**APPENDIX A**

**Proof of \( \hat{\rho}_i \leq \rho_i \)**

We first define the following stochastic processes generated by the queueing process at node \( i \).

\[
T_{i,Q}(t)/T_{i,\overline{Q}}(t) := \text{the total length of real time periods up to time } t \text{ that the queue at node } i \text{ is non-empty/empty (or } i \text{ is busy/idle);}
\]

\[
N_{i,Q}(t)/N_{i,\overline{Q}}(t) := \text{the total number of slots up to time } t \text{ that the queue at node } i \text{ is non-empty/empty at the beginning of slots.}
\]

These processes are well-defined on the same sample space \( \Omega \). Assume that the queue is stable, then due to ergodicity \( \rho_i \) and \( \hat{\rho}_i \) can be expressed respectively as

\[
\rho_i = \lim_{t \to \infty} \frac{T_{i,Q}(\omega,t)}{t} = \lim_{t \to \infty} \frac{T_{i,Q}(\omega,t)}{T_{i,Q}(\omega,t) + T_{i,\overline{Q}}(\omega,t)},
\]

and

\[
\hat{\rho}_i = \lim_{t \to \infty} \frac{N_{i,Q}(\omega,t)}{N_{i,Q}(\omega,t) + N_{i,\overline{Q}}(\omega,t)},
\]

for all \( \omega \in \Omega \). Let \( \Delta_i(t) \) be the total time fragmentation of busy periods in idle slots of node \( i \) up to time \( t \), and let \( S_{i,Q}(k) / S_{i,\overline{Q}}(k) \) be the length of the \( k \)-th busy (resp. idle) slot. Quantities described above are illustrated in Figure 1. Then, we have

\[
T_{i,Q}(t) - \Delta_i(t) = \sum_{k=1}^{N_{i,Q}(t)} S_{i,Q}(k),
\]

and

\[
t = \sum_{k=1}^{N_{i,Q}(t)} S_{i,Q}(k) + \sum_{k=1}^{N_{i,\overline{Q}}(t)} S_{i,\overline{Q}}(k).
\]

Therefore,

\[
\rho_i \geq \lim_{t \to \infty} \frac{T_{i,Q}(t) - \Delta_i(t)}{t} = \lim_{t \to \infty} \frac{\sum_{k=1}^{N_{i,Q}(t)} S_{i,Q}(k)}{\sum_{k=1}^{N_{i,Q}(t)} S_{i,Q}(k) + \sum_{k=1}^{N_{i,\overline{Q}}(t)} S_{i,\overline{Q}}(k)} \left[ \frac{\sum_{k=1}^{N_{i,Q}(t)} S_{i,Q}(k)}{N_{i,Q}(t)} \frac{N_{i,Q}(t)}{N_{i,Q}(t)} \right] = \lim_{t \to \infty} \left[ \frac{\sum_{k=1}^{N_{i,Q}(t)} S_{i,Q}(k)}{N_{i,Q}(t)} \frac{N_{i,Q}(t)}{N_{i,Q}(t)} + \frac{\sum_{k=1}^{N_{i,\overline{Q}}(t)} S_{i,\overline{Q}}(k)}{N_{i,\overline{Q}}(t)} \frac{N_{i,\overline{Q}}(t)}{N_{i,\overline{Q}}(t)} \right],
\]

where we have suppressed the reference to a sample point \( \omega \) in all involved processes for simplicity, or interpreted the equalities as with probability one. Let \( E[S_{i,Q}] \) and \( E[S_{i,\overline{Q}}] \) be the conditional average lengths of an arbitrary slot, given that the queue at node \( i \) is non-empty or empty at the beginning of slot, respectively. We claim that \( E[S_{i,Q}] > E[S_{i,\overline{Q}}] \) (see the next Appendix for justification). Note also that \( N_{i,Q}(t) \to \infty \) and \( N_{i,\overline{Q}}(t) \to \infty \) as \( t \to \infty \) due to the stability assumption. Consequently, following ergodicity, we obtain

\[
\rho_i \geq \lim_{t \to \infty} \frac{N_{i,Q}(t)E[S_{i,Q}]}{N_{i,Q}(t)E[S_{i,Q}] + N_{i,\overline{Q}}(t)E[S_{i,\overline{Q}}]} \geq \lim_{t \to \infty} \frac{N_{i,Q}(t)}{N_{i,Q}(t) + N_{i,\overline{Q}}(t)} = \hat{\rho}_i.
\]

When the queue is unstable, we have \( \rho_i = \hat{\rho}_i = 1 \). In either case, we have \( \rho_i \geq \hat{\rho}_i \). It remains to justify the claim made above, which appears in the next Appendix.

**APPENDIX B**

**Computation of \( E[S_{\{\cdot\}}] \) and Related Quantities**

Given an event \( \{\cdot\} \), let \( P_{\text{idm:}\{\cdot\}} \), \( P_{\text{succ:}\{\cdot\}} \) and \( P_{\text{coll:}\{\cdot\}} \) be the conditional probabilities that a slot is idle, that the transmission attempt in the slot is a success, and that the attempt is a collision, respectively. Notice that \( P_{\text{coll:}\{\cdot\}} = 1 - P_{\text{idm:}\{\cdot\}} - P_{\text{succ:}\{\cdot\}} \). Therefore,

\[
E[S_{\{\cdot\}}] = \sigma \cdot P_{\text{idm:}\{\cdot\}} + T_s \cdot P_{\text{succ:}\{\cdot\}} + T_c \cdot P_{\text{coll:}\{\cdot\}},
\]

where \( \sigma, T_s \) and \( T_c \) are the lengths of an empty system slot, a successful transmission, and a collision, respectively. Define then by \( \tau_{i,Q} \) the conditional probability that node \( i \) transmits in an arbitrary slot, given its queue is non-empty at the beginning of this slot, and hence we have \( \tau_{i,Q} = \frac{1}{W_i} \). Consequently,

\[
P_{\text{idm:}\{i\}} = \prod_{j \neq i} (1 - \tau_j),
\]

\[
P_{\text{suc:}\{i\}} = \sum_{j \neq i} \tau_j \prod_{l \neq i,j} (1 - \tau_l),
\]

\[
P_{\text{idm:}\{i\}} = (1 - \tau_i) \prod_{j \neq i} (1 - \tau_j),
\]

\[
P_{\text{suc:}\{i\}} = \sum_l \tilde{\tau}_l \prod_{j \neq l} (1 - \tilde{\tau}_l),
\]

where

\[
\tilde{\tau}_j = \begin{cases} \tau_{i,Q}, & \text{if } j = i \\ \tau_j, & \text{if } j \neq i. \end{cases}
\]
Since $P_{idle;i,Q} < P_{idle;i,Q}$ and $\sigma < \min\{T_s, T_c\}$, we have $E[S_{i,Q}] \geq E[S_{i,Q}]$ and they are both finite. Explicit expressions for other variations of $E[S_{i,\cdot}]$ can be derived in a similar way, and are thus omitted.

**APPENDIX C**

**APPROXIMATION OF $\hat{\rho}_i$**

Due to the analytical intractability of $\Delta_i(t)$, we are interested in proper approximations of $\hat{\rho}_i$ that can lead to good estimate of $\Lambda$; a good estimate in the context of stability study means a tight underestimation. Recall that $\rho_i \leq \hat{\rho}_i$ and equality holds if and only if $\rho_i = 1$ or $\rho_i = 0$; therefore by replacing $\hat{\rho}_i$ by $\rho_i$ in $\Sigma(c)$, solutions to the resulting system of equations form an underestimation of $\Lambda$ but accurate when $\rho_i = 0$ or 1 for all $i$. Moreover, when $0 < \hat{\rho}_i < 1$, we have

$$\hat{\rho}_i = \lim_{t \to \infty} \frac{T_i,\rho(t) - \Delta_i(t)}{S_{i,\rho}(t)} + \frac{T_i,\rho(t) + \Delta_i(t)}{S_{i,\rho}(t)}$$

$$\leq \lim_{t \to \infty} \frac{T_i,\rho(t)}{T_i,\rho(t) + \Delta_i(t)} \frac{S_{i,\rho}(t)}{T_i,\rho(t) + \Delta_i(t)} + \frac{T_i,\rho(t)}{S_{i,\rho}(t)} \frac{S_{i,\rho}(t)}{S_{i,\rho}(t)}$$

$$= \rho_i E[S_{i,\rho}] + (1 - \rho_i) E[S_{i,Q}]$$

where

$$S_{i,Q}(t) = \frac{1}{N_i,\rho(t)} \sum_{k=1}^{N_i,\rho(t)} S_{i,Q}(k)$$

and defining

$$\hat{\rho}_i = \frac{\rho_i E[S_{i,\rho}]}{\rho_i E[S_{i,\rho}] + (1 - \rho_i) E[S_{i,Q}]}$$

have $\hat{\rho}_i \leq \hat{\rho}_i \leq \rho_i$. Hence, substituting $\hat{\rho}_i$ with $\hat{\rho}_i$ in $\Sigma(c)$, we can obtain a tighter underestimation of $\Lambda$ than with $\rho_i$, thus trading off computational complexity for higher accuracy. Empirical results suggest that $\rho_i$ is sufficiently close to $\hat{\rho}_i$, and we use $\hat{\rho}$ as $\rho$ throughout our computation.

**APPENDIX D**

**PROOF OF PROPOSITION 1**

Substituting (b) in (a), we obtain

$$\tau_i = \frac{2\lambda_i}{P(W + 1)} \left[ \frac{W - 1}{2} \left( \sigma + T \sum_{j \neq i} \tau_j \right) + T \left( 1 + \sum_{j \neq i} \tau_j \right) \right]$$

$$= \frac{2\lambda_i}{P(W + 1)} \left[ \frac{W + 1}{2} \sum_{j \neq i} \tau_j + \frac{W}{2} \sigma + T \right]$$

$$= \frac{\lambda_i T}{P} \sum_{j \neq i} \tau_j + \frac{\lambda_i (W - 1) \sigma + 2T}{P(W + 1)}$$

which can be rewritten as

$$\tau_i = \frac{\lambda_i T}{P} \sum_{j \neq i} \tau_j + \frac{\lambda_i ((W - 1) \sigma + 2T)}{P(W + 1)}$$

Therefore, let $y = \sum_j \tau_j$, $\gamma_1 = \frac{\lambda_i T}{1 + \lambda_i T}$ and

$$\gamma_2 = \frac{\lambda_i ((W - 1) \sigma + 2T)}{P(W + 1)}$$

and we have

$$\tau_i = \gamma_1 y + \gamma_2^2.$$

Then, $\Sigma$ is equivalent to

$$\Sigma : \begin{cases} \tau_i = \gamma_1 y + \gamma_2^2 \\ y = \sum_i (\gamma_1 y + \gamma_2^2) \end{cases}$$

which admits only one solution, namely

$$\tau_i = \gamma_1 \sum_i \gamma_2^2 + \gamma_2^2.$$

**APPENDIX E**

**PROOF OF THEOREM 3**

Using $\Sigma''(a)$, we can rewrite $\Sigma''(b)$ as follows:

$$\rho_i = \frac{\lambda_i}{P} \sum_{k \in C} q(k) \left[ \frac{W - 1}{2} \left( \sigma + T \sum_{j \neq k} \tau_j \right) + T \left( 1 + \sum_{j \neq k} \tau_j \right) \right]$$

$$= \theta_i \sum_{k \in C} q(k) \sum_{j \neq i} \tau_j^2 + \theta_i^2$$

where $\theta_i = \frac{\lambda_i (W + 1) T}{2P}$, $\theta_i^2 = \frac{\lambda_i (W - 1) \sigma + 2T}{2P}$, and

$$\phi_i(q(k); \rho_j, j \neq i) = q(k) \sum_{j \neq i} \tau_j^2$$

with $\phi_i(q(k); \rho_j, j \neq i)$ is a convex function of $q(k)$ given any fixed $\rho_j$ where $j \neq i$, and it is also an increasing function of $\rho_j$'s given any fixed $q(k)$. We then have

$$\rho_i = \theta_i \sum_{k \in C} \phi_i(q(k)) + \theta_i^2$$

where the equality holds when $q_i = \frac{1}{K}$. Therefore, when switching to the equi-occupancy policy from any arbitrary unbiased policy, the utilization factor of each node is always non-increasing. Hence, we conclude that the equi-occupancy scheduling policy is throughput optimal in $G''$. 


### APPENDIX F

**MISCELLANEOUS**

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<tr>
<th>Specification</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>Total bandwidth</td>
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<td>Data packet length $P$</td>
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<td>DIFS</td>
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<tr>
<td>SIFS</td>
<td>10 $\mu$s</td>
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<tr>
<td>ACK packet length (in time units)</td>
<td>200 $\mu$s</td>
</tr>
<tr>
<td>Header length (in time units)</td>
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<td>Empty system slot time $\sigma$</td>
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<td>Propagation delay $\delta$</td>
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<td>Initial backoff window size $W$</td>
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<td>Maximum backoff stage $m$</td>
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<td>Data rate granularity $\Delta\lambda$</td>
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<tr>
<td>Instability threshold constant</td>
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</tr>
<tr>
<td>Total simulated time $T_f$</td>
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</tr>
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**TABLE 1**

Specifications of the implementation of test bench.