Appendix A
Proof of $\hat{\rho}_i \le \rho_i$

We first define the following stochastic processes generated by the queueing process at node i.

$T_{i,Q}(t)/T_{i,\overline{Q}}(t)$:= the total length of real time periods up to time t that the queue at node i is non-empty/empty (or i is busy/idle);

$N_{i,Q}(t)/N_{i,\overline{Q}}(t)$:= the total number of slots up to time t that the queue at node i is non-empty/empty at the beginning of slots.

These processes are well-defined on the same sample space Ω. Assume that the queue is stable, then due to ergodicity ρ_i and $\hat{\rho}_i$ can be expressed respectively as

$$\rho_i = \lim_{t\to\infty} \frac{T_{i,Q}(\omega,t)}{t} = \lim_{t\to\infty} \frac{T_{i,Q}(\omega,t)}{T_{i,Q}(\omega,t) + T_{i,\overline{Q}}(\omega,t)},$$

and

$$\hat{\rho}_i = \lim_{t\to\infty} \frac{N_{i,Q}(\omega,t)}{N_{i,Q}(\omega,t) + N_{i,\overline{Q}}(\omega,t)},$$

for all $\omega \in \Omega$. Let $\Delta_i(t)$ be the total time fragmentation of busy periods in idle slots of node i up to time t, and let $S_{i,Q}(k)$ ($S_{i,\overline{Q}}(k)$) be the length of the kth busy (resp. idle) slot. Quantities described above are illustrated in Figure 1. Then, we have

$$T_{i,Q}(t) - \Delta_i(t) = \sum_{k=1}^{N_{i,Q}(t)} S_{i,Q}(k),$$

and

$$t = \sum_{k=1}^{N_{i,Q}(t)} S_{i,Q}(k) + \sum_{k=1}^{N_{i,\overline{Q}}(t)} S_{i,\overline{Q}}(k).$$

Therefore,

$$\begin{aligned}
\rho_i &\ge \lim_{t\to\infty} \frac{T_{i,Q}(t) - \Delta_i(t)}{t} \\
&= \lim_{t\to\infty} \frac{\sum_{k=1}^{N_{i,Q}(t)} S_{i,Q}(k)}{\sum_{k=1}^{N_{i,Q}(t)} S_{i,Q}(k) + \sum_{k=1}^{N_{i,\overline{Q}}(t)} S_{i,\overline{Q}}(k)} \\
&= \lim_{t\to\infty} \Bigg[\frac{\sum_{k=1}^{N_{i,Q}(t)} S_{i,Q}(k)}{N_{i,Q}(t)} N_{i,Q}(t) \Bigg/ \\
&\qquad \Bigg(\frac{\sum_{k=1}^{N_{i,Q}(t)} S_{i,Q}(k)}{N_{i,Q}(t)} N_{i,Q}(t) + \\
&\qquad + \frac{\sum_{k=1}^{N_{i,\overline{Q}}(t)} S_{i,\overline{Q}}(k)}{N_{i,\overline{Q}}(t)} N_{i,\overline{Q}}(t) \Bigg) \Bigg],
\end{aligned}$$

where we have suppressed the reference to a sample point ω in all involved processes for simplicity, or interpreted the equalities as with probability one. Let $\mathbb{E}[S_{i,Q}]$ and $\mathbb{E}[S_{i,\overline{Q}}]$ be the conditional average lengths of an arbitrary slot, given that the queue at node i is non-empty or empty at the beginning of slot, respectively. We claim that $\mathbb{E}[S_{i,Q}] > \mathbb{E}[S_{i,\overline{Q}}]$ (see the next Appendix for justification). Note also that $N_{i,Q}(t) \to \infty$ and $N_{i,\overline{Q}}(t) \to \infty$ as $t \to \infty$ due to the stability assumption. Consequently, following ergodicity, we obtain

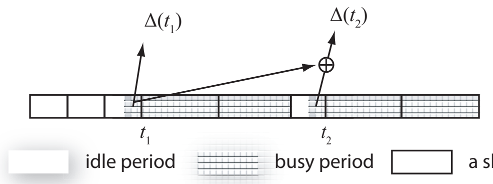

Fig. 1. Slotted time dynamics.

$$\begin{aligned}
\rho_i &\ge \lim_{t\to\infty} \frac{N_{i,Q}(t)\mathbb{E}[S_{i,Q}]}{N_{i,Q}(t)\mathbb{E}[S_{i,Q}] + N_{i,\overline{Q}}(t)\mathbb{E}[S_{i,\overline{Q}}]} \\
&\ge \lim_{t\to\infty} \frac{N_{i,Q}(t)}{N_{i,Q}(t) + N_{i,\overline{Q}}(t)} \\
&= \hat{\rho}_i.
\end{aligned}$$

When the queue is unstable, we have $\rho_i = \hat{\rho}_i = 1$. In either case, we have $\rho_i \ge \hat{\rho}_i$. It remains to justify the claim made above, which appears in the next Appendix.

Appendix B
Computation of $\mathbb{E}[S_{\{\cdot\}}]$ and Related Quantities

Given an event $\{\cdot\}$, let $P_{idle;\{\cdot\}}$, $P_{succ;\{\cdot\}}$ and $P_{coll;\{\cdot\}}$ be the conditional probabilities that a slot is idle, that the transmission attempt in the slot is a success, and that the attempt is a collision, respectively. Notice that $P_{coll;\{\cdot\}} = 1 - P_{idle;\{\cdot\}} - P_{succ;\{\cdot\}}$. Therefore,

$$\mathbb{E}[S_{\{\cdot\}}] = \sigma \cdot P_{idle;\{\cdot\}} + T_s \cdot P_{succ;\{\cdot\}} + T_c \cdot P_{coll;\{\cdot\}}.$$

where σ, T_s and T_c are the lengths of an empty system slot, a successful transmission, and a collision, respectively. Define then by $\tau_{i,Q}$ the conditional probability that node i transmits in an arbitrary slot, given its queue is non-empty at the beginning of this slot, and hence we have $\tau_{i,Q} = \frac{1}{W_i}$. Consequently,

$$\begin{aligned}
P_{idle;i,\overline{Q}} &= \prod_{j\neq i}(1-\tau_j), \\
P_{succ;i,\overline{Q}} &= \sum_{j\neq i} \tau_j \prod_{l\neq i,j}(1-\tau_l), \\
P_{idle;i,Q} &= (1-\tau_{i,Q}) \prod_{j\neq i}(1-\tau_j), \\
P_{succ;i,Q} &= \sum_{l} \tilde{\tau}_l \prod_{j\neq l}(1-\tilde{\tau}_l),
\end{aligned}$$

where

$$\tilde{\tau}_j = \begin{cases} \tau_{i,Q}, & \text{if } j = i \\ \tau_j, & \text{if } j \neq i \end{cases}.$$

Since $P_{idle;i,Q} < P_{idle;i,\overline{Q}}$ and $\sigma < \min\{T_s, T_c\}$, we have $\mathbb{E}[S_{i,Q}] > \mathbb{E}[S_{i,\overline{Q}}]$ and they are both finite. Explicit expressions for other variations of $\mathbb{E}[S_{\{\cdot\}}]$ can be derived in a similar way, and are thus omitted.

Appendix C
Approximation of $\hat{\rho}_i$

Due to the analytical intractability of $\Delta_i(t)$, we are interested in proper approximations of $\hat{\rho}_i$ that can lead to good estimate of Λ; a good estimate in the context of stability study means a tight underestimation. Recall that $\hat{\rho}_i \le \rho_i$ and equality holds if and only if $\rho_i = 1$ or $\rho_i = 0$; therefore by replacing $\hat{\rho}_i$ by ρ_i in Σ(c), solutions to the resulting system of equations form an underestimation of Λ but accurate when $\rho_i = 1$ or 0 for all i. Moreover, when $0 < \hat{\rho}_i < 1$, we have

$$\begin{aligned}\hat{\rho}_i &= \lim_{t\to\infty} \frac{\frac{T_{i,Q}(t)-\Delta_i(t)}{S^{\text{av}}_{i,Q}(t)}}{\frac{T_{i,Q}(t)-\Delta_i(t)}{S^{\text{av}}_{i,Q}(t)} + \frac{T_{i,\overline{Q}}(t)+\Delta_i(t)}{S^{\text{av}}_{i,\overline{Q}}(t)}} \\ &\le \lim_{t\to\infty} \frac{\frac{T_{i,Q}(t)}{T_{i,Q}(t)+T_{i,\overline{Q}}(t)} S^{\text{av}}_{i,\overline{Q}}(t)}{\frac{T_{i,Q}(t)}{T_{i,Q}(t)+T_{i,\overline{Q}}(t)} S^{\text{av}}_{i,\overline{Q}}(t) + \frac{T_{i,\overline{Q}}(\omega,t)}{T_{i,Q}(t)+T_{i,\overline{Q}}(t)} S^{\text{av}}_{i,Q}(t)} \\ &= \frac{\rho_i \mathbb{E}[S_{i,\overline{Q}}]}{\rho_i \mathbb{E}[S_{i,\overline{Q}}] + (1-\rho_i)\mathbb{E}[S_{i,Q}]} \\ &\le \rho_i,\end{aligned}$$

where

$$S^{\text{av}}_{i,Q}(t) = \frac{1}{N_{i,Q}(t)} \sum_{k=1}^{N_{i,Q}(t)} S_{i,Q}(k)$$

and defining

$$\hat{\hat{\rho}}_i = \frac{\rho_i \mathbb{E}[S_{i,\overline{Q}}]}{\rho_i \mathbb{E}[S_{i,\overline{Q}}] + (1-\rho_i)\mathbb{E}[S_{i,Q}]},$$

we have $\hat{\rho}_i \le \hat{\hat{\rho}}_i \le \rho_i$. Hence, substituting $\hat{\rho}_i$ with $\hat{\hat{\rho}}_i$ in Σ(c), we can obtain a tighter underestimation of Λ than with ρ_i, thus trading off computational complexity for higher accuracy. Empirical results suggest that $\hat{\hat{\rho}}$ is sufficiently close to $\hat{\rho}$, and we use $\hat{\hat{\rho}}$ as $\hat{\rho}$ throughout our computation.

Appendix D
Proof of Proposition 1

Substituting $\widetilde{\Sigma}$(b) in (a), we obtain

$$\begin{aligned}\tau_i &= \frac{2\lambda_i}{P(W+1)}\left[\frac{W-1}{2}\left(\sigma + T\sum_{j\ne i}\tau_j\right) + \right. \\ &\qquad \left. + T\left(1 + \sum_{j\ne i}\tau_j\right)\right] \\ &= \frac{2\lambda_i}{P(W+1)}\left[\frac{W+1}{2}T\sum_{j\ne i}\tau_j + \frac{W-1}{2}\sigma + T\right] \\ &= \frac{\lambda_i T}{P}\sum_{j\ne i}\tau_j + \frac{\lambda_i((W-1)\sigma + 2T)}{P(W+1)},\end{aligned}$$

which can be rewritten as

$$\tau_i = \left(\frac{\lambda_i T}{P}\sum_j \tau_j + \frac{\lambda_i((W-1)\sigma+2T)}{P(W+1)}\right)\Big/\left(1 + \frac{\lambda_i T}{P}\right).$$

Therefore, let $y = \sum_j \tau_j$, $\gamma_i^1 = \frac{\lambda_i T}{P}\Big/\left(1 + \frac{\lambda_i T}{P}\right)$ and $\gamma_i^2 = \frac{\lambda_i((W-1)\sigma+2T)}{P(W+1)}\Big/\left(1+\frac{\lambda_i T}{P}\right)$, and we have

$$\tau_i = \gamma_i^1 y + \gamma_i^2.$$

Then, $\widetilde{\Sigma}$ is equivalent to

$$\widetilde{\Sigma} : \begin{cases} \tau_i = \gamma_i^1 y + \gamma_i^2 & \text{(a}')\\ y = \sum_i \left(\gamma_i^1 y + \gamma_i^2\right) & \text{(b}')\end{cases}$$

which admits only one solution, namely

$$\tau_i = \frac{\gamma_i^1 \sum_j \gamma_j^2}{1 - \sum_i \gamma_j^1} + \gamma_i^2.$$

Appendix E
Proof of Theorem 3

Using $\widetilde{\Sigma}^{\mathbf{g}^U}$(a), we can rewrite $\widetilde{\Sigma}^{\mathbf{g}^U}$(b) as follows:

$$\begin{aligned}\rho_i &= \frac{\lambda_i}{P}\sum_{k\in\mathcal{C}}\left\{q^{(k)}\left[\frac{W-1}{2}\left(\sigma + T\sum_{j\ne i}\tau_j^{(k)}\right) + \right.\right. \\ &\qquad \left.\left. + T\left(1 + \sum_{j\ne i}\tau_j^{(k)}\right)\right]\right\} \\ &= \theta_i^1 \sum_{k\in\mathcal{C}}\left(q^{(k)}\sum_{j\ne i}\tau_j^{(k)}\right) + \theta_i^2 \\ &= \theta_i^1 \sum_{k\in\mathcal{C}} \phi_i(q^{(k)}; \rho_j, j\ne i) + \theta_i^2,\end{aligned}$$

where $\theta_i^1 = \frac{\lambda_i(W+1)T}{2P}$, $\theta_i^2 = \frac{\lambda_i(W-1)\sigma+2T}{2P}$, and $\phi_i(q^{(k)}; \rho_j, j\ne i) = q^{(k)}\sum_{j\ne i}\tau_j^{(k)} = \sum_{j\ne i}\alpha_j\left[q^{(k)}\right]^2$ with $\alpha_j = \frac{2\rho_j}{W+1} > 0$ for all j. Notice that $\phi_i(q^{(k)}; \rho_j, j\ne i)$ is a convex function of $q^{(k)}$ given any fixed ρ_j where $j \ne i$, and it is also an increasing function of ρ_j's given any fixed $q^{(k)}$. We then have

$$\begin{aligned}\rho_i &= \theta_i^1 \sum_{k\in\mathcal{C}}\phi_i(q^{(k)}) + \theta_i^2 \\ &= \theta_i^1 \cdot K\sum_{k\in\mathcal{C}}\left(\frac{1}{K}\phi_i(q^{(k)})\right) + \theta_i^2 \\ &\ge \theta_i^1 \cdot K\phi_i\left(\sum_{k\in\mathcal{C}}\left(\frac{1}{K}q^{(k)}\right)\right) + \theta_i^2 \\ &= \theta_i^1 \cdot K\phi_i\left(\frac{1}{K}\right) + \theta_i^2,\end{aligned}$$

where the equality holds when $q_i^{(k)} = \frac{1}{K}$. Therefore, when switching to the equi-occupancy policy from any arbitrary unbiased policy, the utilization factor of each node is always non-increasing. Hence, we conclude that the equi-occupancy scheduling policy is throughput optimal in $\mathcal{G}^U$.

APPENDIX F
MISCELLANEOUS

Total bandwidth	11 Mbps
Data packet length P	1500 Bytes
DIFS	50 μs
SIFS	10 μs
ACK packet length (in time units)	203 μs
Header length (in time units)	192 μs
Empty system slot time σ	20 μs
Propagation delay δ	1 μs
Initial backoff window size W	32
Maximum backoff stage m	5
Data rate granularity $\Delta\lambda$	100 Kbps
Instability threshold constant	1%
Total simulated time T_f	10 seconds

TABLE 1
Specifications of the implementation of test bench.