectual feats accomplished by seven and eight year old children? Is its logical structure essentially more complex? Can it possibly require a neurological mechanism that does not mature until the approach of puberty? I think that a more likely explanation is provided by looking at the nature of the culture. The task of making the families of beads can be looked at as constructing and executing a program, a very common sort of program, in which two loops are nested: Fix a first color and run through all the possible second colors, then repeat until all possible first colors have been run through. For someone who is thoroughly used to computers and programming there is nothing "formal" or abstract about this task. For a child in a computer culture it would be as concrete as matching up knives and forks at the dinner table. Even the common "bug" of including some families twice (for example, red-blue and blue-red) would be well-known. Our culture is rich in pairs, couples, and one-to-one correspondences of all sorts, and it is rich in language for talking about such things. This richness provides both the incentive and a supply of models and tools for children to build ways to think about such issues as whether three large pieces of candy are more or less than four much smaller pieces. For such problems our children acquire an excellent intuitive sense of quantity. But our culture is relatively poor in models of systematic procedures. Until recently there was not even a name in popular language for programming, let alone for the ideas needed to do so successfully. There is no word for "nested loops" and no word for the double-counting bug. Indeed, there are no words for the powerful ideas computerists refer to as "bug" and "debugging."

Without the incentive or the materials to build powerful, concrete ways to think about problems involving systematicity, children are forced to approach such problems in a groping, abstract fashion. Thus cultural factors that are common to both the Ameri-

can city and the African village can explain the difference in age at which children build their intuitive knowledge of quantity and of systematicity.

While still working in Geneva I had become sensitive to the way in which materials from the then very young computer cultures were allowing psychologists to develop new ways to think about thinking. In fact, my entry into the world of computers was motivated largely by the idea that children could also benefit, perhaps even more than the psychologists, from the way in which computer models seemed able to give concrete form to areas of knowledge that had previously appeared so intangible and abstract.

I began to see how children who had learned to program computers could use very concrete computer models to think about thinking and to learn about learning and in doing so, enhance their powers as psychologists and as epistemologists. For example, many children are held back in their learning because they have a model of learning in which you have either "got it" or "got it wrong." But when you learn to program a computer you almost never get it right the first time. Learning to be a master programmer is learning to become highly skilled at isolating and correcting "bugs," the parts that keep the program from working. The question to ask about the program is not whether it is right or wrong, but if it is fixable. If this way of looking at intellectual products were generalized to how the larger culture thinks about knowledge and its acquisition, we all might be less intimidated by our fears of "being wrong." This potential influence of the computer on changing our notion of a black and white version of our successes and failures is an example of using the computer as an "object-to-think-with." It is obviously not necessary to work with computers in order to acquire good strategies for learning. Surely "debugging" strategies were developed by successful learners long before computers existed. But thinking about learning by analogy with developing a program is a powerful and accessible way to get started on becoming more articulate about one's debugging strategies and more deliberate about improving them.

My discussion of a computer culture and its impact on thinking presupposes a massive penetration of powerful computers into peo-



ple's lives. That this will happen there can be no doubt. The calculator, the electronic game, and the digital watch were brought to us by a technical revolution that rapidly lowered prices for electronics in a period when all others were rising with inflation. That same technological revolution, brought about by the integrated circuit, is now bringing us the personal computer. Large computers used to cost millions of dollars because they were assembled out of millions of physically distinct parts. In the new technology a complex circuit is not assembled but made as a whole, solid entity—hence the term “integrated circuit.” The effect of integrated circuit technology on cost can be understood by comparing it to printing. The main expenditure in making a book occurs long before the press begins to roll. It goes into writing, editing, and typesetting. Other costs occur after the printing: binding, distributing, and marketing. The actual cost per copy for printing itself is negligible. And the same is true for a powerful as for a trivial book. So, too, most of the cost of an integrated circuit goes into a preparatory process; the actual cost of making an individual circuit becomes negligible, provided enough are sold to spread the costs of development. The consequences of this technology for the cost of computation are dramatic. Computers that would have cost hundreds of thousands in the 1960s and tens of thousands in the early 1970s can now be made for less than a dollar. The only limiting factor is whether the particular circuit can fit onto what corresponds to a “page”—that is to say the “silicon chips” on which the circuits are etched.

But each year in a regular and predictable fashion the art of etching circuits on silicon chips is becoming more refined. More and more complex circuitry can be squeezed onto a chip, and the computer power that can be produced for less than a dollar increases. I predict that long before the end of the century, people will buy children toys with as much computer power as the great IBM computers currently selling for millions of dollars. And as for computers to be used as such, the main cost of these machines will be the peripheral devices, such as the keyboard. Even if these do not fall in price, it is likely that a supercomputer will be equivalent in price to a typewriter and a television set.

There really is no disagreement among experts that the cost of

computers will fall to a level where they will enter everyday life in vast numbers. Some will be there as computers proper, that is to say, programmable machines. Others might appear as games of ever-increasing complexity and in automated supermarkets where the shelves, maybe even the cans, will talk. One really can afford to let one's imagination run wild. There is no doubt that the material surface of life will become very different for everyone, perhaps most of all for children. But there has been significant difference of opinion about the effects this computer presence will produce. I would distinguish my thinking from two trends of thinking which I refer to here as the “skeptical” and the “critical.”

Skeptics do not expect the computer presence to make much difference in how people learn and think. I have formulated a number of possible explanations for why they think as they do. In some cases I think the skeptics might conceive of education and the effect of computers on it too narrowly. Instead of considering general cultural effects, they focus attention on the use of the computer as a device for programmed instruction. Skeptics then conclude that while the computer might produce some improvements in school learning, it is not likely to lead to fundamental change. In a sense, too, I think the skeptical view derives from a failure to appreciate just how much Piagetian learning takes place as a child grows up. If a person conceives of children's intellectual development (or, for that matter, moral or social development) as deriving chiefly from deliberate teaching, then such a person would be likely to underestimate the potential effect that a massive presence of computers and other interactive objects might have on children.

The critics,<sup>2</sup> on the other hand, do think that the computer presence will make a difference and are apprehensive. For example, they fear that more communication via computers might lead to less human association and result in social fragmentation. As knowing how to use a computer becomes increasingly necessary to effective social and economic participation, the position of the underprivileged could worsen, and the computer could exacerbate existing class distinctions. As to the political effect computers will have, the critics' concerns resonate with Orwellian images of a 1984 where home computers will form part of a complex system of

surveillance and thought control. Critics also draw attention to potential mental health hazards of computer penetration. Some of these hazards are magnified forms of problems already worrying many observers of contemporary life; others are problems of an essentially new kind. A typical example of the former kind is that our grave ignorance of the psychological impact of television becomes even more serious when we contemplate an epoch of super TV. The holding power and the psychological impact of the television show could be increased by the computer in at least two ways. The content might be varied to suit the tastes of each individual viewer, and the show might become interactive, drawing the "viewer" into the action. Such things belong to the future, but people who are worried about the impact of the computer on people already cite cases of students spending sleepless nights riveted to the computer terminal, coming to neglect both studies and social contact. Some parents have been reminded of these stories when they observe a special quality of fascination in their own children's reaction to playing with the still rudimentary electronic games.

In the category of problems that are new rather than aggravated versions of old ones, critics have pointed to the influence of the allegedly mechanized thought processes of computers on how people think. Marshall McLuhan's dictum that "the medium is the message" might apply here: If the medium is an interactive system that takes in words and speaks back like a person, it is easy to get the message that machines are like people and that people are like machines. What this might do to the development of values and self-image in growing children is hard to assess. But it is not hard to see reasons for worry.

Despite these concerns I am essentially optimistic—some might say utopian—about the effect of computers on society. I do not dismiss the arguments of the critics. On the contrary, I too see the computer presence as a potent influence on the human mind. I am very much aware of the holding power of an interactive computer and of how taking the computer as a model can influence the way we think about ourselves. In fact the work on LOGO to which I have devoted much of the past ten years consists precisely of developing such forces in positive directions. For example, the critic is

horrified at the thought of a child hypnotically held by a futuristic, computerized super-pinball machine. In the LOGO work we have invented versions of such machines in which powerful ideas from physics or mathematics or linguistics are embedded in a way that permits the player to learn them in a natural fashion, analogous to how a child learns to speak. The computer's "holding power," so feared by critics, becomes a useful educational tool. Or take another, more profound example. The critic is afraid that children will adopt the computer as model and eventually come to "think mechanically" themselves. Following the opposite tack, I have invented ways to take educational advantage of the opportunities to master the art of *deliberately* thinking like a computer, according, for example, to the stereotype of a computer program that proceeds in a step-by-step, literal, mechanical fashion. There are situations where this style of thinking is appropriate and useful. Some children's difficulties in learning formal subjects such as grammar or mathematics derive from their inability to see the point of such a style.

A second educational advantage is indirect but ultimately more important. By deliberately learning to imitate mechanical thinking, the learner becomes able to articulate what mechanical thinking is and what it is not. The exercise can lead to greater confidence about the ability to choose a cognitive style that suits the problem. Analysis of "mechanical thinking" and how it is different from other kinds and practice with problem analysis can result in a new degree of intellectual sophistication. By providing a very concrete, down-to-earth model of a particular style of thinking, work with the computer can make it easier to understand that there is such a thing as a "style of thinking." And giving children the opportunity to choose one style or another provides an opportunity to develop the skill necessary to choose between styles. Thus instead of inducing mechanical thinking, contact with computers could turn out to be the best conceivable antidote to it. And for me what is most important in this is that through these experiences these children would be serving their apprenticeships as epistemologists, that is to say learning to think articulately about thinking.

The intellectual environments offered to children by today's cul-

most cases the computer is being used either as a versatile video game or as a "teaching machine" programmed to put children through their paces in arithmetic or spelling. And even when children are taught by a parent, a peer, or a professional teacher to write simple programs in a language like BASIC, this activity is not accompanied at all by the kind of epistemological reflection that we see in the LOGO environments. So I share a skepticism with the critics about what is being done with computation now. But I am interested in stimulating a major change in how things can be. The bottom line for such changes is political. What is happening now is an empirical question. What can happen is a technical question. But what will happen is a political question, depending on social choices.

The central open questions about the effect of computers on children in the 1980s are these: Which people will be attracted to the world of computers, what talents will they bring, and what tastes and ideologies will they impose on the growing computer culture? I have described children in LOGO environments engaged in self-referential discussions about their own thinking. This could happen because the LOGO language and the Turtle were designed by people who enjoy such discussion and worked hard to design a medium that would encourage it. Other designers of computer systems have different tastes and different ideas about what kinds of activities are suitable for children. Which design will prevail, and in what sub-culture, will not be decided by a simple bureaucratic decision made, for example, in a government Department of Education or by a committee of experts. Trends in computer style will emerge from a complex web of decisions by Foundations with resources to support one or another design, by corporations who may see a market, by schools, by individuals who will decide to make their career in the new field of activity, and by children who will have their own say in what they pick up and what they make of it. People often ask whether in the future children will program computers or become absorbed in pre-programmed activities. The answer must be that some children will do the one, some the other, some both and some neither. But which children, and most importantly, which social classes of children, will fall into each category will be influenced by

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tures are poor in opportunities to bring their thinking about thinking into the open, to learn to talk about it and to test their ideas by externalizing them. Access to computers can dramatically change this situation. Even the simplest Turtle work can open new opportunities for sharpening one's thinking about thinking: Programming the Turtle starts by making one reflect on how one does oneself what one would like the Turtle to do. Thus teaching the Turtle to act or to "think" can lead one to reflect on one's own actions and thinking. And as children move on, they program the computer to make more complex decisions and find themselves engaged in reflecting on more complex aspects of their own thinking.

In short, while the critic and I share the belief that working with computers can have a powerful influence on how people think, I have turned my attention to exploring how this influence could be turned in positive directions.

I see two kinds of counterarguments to my arguments against the critics. The first kind challenges my belief that it is a good thing for children to be epistemologists. Many people will argue that overly analytic, verbalized thinking is counterproductive even if it is deliberately chosen. The second kind of objection challenges my suggestion that computers are likely to lead to more reflective self-conscious thinking. Many people will argue that work with computers usually has the opposite effect. These two kinds of objections call for different kinds of analysis and cannot be discussed simultaneously. The first kind raises technical questions about the psychology of learning which will be discussed in chapters 4 and 6. The second kind of objection is most directly answered by saying that there is absolutely no inevitability that computers will have the effects I hope to see. Not all computer systems do. Most in use today do not. In LOGO environments I have seen children engaged in animated conversations about their own personal knowledge as they try to capture it in a program to make a Turtle carry out an action that they themselves know very well how to do. But of course the physical presence of a computer is not enough to insure that such conversations will come about. Far from it. In thousands of schools and in tens of thousands of private homes children are right now living through very different computer experiences. In

the kind of computer activities and the kind of environments created around them.

As an example, we consider an activity which may not occur to most people when they think of computers and children: the use of a computer as a writing instrument. For me, writing means making a rough draft and refining it over a considerable period of time. My image of myself as a writer includes the expectation of an "unacceptable" first draft that will develop with successive editing into presentable form. But I would not be able to afford this image if I were a third grader. The physical act of writing would be slow and laborious. I would have no secretary. For most children rewriting a text is so laborious that the first draft is the final copy, and the skill of rereading with a critical eye is never acquired. This changes dramatically when children have access to computers capable of manipulating text. The first draft is composed at the keyboard. Corrections are made easily. The current copy is always neat and tidy. I have seen a child move from total rejection of writing to an intense involvement (accompanied by rapid improvement of quality) within a few weeks of beginning to write with a computer. Even more dramatic changes are seen when the child has physical hand-caps that make writing by hand more than usually difficult or even impossible.

This use of computers is rapidly becoming adopted wherever adults write for a living. Most newspapers now provide their staff with "word processing" computer systems. Many writers who work at home are acquiring their own computers, and the computer terminal is steadily displacing the typewriter as the secretary's basic tool. The image of children using the computer as a writing instrument is a particularly good example of my general thesis that what is good for professionals is good for children. But this image of how the computer might contribute to children's mastery of language is dramatically opposed to the one that is taking root in most elementary schools. There the computer is seen as a teaching instrument. It gives children practice in distinguishing between verbs and nouns, in spelling, and in answering multiple-choice questions about the meaning of pieces of text. As I see it, this difference is not a matter of a small and technical choice between two teaching

strategies. It reflects a fundamental difference in educational philosophies. More to the point, it reflects a difference in views on the nature of childhood. I believe that the computer as writing instrument offers children an opportunity to become more like adults, indeed like advanced professionals, in their relationship to their intellectual products and to themselves. In doing so, it comes into head-on collision with the many aspects of school whose effect, if not whose intention, is to "infantilize" the child.

Word processors *can* make a child's experience of writing more like that of a real writer. But this can be undermined if the adults surrounding that child fail to appreciate what it is like to be a writer. For example, it is only too easy to imagine adults, including teachers, expressing the view that editing and re-editing a text is a waste of time ("Why don't you get on to something new?" or "You aren't making it any better, why don't you fix your spelling?").

As with writing, so with music-making, games of skill, complex graphics, whatever: The computer is not a culture unto itself but it can serve to advance very different cultural and philosophical outlooks. For example, one could think of the Turtle as a device to teach elements of the traditional curriculum, such as notions of angle, shape, and coordinate systems. And in fact, most teachers who consult me about its use are, quite understandably, trying to use it in this way. Their questions are about classroom organization, scheduling problems, pedagogical issues raised by the Turtle's introduction, and especially, about how it relates conceptually to the rest of the curriculum. Of course the Turtle can help in the teaching of traditional curriculum, but I have thought of it as a vehicle for Piagetian learning, which to me is learning without curriculum.

There are those who think about creating a "Piagetian curriculum" or "Piagetian teaching methods." But to my mind these phrases and the activities they represent are contradictions in terms. I see Piaget as the theorist of learning without curriculum and the theorist of the kind of learning that happens without deliberate-teaching. To turn him into the theorist of a new curriculum is to stand him on his head.

But "teaching without curriculum" does not mean spontaneous, free-form classrooms or simply "leaving the child alone." It means

supporting children as they build their own intellectual structures with materials drawn from the surrounding culture. In this model, educational intervention means changing the culture, planting new constructive elements in it and eliminating noxious ones. This is a more ambitious undertaking than introducing a curriculum change, but one which is feasible under conditions now emerging.

Suppose that thirty years ago an educator had decided that the way to solve the problem of mathematics education was to arrange for a significant fraction of the population to become fluent in (and enthusiastic about) a new mathematical language. The idea might have been good in principle, but in practice it would have been absurd. No one had the power to implement it. Now things are different. Many millions of people are learning programming languages for reasons that have nothing to do with the education of children. Therefore, it becomes a practical proposition to influence the form of the languages they learn and the likelihood that their children will pick up these languages.

The educator must be an anthropologist. The educator as anthropologist must work to understand which cultural materials are relevant to intellectual development. Then, he or she needs to understand which trends are taking place in the culture. Meaningful intervention must take the form of working with these trends. In my role of educator as anthropologist I see new needs being generated by the penetration of the computer into personal lives. People who have computers at home or who use them at work will want to be able to talk about them to their children. They will want to be able to teach their children to use the machines. Thus there could be a cultural demand for something like Turtle graphics in a way there never was, and perhaps never could be, a cultural demand for the New Math.

Throughout the course of this chapter I have been talking about the ways in which choices made by educators, foundations, governments, and private individuals can affect the potentially revolutionary changes in how children learn. But making good choices is not always easy, in part because past choices can often haunt us. There is a tendency for the first usable, but still primitive, product of a new technology to dig itself in. I have called this phenomenon the QWERTY phenomenon.

The top row of alphabetic keys of the standard typewriter reads QWERTY. For me this symbolizes the way in which technology can all too often serve not as a force for progress but for keeping things stuck. The QWERTY arrangement has no rational explanation, only a historical one. It was introduced in response to a problem in the early days of the typewriter: The keys used to jam. The idea was to minimize the collision problem by separating those keys that followed one another frequently. Just a few years later, general improvements in the technology removed the jamming problem, but QWERTY stuck. Once adopted, it resulted in many millions of typewriters and a method (indeed a full-blown curriculum) for learning typing. The social cost of change (for example, putting the most used keys *together* on the keyboard) mounted with the vested interest created by the fact that so many fingers now knew how to follow the QWERTY keyboard. QWERTY has stayed on despite the existence of other, more "rational" systems. On the other hand, if you talk to people about the QWERTY arrangement they will justify it by "objective" criteria. They will tell you that it "optimizes this" or it "minimizes that." Although these justifications have no rational foundation, they illustrate a process, a social process, of myth construction that allows us to build a justification for primitivity into any system. And I think that we are well on the road to doing exactly the same thing with the computer. We are in the process of digging ourselves into an anachronism by preserving practices that have no rational basis beyond their historical roots in an earlier period of technological and theoretical development.

The use of computers for drill and practice is only one example of the QWERTY phenomenon in the computer domain. Another example occurs even when attempts are made to allow students to learn to program the computer. As we shall see in later chapters, learning to program a computer involves learning a "programming language." There are many such languages—for example, FORTRAN, PASCAL, BASIC, SMALLTALK, and LISP, and the lesser known language LOGO, which our group has used in most of our experiments with computers and children. A powerful QWERTY phenomenon is to be expected when we choose the language in which children are to learn to program computers. I shall

argue in detail that the issue is consequential. A programming language is like a natural, human language in that it favors certain metaphors, images, and ways of thinking. The language used strongly colors the computer culture. It would seem to follow that educators interested in using computers and sensitive to cultural influences would pay particular attention to the choice of language. But nothing of the sort has happened. On the contrary, educators, too timid in technological matters or too ignorant to attempt to influence the languages offered by computer manufacturers, have accepted certain programming languages in much the same way as they accepted the QWERTY keyboard. An informative example is the way in which the programming language BASIC<sup>3</sup> has established itself as the obvious language to use in teaching American children how to program computers. The relevant technical information is this: A very small computer can be made to understand BASIC, while other languages demand more from the computer. Thus, in the early days when computer power was extremely expensive, there was a genuine technical reason for the use of BASIC, particularly in schools where budgets were always tight. Today, and in fact for several years now, the cost of computer memory has fallen to the point where any remaining economic advantages of using BASIC are insignificant. Yet in most high schools, the language remains almost synonymous with programming, despite the existence of other computer languages that are demonstrably easier to learn and are richer in the intellectual benefits that can come from learning them. The situation is paradoxical. The computer revolution has scarcely begun, but is already breeding its own conservatism. Looking more closely at BASIC provides a window on how a conservative social system appropriates and tries to neutralize a potentially revolutionary instrument.

BASIC is to computation what QWERTY is to typing. Many teachers have learned BASIC, many books have been written about it, many computers have been built in such a way that BASIC is "hardwired" into them. In the case of the typewriter, we noted how people invent "rationalizations" to justify the status quo. In the case of BASIC, the phenomenon has gone much further, to the point where it resembles ideology formation. Complex arguments

are invented to justify features of BASIC that were originally included because the primitive technology demanded them or because alternatives were not well enough known at the time the language was designed.

An example of BASIC ideology is the argument that BASIC is easy to learn because it has a very small vocabulary. The surface validity of the argument is immediately called into question if we apply it to the context of how children learn natural languages. Imagine a suggestion that we invent a special language to help children learn to speak. This language would have a small vocabulary of just fifty words, but fifty words so well chosen that all ideas could be expressed using them. Would this language be easier to learn? Perhaps the vocabulary might be easy to learn, but the use of the vocabulary to express what one wanted to say would be so contorted that only the most motivated and brilliant children would learn to say more than "hi." This is close to the situation with BASIC. Its small vocabulary can be learned quickly enough. But using it is a different matter. Programs in BASIC acquire so labyrinthine a structure that in fact only the most motivated and brilliant ("mathematical") children do learn to use it for more than trivial ends.

One might ask why the teachers do not notice the difficulty children have in learning BASIC. The answer is simple: Most teachers do not expect high performance from most students, especially in a domain of work that appears to be as "mathematical" and "formal" as programming. Thus the culture's general perception of mathematics as inaccessible bolsters the maintenance of BASIC, which in turn confirms these perceptions. Moreover, the teachers are not the only people whose assumptions and prejudices feed into the circuit that perpetuates BASIC. There are also the computerists, the people in the computer world who make decisions about what languages their computers will speak. These people, generally engineers, find BASIC quite easy to learn, partly because they are accustomed to learning such very technical systems and partly because BASIC's sort of simplicity appeals to their system of values. Thus, a particular subculture, one dominated by computer engineers, is influencing the world of education to favor those school

students who are most like that subculture. The process is tacit, unintentional: It has never been publicly articulated, let alone evaluated. In all of these ways, the social embedding of BASIC has far more serious consequences than the "digging in" of QWERTY.

There are many other ways in which the attributes of the subcultures involved with computers are being projected onto the world of education. For example, the idea of the computer as an instrument for drill and practice that appeals to teachers because it resembles traditional teaching methods also appeals to the engineers who design computer systems: Drill and practice applications are predictable, simple to describe, efficient in use of the machine's resources. So the best engineering talent goes into the development of computer systems that are biased to favor this kind of application. The bias operates subtly. The machine designers do not actually decide what will be done in the classrooms. That is done by teachers and occasionally even by carefully controlled comparative research experiments. But there is an irony in these controlled experiments. They are very good at telling whether the small effects seen in best scores are real or due to chance. But they have no way to measure the undoubtedly real (and probably more massive) effects of the biases built into the machines.

We have already noted that the conservative bias being built into the use of computers in education has also been built into other new technologies. The first use of the new technology is quite naturally to do in a slightly different way what had been done before without it. It took years before designers of automobiles accepted the idea that they were cars, not "horseless carriages," and the precursors of modern motion pictures were plays acted as if before a live audience but actually in front of a camera. A whole generation was needed for the new art of motion pictures to emerge as something quite different from a linear mix of theater plus photography. Most of what has been done up to now under the name of "educational technology" or "computers in education" is still at the stage of the linear mix of old instructional methods with new technologies. The topics I shall be discussing are some of the first probings toward a more organic interaction of fundamental educational principles and new methods for translating them into reality.

We are at a point in the history of education when radical

change is possible, and the possibility for that change is directly tied to the impact of the computer. Today what is offered in the education "market" is largely determined by what is acceptable to a sluggish and conservative system. But this is where the computer presence is in the process of creating an environment for change. Consider the conditions under which a new educational idea can be put into practice today and in the near future. Let us suppose that today I have an idea about how children could learn mathematics more effectively and more humanely. And let us suppose that I have been able to persuade a million people that the idea is a good one. For many products such a potential market would guarantee success. Yet in the world of education today this would have little clout: A million people across the nation would still mean a minority in every town's school system, so there might be no effective channel for the million voices to be expressed. Thus, not only do good educational ideas sit on the shelves, but the process of invention is itself stymied. This inhibition of invention in turn influences the selection of people who get involved in education. Very few with the imagination, creativity, and drive to make great new inventions enter the field. Most of those who do are soon driven out in frustration. Conservatism in the world of education has become a self-perpetuating *social* phenomenon.

Fortunately, there is a weak link in the vicious circle. Increasingly, the computers of the very near future will be the private property of individuals, and this will gradually return to the individual the power to determine patterns of education. Education will become more of a private act, and people with good ideas, different ideas, exciting ideas will no longer be faced with a dilemma where they either have to "sell" their ideas to a conservative bureaucracy or shelve them. They will be able to offer them in an open marketplace directly to consumers. There will be new opportunities for imagination and originality. There might be a renaissance of thinking about education.

## Chapter 2

# Mathophobia:

# The Fear of

# Learning

presence might, I think, plant seeds that could grow into a less dissociated cultural epistemology.

The status of mathematics in contemporary culture is one of the most acute symptoms of its dissociation. The emergence of a "humanistic" mathematics, one that is not perceived as separated from the study of man and "the humanities," might well be the sign that a change is in sight. So in this book I try to show how the computer presence can bring children into a more humanistic as well as a more humane relationship with mathematics. In doing so I shall have to go beyond discussion of mathematics. I shall have to develop a new perspective on the process of learning itself.

It is not uncommon for intelligent adults to turn into passive observers of their own incompetence in anything but the most rudimentary mathematics. Individuals may see the direct consequences of this intellectual paralysis in terms of limiting job possibilities. But the indirect, secondary consequences are even more serious. One of the main lessons learned by most people in math class is a sense of having rigid limitations. They learn a balkanized image of human knowledge which they come to see as a patchwork of territories separated by impassable iron curtains. My challenge is not to the sovereignty of the intellectual territories but to the restrictions imposed on easy movement among them. I do not wish to reduce mathematics to literature or literature to mathematics. But I do want to argue that their respective ways of thinking are not as separate as is usually supposed. And so, I use the image of a Mathland—where mathematics would become a natural vocabulary—to develop my idea that the computer presence could bring the humanistic and mathematical/scientific cultures together. In this book, Mathland is the first step in a larger argument about how the computer presence can change not only the way we teach children mathematics, but, much more fundamentally, the way in which our culture as a whole thinks about knowledge and learning.

To my ear the word "mathophobia" has two associations. One of these is a widespread fear of mathematics, which often has the intensity of a real phobia. The other comes from the meaning of the stem "math." In Greek it means "learning" in a general sense.\* In

\*The original meaning is present in the word "polymath," a person of many learnings. A less well-known word with the same stem which I shall use in later chapters is "mathetic," having to do with learning.

PLATO WROTE over his door, "Let only geometers enter." Times have changed. Most of those who now seek to enter Plato's intellectual world neither know mathematics nor sense the least contradiction in their disregard for his injunction. Our culture's schizophrenic split between "humanities" and "science" supports their sense of security. Plato was a philosopher, and philosophy belongs to the humanities as surely as mathematics belongs to the sciences.

This great divide is thoroughly built into our language, our worldview, our social organization, our educational system, and, most recently, even our theories of neurophysiology. It is self-perpetuating: The more the culture is divided, the more each side builds separation into its new growth.

I have already suggested that the computer may serve as a force to break down the line between the "two cultures." I know that the humanist may find it questionable that a "technology" could change his assumptions about what kind of knowledge is relevant to his or her perspective of understanding people. And to the scientist dilution of rigor by the encroachment of "wishy-washy" humanistic thinking can be no less threatening. Yet the computer



For an adult it is obvious that pouring liquid from one glass to another does not change the volume (ignoring such little effects as drops that spilled or remained behind). The conservation of volume is so obvious that it seems not to have occurred to anyone before Piaget that children of four might not find it obvious at all.\* A substantial intellectual growth is needed before children develop the "conservationist" view of the world. The conservation of volume is only one of many conservations they all learn. Another is the conservation of numbers. Again, it does not occur to most adults that a child must learn that counting a collection of objects in a different order should yield the same result. For adults counting is simply a method of determining how many objects "there are." The result of the operation is an "objective fact" independent of the act of counting. But the separation of number from counting (of product from process) rests on epistemological presuppositions not only unknown to preconservationist children, but alien to their worldview. These conservations are only part of a vast structure of "hidden" mathematical knowledge that children learn by themselves. In the intuitive geometry of the child of four or five, a straight line is not necessarily the shortest distance between two points, and walking slowly between two points does not necessarily take more time than walking fast. Here, too, it is not merely the "item" of knowledge that is missing, but the epistemological presupposition underlying the idea of "shortest" as a property of the path rather than of the action of traversing it.

None of this should be understood as mere *lack* of knowledge on the part of the children. Piaget has demonstrated how young children hold theories of the world that, in their own terms, are perfectly coherent. These theories, spontaneously "learned" by all children, have well-developed components that are not less "mathematical," though expressing a different mathematics, than the one generally accepted in our (adult) culture. The hidden learning process has at least two phases: Already in the preschool years every child first constructs one or more preadult theorizations of the

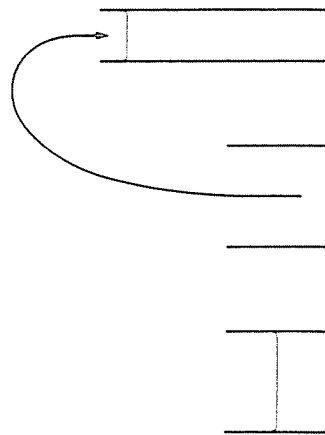
\*People have lived with children for a long time. The fact that we had to wait for Piaget to tell us how children think and *what we all forget about our thinking as children* is so remarkable that it suggests a Freudian model of "cognitive repression."

**MINDSTORMS**

our culture, fear of learning is no less endemic (although more frequently disguised) than fear of mathematics. Children begin their lives as eager and competent learners. They have to *learn* to have trouble with learning in general and mathematics in particular. In both senses of "math" there is a shift from mathophile to mathophobe, from lover of mathematics and of learning to a person fearful of both. We shall look at how this shift occurs and develop some idea of how the computer presence could serve to counteract it. Let me begin with some reflections on what it is like to learn as a child.

That children learn a great deal seems so obvious to most people that they believe it is scarcely worth documenting. One area in which a high rate of learning is very plain is the acquisition of a spoken vocabulary. At age two very few children have more than a few hundred words. By the time they enter first grade, four years later, they know thousands of words. They are evidently learning many new words every day.

While we can "see" that children learn words, it is not quite as easy to see that they are learning mathematics at a similar or greater rate. But this is precisely what has been shown by Piaget's life-long study of the genesis of knowledge in children. One of the more subtle consequences of his discoveries is the revelation that adults fail to appreciate the extent and the nature of what children are learning, because knowledge structures we take for granted have rendered much of that learning invisible. We see this most clearly in what have come to be known as Piagetian "conservations" (see Figure 2).



**Figure 2** The Conservation of Liquids

world and then moves toward more adultlike views. And all this is done through what I have called Piagetian learning, a learning process that has many features the schools should envy: It is effective (all the children get there), it is inexpensive (it seems to require neither teacher nor curriculum development), and it is humane (the children seem to do it in a carefree spirit without explicit external rewards and punishments).

The extent to which adults in our society have lost the child's positive stance toward learning varies from individual to individual. An unknown but certainly significant proportion of the population has almost completely given up on learning. These people seldom, if ever, engage in deliberate learning and see themselves as neither competent at it nor likely to enjoy it. The social and personal cost is enormous: Mathphobia can, culturally and materially, limit people's lives. Many more people have not completely given up on learning but are still severely hampered by entrenched negative beliefs about their capacities. Deficiency becomes identity: "I can't learn French, I don't have an ear for languages;" "I could never be a businessman, I don't have a head for figures;" "I can't get the hang of parallel skiing, I never was coordinated." These beliefs are often repeated ritualistically, like superstitions. And, like superstitions, they create a world of taboos; in this case, taboos on learning. In this chapter and chapter 3, we discuss experiments that demonstrate that these self-images often correspond to a very limited reality—usually to a person's "school reality." In a learning environment with the proper emotional and intellectual support, the "uncoordinated" can learn circus arts like juggling and those with "no head for figures" learn not only that they can do mathematics but that they can enjoy it as well.

Although these negative self-images can be overcome, in the life of an individual they are extremely robust and powerfully self-reinforcing. If people believe firmly enough that they cannot do math, they will usually succeed in preventing themselves from doing whatever they recognize as math. The consequences of such self-sabotage is personal failure, and each failure reinforces the original belief. And such beliefs may be most insidious when held not only by individuals, but by our entire culture.

Our children grow up in a culture permeated with the idea that there are "smart people" and "dumb people." The social construction of the individual is as a bundle of aptitudes. There are people who are "good at math" and people who "can't do math." Everything is set up for children to attribute their first unsuccessful or unpleasant learning experiences to their own disabilities. As a result, children perceive failure as relegating them either to the group of "dumb people" or, more often, to a group of people "dumb at  $x$ " (where, as we have pointed out,  $x$  often equals mathematics). Within this framework children will define themselves in terms of their limitations, and this definition will be consolidated and reinforced throughout their lives. Only rarely does some exceptional event lead people to reorganize their intellectual self-image in such a way as to open up new perspectives on what is learnable.

This belief about the structure of human abilities is not easy to undermine. It is never easy to uproot popular beliefs. But here the difficulty is compounded by several other factors. First, popular theories about human aptitudes seem to be supported by "scientific" ones. After all, psychologists talk in terms of measuring aptitudes. But the significance of what is measured is seriously questioned by our simple thought experiment of imagining Mathland.

Although the thought experiment of imagining a Mathland leaves open the question of how a Mathland can actually be created, it is completely rigorous as a demonstration that the accepted beliefs about mathematical aptitude do not follow from the available evidence. But since truly mathophobic readers might have trouble making this experiment their own, I shall reinforce the argument by casting it in another form. Imagine that children were forced to spend an hour a day drawing dance steps on squared paper and had to pass tests in these "dance facts" before they were allowed to dance physically. Would we not expect the world to be full of "dancophobes"? Would we say that those who made it to the dance floor and music had the greatest "aptitude for dance"? In my view, it is no more appropriate to draw conclusions about mathematical aptitude from children's unwillingness to spend many hundreds of hours doing sums.

One might hope that if we pass from parables to the more rigor-

ous methods of psychology we could get some "harder" data on the problem of the true ceilings of competence attainable by individuals. But this is not so: The paradigm in use by contemporary educational psychology is focused on investigations of how children learn or (more usually) don't learn mathematics in the "anti-Mathland" in which we all live. The direction of such research has an analogy in the following parable:

Imagine someone living in the nineteenth century who felt the need to improve methods of transportation. He was persuaded that the route to new methods started with a deep understanding of the existing problems. So he began a careful study of the differences among horse-drawn carriages. He carefully documented by the most refined methods how speed varied with the form and substance of various kinds of axles, bearings, and harnessing techniques.

In retrospect, we know that the road that led from nineteenth-century transportation was quite different. The invention of the automobile and the airplane did not come from a detailed study of how their predecessors, such as horse-drawn carriages, worked or did not work. Yet, this is the model for contemporary educational research. The standard paradigms for education research take the existing classroom or extracurricular culture as the primary object of study. There are many studies concerning the poor notions of math or science students acquire from today's schooling. There is even a very prevalent "humanistic" argument that "good" pedagogy should take these poor ways of thinking as its starting point. It is easy to sympathize with the humane intent. Nevertheless I think that the strategy implies a commitment to preserving the traditional system. It is analogous to improving the axle of the horse-drawn cart. But the real question, one might say, is whether we can invent the "educational automobile." Since this question (the central theme of this book) has not been addressed by educational psychology, we must conclude that the "scientific" basis for beliefs about aptitudes is really very shaky. But these beliefs are institutionalized in schools, in testing systems, and in college admissions criteria and consequently, their social basis is as firm as their scientific basis is weak.

From kindergarten on, children are tested for verbal and quanti-

tative aptitudes, conceived of as "real" and separable entities. The results of these tests enter into the social construction of each child as a bundle of aptitudes. Once Johnny and his teacher have a shared perception of Johnny as a person who is "good at" art and "poor at" math, this perception has a strong tendency to dig itself in. This much is widely accepted in contemporary educational psychology. But there are deeper aspects to how school constructs aptitudes. Consider the case of a child I observed through his eighth and ninth years. Jim was a highly verbal and mathophobic child from a professional family. His love for words and for talking showed itself very early, long before he went to school. The mathophobia developed at school. My theory is that it came as a direct result of his verbal precocity. I learned from his parents that Jim had developed an early habit of describing in words, often aloud, whatever he was doing as he did it. This habit caused him minor difficulties with parents and preschool teachers. The real trouble came when he hit the arithmetic class. By this time he had learned to keep "talking aloud" under control, but I believe that he still maintained his inner running commentary on his activities. In his math class he was stymied: He simply did not know how to talk about doing sums. He lacked a vocabulary (as most of us do) and a sense of purpose. Out of this frustration of his verbal habits grew a hatred of math, and out of the hatred grew what the tests later confirmed as poor aptitude.

For me the story is poignant. I am convinced that what shows up as intellectual weakness very often grows, as Jim's did, out of intellectual strengths. And it is not only verbal strengths that undermine others. Every careful observer of children must have seen similar processes working in different directions: For example, a child who has become enamored of logical order is set up to be turned off by English spelling and to go on from there to develop a global dislike for writing.

The Mathland concept shows how to use computers as vehicles to escape from the situation of Jim and his dyslexic counterpart. Both children are victims of our culture's hard-edged separation between the verbal and the mathematical. In the Mathland we shall describe in this chapter, Jim's love and skill for language

could be mobilized to serve his formal mathematical development instead of opposing it, and the other child's love for logic could be recruited to serve the development of interest in linguistics.

The concept of mobilizing a child's multiple strengths to serve all domains of intellectual activity is an answer to the suggestion that differing aptitudes may reflect actual differences in brain development. It has become commonplace to talk as if there are separate brains, or separate "organs" in the brain, for mathematics and for language. According to this way of thinking, children split into the verbally and the mathematically apt depending on which brain organs are strongest. But the argument from anatomy to intellect reflects a set of epistemological assumptions. It assumes, for example, that there is only one route to mathematics and that if this route is "anatomically blocked," the child cannot get to the destination. Now, in fact, for most children in contemporary societies there may indeed be only one route into "advanced" mathematics, the route via school math. But even if further research in brain biology confirms that this route depends on anatomical brain organs that might be missing in some children, it would not follow that mathematics itself is dependent on these brain organs. Rather, it would follow that we should seek out other routes. Since this book is an argument that alternate routes do exist, it can be read as showing how the dependency of function on brain is itself a social construct.

Let us grant, for the sake of argument, that there is a special part of the brain especially good at performing the mental manipulations of numbers we teach children in school, and let's call it the MAD, or "math acquisition device."<sup>2</sup> On this assumption it would make sense that in the course of history humankind would have evolved methods of doing and of teaching arithmetic that take full advantage of the MAD. But while these methods would work for most of us, and so for society as a whole, reliance on them would be catastrophic for an individual whose MAD happened to be damaged or inaccessible for some other (perhaps "neurotic") reason. Such a person would fail at school and be diagnosed as a victim of "dyscalculia." And as long as we insist on making children learn arithmetic by the standard route, we will continue to "prove" by

objective tests that these children really cannot "do arithmetic." But this is like proving that the deaf children cannot have language because they don't hear. Just as sign languages use hands and eyes to bypass the more usual speaking organs so, too, alternative ways of doing mathematics that bypass the MAD may be as good as, even if different from, the usual ones.

But we do not have to appeal to neurology to explain why some children do not become fluent in mathematics. The analogy of the dance class without music or dance floor is a serious one. Our education culture gives mathematics learners scarce resources for making sense of what they are learning. As a result our children are forced to follow the very worst model for learning mathematics. This is the model of rote learning, where material is treated as meaningless; it is a *dissociated* model. Some of our difficulties in teaching a more culturally integrable mathematics have been due to an objective problem: Before we had computers there were very few good points of contact between what is most fundamental and engaging in mathematics and anything firmly planted in everyday life. But the computer—a mathematics-speaking being in the midst of the everyday life of the home, school, and workplace—is able to provide such links. The challenge to education is to find ways to exploit them.

Mathematics is certainly not the only example of dissociated learning. But it is a very good example for precisely the reason that many readers are probably now wishing that I would talk about something else. Our culture is so mathophobic, so math-fearing, that if I could demonstrate how the computer can bring us into a new relationship to mathematics, I would have a strong foundation for claiming that the computer has the ability to change our relation to other kinds of learning we might fear. Experiences in Mathland, such as entering into a "mathematical conversation," give the individual a liberating sense of the possibilities of doing a variety of things that may have previously seemed "too hard." In this sense, contact with the computer can open access to knowledge for people, not instrumentally by providing them with processed information, but by challenging some constraining assumptions they make about themselves.

## MINDSTORMS

The computer-based Mathland I propose extends the kind of natural, Piagetian learning that accounts for children's learning a first language to learning mathematics. Piagetian learning is typically deeply embedded in other activities. For example, the infant does not have periods set aside for "learning talking." This model of learning stands in opposition to dissociated learning, learning that takes place in relative separation from other kinds of activities, mental and physical. In our culture, the teaching of mathematics in schools is paradigmatic of dissociated learning. For most people, mathematics is taught and taken as medicine. In its dissociation of mathematics, our culture comes closest to caricaturing its own worst habits of epistemological alienation. In LOGO environments we have done some blurring of boundaries: No particular computer activities are set aside as "learning mathematics."

The problem of making mathematics "make sense" to the learner touches on the more general problem of making a language of "formal description" make sense. So before turning to examples of how the computer helps give meaning to mathematics, we shall look at several examples where the computer helped give meaning to a language of formal description in domains of knowledge that people do not usually count as mathematics. In our first example the domain is grammar, for many people a subject only a little less threatening than math.

Well into a year-long study that put powerful computers in the classrooms of a group of "average" seventh graders, the students were at work on what they called "computer poetry." They were using computer programs to generate sentences. They gave the computer a syntactic structure within which to make random choices from given lists of words. The result is the kind of concrete poetry we see in the illustration that follows. One of the students, a thirteen-year-old named Jenny, had deeply touched the project's staff by asking on the first day of her computer work, "Why were we chosen for this? We're not the brains." The study had deliberately chosen children of "average" school performance. One day Jenny came in very excited. She had made a discovery. "Now I know why we have nouns and verbs," she said. For many years in school Jenny had been drilled in grammatical categories. She had

## Mathophobia: The Fear of Learning

never understood the differences between nouns and verbs and adverbs. But now it was apparent that her difficulty with grammar was not due to an inability to work with logical categories. It was something else. She had simply seen no purpose in the enterprise. She had not been able to make any sense of what grammar was about in the sense of what it might be *for*. And when she had asked what it was for, the explanations that her teachers gave seemed manifestly dishonest. She said she had been told that "grammar helps you talk better."

INSANE RETARD MAKES BECAUSE SWEET SNOOPY SCREAMS  
SEXY WOLF LOVES THATS WHY THE SEXY LADY HATES  
UGLY MAN LOVES BECAUSE UGLY DOG HATES  
MAD WOLF HATES BECAUSE INSANE WOLF SKIPS  
SEXY RETARD SCREAMS THATS WHY THE SEXY RETARD  
HATES  
THIN SNOOPY RUNS BECAUSE FAT WOLF HOPS  
SWEET FOGINY SKIPS A FAT LADY RUNS

### Jenny's Concrete Poetry

In fact, tracing the connection between learning grammar and improving speech requires a more distanced view of the complex process of learning language than Jenny could have been given at the age she first encountered grammar. She certainly didn't see any way in which grammar could help talking, nor did she think her talking needed any help. Therefore she learned to approach grammar with resentment. And, as is the case for most of us, resentment guaranteed failure. But now, as she tried to get the computer to generate poetry, something remarkable happened. She found herself classifying words into categories, not because she had been told she had to but because she needed to. In order to "teach" her computer to make strings of words that would look like English, she had to "teach" it to choose words of an appropriate class. What she learned about grammar from this experience with a machine was anything but mechanical or routine. Her learning was deep and meaningful. Jenny did more than learn definitions for particular

grammatical classes. She understood the general idea that words (like things) can be placed in different groups or sets, and that doing so could work for her. She not only "understood" grammar, she changed her relationship to it. It was "hers," and during her year with the computer, incidents like this helped Jenny change her image of herself. Her performance changed too; her previously low to average grades became "straight A's" for her remaining years of school. She learned that she could be "a brain" after all.

It is easy to understand why math and grammar fail to make sense to children when they fail to make sense to everyone around them and why helping children to make sense of them requires more than a teacher making the right speech or putting the right diagram on the board. I have asked many teachers and parents what they thought mathematics to be and why it was important to learn it. Few held a view of mathematics that was sufficiently coherent to justify devoting several thousand hours of a child's life to learning it, and children sense this. When a teacher tells a student that the reason for those many hours of arithmetic is to be able to check the change at the supermarket, the teacher is simply not believed. Children see such "reasons" as one more example of adult double talk. The same effect is produced when children are told school math is "fun" when they are pretty sure that teachers who say so spend their leisure hours on anything except this allegedly fun-filled activity. Nor does it help to tell them that they need math to become scientists—most children don't have such a plan. The children can see perfectly well that the teacher does not like math any more than they do and that the reason for doing it is simply that it has been inscribed into the curriculum. All of this erodes children's confidence in the adult world and the process of education. *And I think it introduces a deep element of dishonesty into the educational relationship.*

Children perceive the school's rhetoric about mathematics as double talk. In order to remedy the situation we must first acknowledge that the child's perception is fundamentally correct. The *kind of mathematics* foisted on children in schools is not meaningful, fun, or even very useful. This does not mean that an individual child cannot turn it into a valuable and enjoyable personal game.

For some the game is scoring grades; for others it is outwitting the teacher and the system. For many, school math is enjoyable in its repetitiveness, precisely because it is so mindless and dissociated that it provides a shelter from having to think about what is going on in the classroom. But all this proves is the ingenuity of children. It is not a justification for school math to say that *despite* its intrinsic dullness, inventive children can find excitement and meaning in it.

It is important to remember the distinction between *mathematics*—a vast domain of inquiry whose beauty is rarely suspected by most nonmathematicians—and something else which I shall call *math* or *school math*.

I see "school math" as a social construction, a kind of QWERTY. A set of historical accidents (which shall be discussed in a moment) determined the choice of certain mathematical topics as *the* mathematical baggage that citizens should carry. Like the QWERTY arrangement of typewriter keys, school math did make some sense in a certain historical context. But, like QWERTY, it has dug itself in so well that people take it for granted and invent rationalizations for it long after the demise of the historical conditions that made sense of it. Indeed, for most people in our culture it is inconceivable that school math could be very much different. This is the only mathematics they know. In order to break this vicious circle I shall lead the reader into a new area of mathematics, Turtle geometry, that my colleagues and I have created as a better, more meaningful first area of formal mathematics for children. The design criteria of Turtle geometry are best understood by looking a little more closely at the historical conditions responsible for the shape of school math.

Some of these historical conditions were pragmatic. Before electronic calculators existed it was a practical social necessity that many people be "programmed" to perform such operations as long division quickly and accurately. But now that we can purchase calculators cheaply we should reconsider the need to expend several hundred hours of every child's life on learning such arithmetic functions. I do not mean to deny the intellectual value of some knowledge, indeed, of a lot of knowledge, about numbers. Far from

construction of school math is strongly influenced by what seemed to be teachable when math was taught as a "dead" subject, using the primitive, passive technologies of sticks and sand, chalk and blackboard, pencil and paper. The result was an intellectually incoherent set of topics that violates the most elementary mathematic principles of what makes certain material easy to learn and some almost impossible.

Faced with the heritage of school, math education can take two approaches. The traditional approach accepts school math as a given entity and struggles to find ways to teach it. Some educators use computers for this purpose. Thus, paradoxically, the most common use of the computer in education has become force-feeding indigestible material left over from the precomputer epoch. In Turtle geometry the computer has a totally different use. There the computer is used as a mathematically expressive medium, one that frees us to design personally meaningful and intellectually coherent and easily learnable mathematical topics for children. Instead of posing the educational problem as "how to teach the existing school math," we pose it as "reconstructing mathematics," or more generally, as reconstructing knowledge in such a way that no great effort is needed to teach it.

All "curriculum development" could be described as "reconstructing knowledge." For example, the New Math curriculum reform of the sixties made some attempt to change the content of school math. But it could not go very far. It was stuck with having to do sums, albeit different sums. The fact that the new sums dealt with sets instead of numbers, or arithmetic in base two instead of base ten made little difference. Moreover, the math reform did not provide a challenge to the inventiveness of creative mathematicians and so never acquired the sparkle of excitement that marks the product of new thought. The name itself—"New Math"—was a misnomer. There was very little new about its mathematical content. It did not come from a process of invention of children's mathematics but from a process of trivialization of mathematician's mathematics. Children need and deserve something better than selecting out pieces of old mathematics. Like clothing passed down to the younger siblings, it never fits comfortably.

Turtle geometry started with the goal of fitting children. Its pri-

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it. But we can now select this knowledge on coherent, rational grounds. We can free ourselves from the tyranny of the superficial, pragmatic considerations that dictated past choices about what knowledge should be learned and at what age.

But utility was only one of the historical reasons for school math. Others were of a *mathetic* nature. Mathematics is the set of guiding principles that govern learning. Some of the historical reasons for school math had to do with what was learnable and teachable in the precomputer epoch. As I see it, a major factor that determined what mathematics went into school math was what could be done in the setting of school classrooms with the primitive technology of pencil and paper. For example, children can draw graphs with pencil and paper. So it was decided to let children draw many graphs. The same considerations influenced the emphasis on certain kinds of geometry. For example, in school math "analytic geometry" has become synonymous with the representation of curves by equations. As a result every educated person vaguely remembers that  $y = x^2$  is the equation of a parabola. And although most parents have very little idea of why anyone should know this, they become indignant when their children do not. They assume that there must be a profound and objective reason known to those who better understand these things. Ironically, their mathophobia keeps most people from trying to examine those reasons more deeply and thus places them at the mercy of the self-appointed math specialists. Very few people ever suspect that the reason for what is included and what is not included in school math might be as crudely technological as the ease of production of parabolas with pencils! This is what could change most profoundly in a computer-rich world: The range of easily produced mathematical constructs will be vastly expanded.

Another mathetic factor in the social construction of school math is the technology of grading. A living language is learned by speaking and does not need a teacher to verify and grade each sentence. A dead language requires constant "feedback" from a teacher. The activity known as "sums" performs this feedback function in school math. These absurd little repetitive exercises have only one merit: They are easy to grade. But this merit has bought them a firm place at the center of school math. In brief, I maintain that

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mary design criterion was to be *appropriate*. Of course it had to have serious mathematical content, but we shall see that appropriability and serious mathematical thinking are not at all incompatible. On the contrary: We shall end up understanding that some of the most personal knowledge is also the most profoundly mathematical. In many ways mathematics—for example the mathematics of space and movement and repetitive patterns of action—is what comes most naturally to children. It is into this mathematics that we sink the tap-root of Turtle geometry. As my colleagues and I have worked through these ideas, a number of principles have given more structure to the concept of an appropriate mathematics. First, there was the *continuity principle*: The mathematics must be continuous with well-established personal knowledge from which it can inherit a sense of warmth and value as well as “cognitive” competence. Then there was the *power principle*: It must empower the learner to perform personally meaningful projects that could not be done without it. Finally there was a *principle of cultural resonance*: The topic must make sense in terms of a larger social context. I have spoken of Turtle geometry making sense to children. But it will not truly make sense to children unless it is accepted by adults too. A dignified mathematics for children cannot be something we permit ourselves to inflict on children, like unpleasant medicine, although we see no reason to take it ourselves.



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tell us about how to create conditions for more knowledge to be acquired by children through this marvelous process of "Piagetian learning." I saw the popular idea of designing a "Piagetian Curriculum" as standing Piaget on his head: Piaget is par excellence the theorist of learning without curriculum. As a consequence, I began to formulate two ideas that run through this book: (1) significant change in patterns of intellectual development will come about through cultural change, and (2) the most likely bearer of potentially relevant cultural change in the near future is the increasingly pervasive computer presence. Although these perspectives had informed the LOGO project from its inception, for a long time I could not see how to give them a theoretical framework.

I was helped in this, as in many other ways, by my wife Sherry Turkle. Without her, this book could not have been written. Ideas borrowed from Sherry turned out to be missing links in my attempts to develop ways of thinking about computers and cultures. Sherry is a sociologist whose particular concerns center on the interaction of ideas and culture formation, in particular how complexes of ideas are adopted by and articulated throughout cultural groups. When I met her she had recently completed an investigation of a new French psychoanalytic culture, of how psychoanalysis had "colonized" France, a country that had fiercely resisted Freudian influence. She had turned her attention to computer cultures and was thinking about how people's relationships with computation influence their language, their ideas about politics, and their views of themselves. Listening to her talk about both projects helped me to formulate my own approach and to achieve a sufficient sense of closure in my ideas to embark on this writing project.

Over the years Sherry has given me every kind of support. When the writing would not work out she gave me hours of conversation and editorial help. But her support was most decisive on the many occasions when I fell out of love with the book or when my confidence in my resolution to write it flagged. Then, her commitment to the project kept it alive and her love for me helped me find my way back to being in love with the work.

SEYMOUR PAPERT  
*Cambridge, Massachusetts*  
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Precisely because LOGO is not a toy, but a powerful computer language, it requires considerably larger memory than less powerful languages such as BASIC. This has meant that until recently LOGO was only to be implemented on relatively large computers. With the lowering cost of memory this situation is rapidly changing. As this book goes to press, prototypes of LOGO systems are running on a 48K Apple II system and on a TI 99/4 with extended memory. Readers who would like to be kept informed of the status of LOGO implementations can write to me at LOGO project, MIT Artificial Intelligence Laboratory, 545 Technology Square, Cambridge, Mass. 02139. See S. Papert et al., *LOGO: A Language For Learning* (Morristown, N.J.: Creative Computing Press, forthcoming, Summer 1981).

3. The history of the Turtle in the LOGO project is as follows. In 1968-1969, the first class of twelve "average" seventh-grade students at the Muzzy Junior High School in Lexington, Massachusetts, worked with LOGO through the whole school year in place of their normal mathematics curriculum. At that time the LOGO system had no graphics. The students wrote programs that could translate English to "Pig Latin," programs that could play games of strategy, and programs to generate concrete poetry. This was the first confirmation that LOGO was a learnable language for computer "novices." However, I wanted to see the demonstration extended to fifth graders, third graders, and ultimately to preschool children. It seemed obvious that even if the LOGO language was learnable at these ages, the programming topics would not be. I proposed the Turtle as a programming domain that could be interesting to people at all ages. This expectation has subsequently been borne out by experience, and the Turtle as a learning device has been widely adopted. Pioneer work in using the Turtle to teach very young children was done by Radia Perlman who demonstrated, while she was a student at MIT, that four-year-old children could learn to control mechanical Turtles. Cynthia Solomon used screen Turtles in the first demonstration that first graders could learn to program. At the other end of the age spectrum, it is encouraging to see that Turtle programming is being used at a college level to teach PASCAL. See Kenneth L. Bowles, *Problem Solving Using PASCAL* (New York: Springer-Verlag, 1977). Controlling Turtles has proven to be an engaging activity for retarded children, for autistic children, and for children with a variety of "learning disorders." See for example, Paul Goldenberg, *Special Technology for Special Children* (Baltimore: University Park Press, 1979). Turtles have been incorporated into the SMALLTALK computer system at the Xerox Palo Alto Research Center. See Alan Kay and Adele Goldberg, "Personal Dynamic Media" (Palo Alto, Calif.: Xerox, Palo Alto Research Center, 1976).

4. *Touch Sensor Turtle*. The simplest touch sensor program in LOGO is as follows:

```
TO BOUNCE
REPEAT
FORWARD 1
TEST FRONT TOUCH
IF TRUE RIGHT 180
END
```

#### Comments

This means repeat all the individual steps  
The turtle keeps moving  
It checks whether it has run into something  
If so, it does an about turn

This will make the Turtle turn about when it encounters an object. A more subtle and more instructive program using the Touch Sensor Turtle is as follows:

```
TO FOLLOW
REPEAT
FORWARD 1
TEST LEFT TOUCH
IF TRUE RIGHT 1
IF FALSE LEFT 1
END
```

#### Comments

Check. Is it touching?  
It thinks it's too close and turns away  
It thinks it might lose the object so it turns toward

This program will cause the Turtle to circumnavigate an object of any shape, provided that it starts with its left side in contact with the object (and provided that the object and any irregularities in its contour are large compared to the Turtle).

It is a very instructive project for a group of students to develop this (or an equivalent) program from first principles by acting out how they think they would use touch to get around an object and by translating their strategies into Turtle commands.

## Chapter 1

1. The program FOLLOW (See Introduction, note 4) is a very simple example of how a powerful cybernetic idea (control by negative feedback) can be used to elucidate a biological or psychological phenomenon. Simple as it is, the example helps bridge the gap between physical models of "causal mechanism" and psychological phenomenon such as "purpose." Theoretical psychologists have used more complex programs in the same spirit to construct models of practically every known psychological phenomenon. A bold formulation of the spirit of such inquiry is found in Herbert A. Simon, *Sciences of the Artificial* (Cambridge: MIT Press, 1969).

2. The critics and skeptics referred to here are distillations from years of public and private debates. These attitudes are widely held, but, unfortunately, seldom published and therefore seldom discussed with any semblance of rigor. One critic who has set a good example by publishing his views is Joseph Weizenbaum in *Computer Power and Human Reason: From Judgment to Calculations* (San Francisco: W.H. Freeman, 1976).

Unfortunately Weizenbaum's book discusses two separate (though related) questions: whether computers harm the way people think and whether computers themselves can think. Most critical reviews of Weizenbaum have focused on the latter question, on which he joins company with Hubert L. Dreyfus, *What Computers Can't Do: A Critique of Artificial Reason* (New York: Harper & Row, 1972).

A lively description of some of the principal participants in the debate about whether computers can or cannot think is found in Pamela McCorduck, *Machines Who Think* (San Francisco: W.H. Freeman, 1979).

There is little published data on whether computers actually affect how people think. This question is being studied presently by S. Turkle.

3. Many versions of BASIC would allow a program to produce a shape like that made by the LOGO program HOUSE. The simplest example would look something like this:

```
10 PLOT (0,0)
20 PLOT (100,0)
30 PLOT (100,100)
40 PLOT (75,150)
50 PLOT (0,100)
60 PLOT (0,0)
70 END
```

Writing such a program falls short of the LOGO program as a beginning programming experience in many ways. It demands more of the beginner, in particular, it demands knowledge of cartesian coordinates. This demand would be less serious if the program, once written, could become a powerful tool for other projects. The LOGO programs SQ, TRI, and HOUSE can be used to draw squares, triangles, and houses in any position and orientation on the screen. The BASIC program allows one particular house to be drawn in one position. In order to make a BASIC program that will draw houses in many positions, it is necessary to use algebraic variables as in PLOT (x, y), PLOT (x + 100, y), and so on. As for defining new commands, such as SQ, TRI, and HOUSE, the commonly used versions of BASIC either do not allow this at all or, at best, allow something akin to it to be achieved through the

use of advanced technical programming methods. Advocates of BASIC might reply that: (1) these objections refer only to a beginner's experience and (2) these deficiencies of BASIC could be fixed. The first argument is simply not true: The intellectual and practical primitivity of BASIC extends all along the line up to the most advanced programming. The second misses the point of my complaint. Of course one could turn BASIC into LOGO or SMALL-TALK or anything else and still call it "BASIC." My complaint is that what is being foisted on the education world has not been so "fixed." Moreover, doing so would be a little like "re-modelling" a wooden house to become a skyscraper.

## Chapter 2

1. "Gedanken experiments" have played an important role in science, particularly in physics. These experiments would encourage more critical attitudes if used more often in thinking about education.

2. There is a joke here. Readers who are not familiar with Noam Chomsky's recent work may not get it. Noam Chomsky believes that we have a language acquisition device. I don't: the MAD seems no more improbable than the LAD. See N. Chomsky, *Reflections on Language* (New York: Pantheon, 1976) for his view of the brain as made up of specialized neurological organs matched to specific intellectual functions. I think that the fundamental question for the future of education is not whether the brain is "a general purpose computer" or a collection of specialized devices, but whether our intellectual functions are reducible in a one-to-one fashion to neurologically given structures.

It seems to be beyond doubt that the brain has numerous inborn "gadgets." But surely these "gadgets" are much more primitive than is suggested by names like LAD and MAD. I see learning language or learning mathematics as harnessing to this purpose numerous "gadgets" whose original purpose bears no resemblance to the complex intellectual functions they come to serve.

## Chapter 3

1. Since this book is written for readers who may not know much mathematics, references to specific mathematics are as restrained as possible. The following remarks will flesh out the discussion for mathematically sophisticated readers.

The isomorphism of different Turtle systems is one of many examples of "advanced" mathematical ideas that come up in Turtle geometry in forms that are both *concrete* and *useful*. Among these, concepts from "calculus" are especially important.

*Example 1: Integration.* Turtle geometry prepares the way for the concept of line integral by the frequent occurrence of situations where the Turtle has to integrate some quantity as it goes along. Often the first case encountered by children comes from the need to have the Turtle keep track of how much it has turned or of how far it has moved. An excellent Turtle project is simulating tropisms that would cause an animal to seek such conditions as warmth, or light, or nutrient concentration represented as a field in the form of a numerical function of position. It is natural to think of comparing two algorithms by integrating the field quantity along the Turtle's path. A simple version is achieved by inserting into a program a single line such as: CALL (:TOTAL + FIELD) "TOTAL", which means: take the quantity previously called "TOTAL," add to it the quantity FIELD and call the result "TOTAL." This version has a "bug" if the steps taken by the Turtle are too large or of variable size. By debugging when such problems are encountered the student moves in a meaningful progression to a more sophisticated concept of integral.

The early introduction of simple version of integration along a path illustrates a frequent phenomenon of reversal of what seemed to be "natural" pedagogic ordering. In the tradi-

tional curriculum, line integration is an advanced topic to which students come after having been encouraged for several years to think of the definite integral as the area under a curve, a concept that seemed to be more concrete in a mathematical world of pencil and paper technology. But the effect is to develop a misleading image of integration that leaves many students with a sense of being lost when they encounter integrals for which the representation as area under a curve is quite inappropriate.

*Example 2: Differential Equation.* "Touch Sensor Turtle" (See introduction, note 4) used a method that strikes many children as excitingly powerful. A typical first approach to programming a Turtle to circumnavigate an object is to measure the object and build its dimensions into the program. Thus if the object is a square with side 150 Turtle steps, the program will include the instruction FORWARD 150. Even if it works (which it usually does not) this approach lacks generality. The program cited in the earlier note works by taking tiny steps that depend only on conditions in the Turtle's immediate vicinity. Instead of the "global" operation FORWARD 150 it uses only "local" operations such as FORWARD 1. In doing so it captures an essential core of the notion of differential equation. I have seen elementary school children who understand clearly why differential equations are the natural form of laws of motion. Here we see another dramatic pedagogic reversal: The power of the differential equation is understood before the analytic formalism of calculus. Much of what is known about Turtle versions of mathematical ideas is brought together in H. Abelson and A. diSessa, *Turtle Geometry: Computation as a Medium for Exploring Mathematics* (Cambridge: MIT Press, in press).

*Example 3: Topological Invariant.* Let a Turtle crawl around an object "totalizing" its turns as it goes: right turns counting as positive, left turns as negative. The result will be  $360^\circ$  whatever the shape of the object. We shall see that this Total Turtle Trip Theorem is useful as well as wonderful.

2. The phrase "ego-syntonic" is used by Freud. It is a "term used to describe instincts or ideas that are acceptable to the ego: i.e., compatible with the ego's integrity and with its demands." (See J. Laplanche and J.-B. Pontalis, *The Language of Psycho-analysis* (New York: Norton, 1973).)

3. G. Polya, *How to Solve It* (Garden City, N.Y.: Doubleday-Anchor, 1954); *Induction and Analogy in Mathematics* (Princeton, N.J.: Princeton University Press, 1954); and *Patterns of Plausible Inference* (Princeton, N.J.: Princeton, 1969).

4. Usual definitions of curvature look more complex but are equivalent to this one. Thus we have another example of an "advanced" concept in graspable form.

5. If turns can be right or left, one direction must be treated as negative. "Boundary of (connected) area" is a simple way of saying "simple closed curve." If the restriction is lifted, the sum of turns must still be an integral multiple of  $360^\circ$ .

## Chapter 4

1. Here I am picking a little quarrel with Jerry Bruner. But I share much of what he thinks, and this is true not only about language and action, but also about the relationship to learning of cultural materials and of teaching.

The systematic difference between us is seen most clearly by comparing our approaches to mathematics education. Bruner, as a psychologist, takes mathematics as a given entity and considers, in his particular rich way, the processes of teaching it and learning it. I try to *make* a learnable mathematics. I think that something of the same sort separates us in regard to language and to culture and leads us to different paradigms for a "theory of learning." See J.S. Bruner, *Toward a Theory of Instruction* (Cambridge: Harvard University Press, 1966) and J.S. Bruner et al., *Studies in Cognitive Growth* (New York: John Wiley, 1966).

2. The most systematic study is in H. Austin, "A Computational Theory of Physical Skill" (Ph.D. thesis, MIT, 1976).
3. These procedures introduce a further expansion in our image of programming. They are capable of running simultaneously, "in parallel." An image of programming that fails to include this expansion is quite out of touch with the modern world of computation. And a child who is restricted to serial programming is deprived of a source of practical and of conceptual power. This deprivation is felt as soon as the child tries to introduce motion into a program.

Suppose, for example, that a child wishes to create a movie on the computer screen with three separate moving objects. The "natural" way to do this would be to create a separate procedure for each object and set the three going. "Serial" computer systems force a less logical way to do this. Typically, the motions of each object would be broken up into steps and a procedure created to run a step of each motion in cyclic order.

The example shows two reasons why a computer system for children should allow parallel computation or "multi-processing." First, from an instrumental point of view, multi-processing makes programming complex systems easier and conceptually clearer. Serial programming breaks up procedural entities that ought to have their own integrity. Second, as a model of learning serial programming does something worse: It betrays the principle of modularity and precludes truly structured programming. The child ought to be able to construct each motion separately, try it out, debug it, and know that it will work (or almost work) as a part of the larger system.

Multi-processing is more demanding of computational resources than simple serial processing. None of the computers commonly found in schools and homes are powerful enough to allow it. Early LOGO systems were "purely serial." More recent ones allow restricted forms of multi-processing (such as the WHEN DEMONS described later in this chapter) tailored for purposes of programming dynamic graphics, games, and music. The development of a much less restrictive multi-processing language for children is a major research goal of the MIT LOGO Group at the time of writing this book. In the work we draw heavily on ideas that have been developed in Alan Kay's SMALLTALK language, on Carl Hewitt's concepts of "ACTOR" languages and on the Minsky-Papert "Society Theory of Mind." But the technical problems inherent in such systems are not fully understood and much more research may be needed before a consensus emerges about the right way (or set of ways) to achieve a really good multi-processing system suitable for children.

## Chapter 5

1. The most prolific contributor to the development of such systems is Andrea diSessa, who is responsible, among many other things, for the term "Dynamturtle." H. Abelson and A. diSessa, *Turtle Geometry: Computation as a Medium for Exploring Mathematics* (Cambridge: MIT Press, in press).

2. The discussion of the Monkey Problem uses a computational model. However this model is very far from fitting the notion of computation as algorithmic programming built into most programming languages. Making this model consists of creating a collection of objects and setting up interactions between them. This image of computation, which has come to be known as "object-oriented" or "message-passing" programming, was first developed as a technical method for simulation programs and implemented as a language called SIMULA. Recently it has drawn much broader interest and, in particular, has become a focus of attention in Artificial Intelligence research where it has been most extensively developed by Carl Hewitt and his students. Alan Kay has for a long time been the most active advocate of object-oriented languages in education.

## Chapter 6

1. Martin Gardner, *Mathematical Carnival* (New York: Random House, 1977).

## Chapter 7

1. For remarks by Piaget on Bourbaki see "Logique et connaissance scientifique," ed. J. Piaget, *Encyclopédie de la Pleiade*, vol. 22 (Paris: Gallimard, 1967).

2. C. Lévi-Strauss, *Structural Anthropology*, 2 vols. (New York: Basic Books, 1963-76).

3. Lévi-Strauss uses the word *bricolage* as a technical term for the tinkering-like process we have been discussing. *Bricoleur* is the word for someone who engages in *bricolage*. These concepts have been developed in a computational context in Robert Lawler, "One Child's Learning: An Intimate Study" (Ph.D. thesis, MIT, 1979).

4. Of course our culture provides everyone with plenty of occasions to *practice* particular systematic procedures. Its poverty is in materials for *thinking about* and *talking about* procedures. When children come to LOGO they often have trouble recognizing a procedure as an entity. Coming to do so, is, in my view, analogous to the process of formation of permanent objects in infancy and of all the Piagetionly-conserved entities such as number, weight, and length. In LOGO, procedures are manipulable entities. They can be named, stored away, retrieved, changed, used as building blocks for superprocedures and analyzed into subprocedures. In this process they are assimilated to schematic or frames of more familiar entities. Thus they acquire the quality of "being entities." They inherit "concreteness." They also inherit specific knowledge.