Sound Mobility Models

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ABSTRACT

Simulation has become an indispensable tool in the construction and evaluation of mobile systems. By using *mobility models* that describe constituent movement, one can explore large systems, producing repeatable results for comparison between alternatives. Unfortunately, the vast majority of mobility models—including all those in which nodal speed and distance or destination are chosen independently—suffer from *decay*; average speed decreases until converging to some long-term average. Such decay provides an unsound basis for simulation studies that collect results averaged over time, complicating the experimental process.

This paper shows via analysis that such decay is inevitable in a wide variety of mobility models, including the most common in use today. We derive a general framework for describing this decay, and apply it to a number of practical cases. Furthermore, this framework allows us to transform any given mobility model into a *stationary* one: choose initial speeds from the steady-state distribution, and subsequent speeds from the original. This transformation provides sound models for simulation, eliminating variations in average nodal speed.

Categories and Subject Descriptors

I.6 [**SIMULATION AND MODELING**]

General Terms

Experimentation, Theory, Performance

Keywords

Mobility Model, Stationary Distribution, Renewal Process

1. INTRODUCTION

Simulation has become an indispensable tool in the construction and evaluation of mobile systems. Simulations allow study of larger scale systems than can be built practically. Furthermore, they enable the evaluation of systems not amenable to analysis. By carefully controlling the movement of nodes and wireless conditions between them, simulations provide excellent reproducibility across experimental trials.

Typically, simulations of mobile systems rely upon *random mobility models*. Such models are characterized by a collection of nodes placed randomly within a confined simulation space U. Each node selects two or more of the following according to some random distributions: a destination d in U, a travelling speed v, an angle α, and a travel time t. It then travels to d at v, or travels at v along α for t, and so on. After reaching d or having travelled for t, the node may pause before repeating the process. The precise means of selecting U, d, v, α, and t differ from model to model. Camp [6] categorizes these models into *entity* mobility models, where individual nodes move independently of each other, and *group* mobility models, where the movement of a group of nodes may be correlated [10]. In this paper we focus exclusively on entity mobility models; in subsequent discussions, the term *random mobility model* will refer to this class of models where each node has an independent, identically distributed movement pattern.

The behavior of most mobile systems depends heavily on the movement of constituent nodes [19]. Therefore it is highly desirable to have a mobility model that generates stable nodal movement so that the mobile system maintains a steady level of mobility over time, e.g., a fixed average nodal speed and a fixed speed variance. This is especially critical for simulation studies that present performance metrics as time averages.

Our recent work [21], shows that one of the most widely used, the *random waypoint* model, suffers from *speed decay* in that as the simulation goes on, the average nodal speed decays to a steady-state level that falls below the initial average nodal speed. Such speed decay can have a dramatic influence on measured performance and overhead. Consequently, one cannot present time-averaged metrics during this period of decay, as the underlying process is not stationary.

Narrowing the range from which to select speeds can reduce the degree of decay and the time required to reach a steady state. However, it limits the speed variation and does not remove decay entirely; the core problem remains unsolved. One method of removing the negative effect of this decay [6, 21] is to *warm-up* every simulation by running it until steady state is reached and then by deleting the initial data. While this is valid, it can be cumbersome,

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especially because the duration of this settling period is case-dependent in general, rendering the simulation process prone to error.

In this paper, we will give a general derivation of the steady-state average speed distribution for several classes of random mobility models, and show that speed decay is not a property exclusive to the random waypoint model, but rather a much more common phenomenon. Indeed, *any* random mobility model that chooses speed and destination independently suffers from the speed decay problem. The intuition is that nodes travel for longer times at lower speeds if the destination is chosen independently of nodal speed. This result is true *independently* of the specific distribution from which speeds are chosen, or the mechanism with which destinations are determined. Furthermore, if pause time between successive trips is set to zero, the distribution governing the steady-state average speed is *independent* of the mechanisms used to determine destination; it depends only on the distribution from which speeds are chosen.

Following this result we show how speed decay can be *completely* eliminated in a fundamental way, by constructing a composite random mobility model from any random mobility model that exhibits speed decay. The result is a sound model with a stationary speed distribution. The key insight is that the speed of the *initial* trips selected by each node is independent of travel times, while the steady-state average speed is *weighted* by travel times (i.e., travel time is longer for lower speed). By choosing speeds for this initial trip from the steady-state distribution, and choosing speeds of later trips from the original speed distribution, decay is completely eliminated, resulting in a stationary mobility process.

This method is analogous to the construction of an equilibrium renewal process found in renewal theory (e.g., [8]), which is done by deriving the limiting distribution of forward recurrence time of a simple renewal point process and applying it to the first renewal epoch. However, mobility models are different from simple renewal point processes in that they are indeed *marked renewal processes* [3], with a speed distribution in addition to the renewal points. It is also worth pointing out that this method is orthogonal to any modification to a random mobility model to obtain desired *spatial* distributions of nodes, e.g., uniform distribution within the movement area. Thus it is equally applicable. Finally, warm-up may still be needed if the simulated mobile system starts from a “cold state”. However by having such stationary mobility models, warm-up is no longer needed for nodal movement, freeing the experimenter to consider other matters.

We note that the speed decay problem of entity mobility models outlined in this paper exists in group mobility models as well, if the group movement follows a random independent selection of speed and distance. Because of this, the construction method we present here can be applied to group mobility models in a similar way.

The rest of the paper is organized as follows. Section 2 gives an overview of related work. Section 3 presents a taxonomy of random mobility models and derives their steady-state average speed distribution. Section 4 shows via a few special cases how one can apply the results from Section 3 to analytically determine the steady-state distribution of nodal speed. Section 5 presents the methodology of constructing a stationary mobility model without speed decay from any random mobility model, while Section 6 demonstrates its effectiveness for a variety of mobility models via simulation. Section 7 concludes.

2. BACKGROUND AND RELATED WORK

Mobility models are essential to mobile system research, and so they have been extensively studied. One can find a thorough and insightful survey by Camp et al in [6]. It included a variety of entity random mobility models used in ad hoc network simulations. It also covers group mobility models such as the *reference point group mobility model* [10, 11].

Of all available mobility models, the *random waypoint model* is perhaps the most extensively used [14, 5, 18, 12, 19]. It is implemented and widely distributed with the ns-2 [1] simulator. Most of the studies on the random waypoint model have focused on its *spatial* properties such as node distribution within the simulated area U. Bettstetter [4] showed by simulation that the random waypoint model does not have a uniform distribution of nodes. Chu and Nikolaidis [7] mathematically proved it and also showed that there is a relationship between the node distribution and node speed. Due to the boundary effect, nodes are more likely to be near the center of U, and thus the node distribution becomes bell-shaped. Royer et al. [20] pointed out that the boundary effect not only causes a non-uniform node distribution but also causes the node density to fluctuate with time. To eliminate both problems, they proposed a *random direction model* and showed satisfying results.

Our recent work [21] studied the *temporal* properties of nodal movement under the random waypoint model. We showed that the average node speed decreases with time before reaching a steady state. The settling time it takes to reach the steady state increases as the minimum speed decreases. In particular, if the minimum speed is zero, which is the default value in ns-2, then the steady-state average node speed is zero, meaning that the average node speed will consistently decrease over time, resulting in an unstable mobility model. Interestingly, this is a special renewal process where the failure time or renewal period has infinite mean. Simulation results showed how such speed decay affects ad hoc routing protocols such as DSR [14] and AODV [17]. One suggested solution was to use a positive minimum speed, combined with simulation warm-up or initial data deletion to remove the negative effect of speed decay.

However, this does not remove the speed decay. In addition, simulation warm-up is inconvenient, and the duration of the warm-up period can be difficult to determine. Fig.1 illustrates a case where the speed decay period of the mobility model outlasts the system warm-up period. We measured the number of overhead packets of DSR using random waypoint model with a speed range [0.3, 19.7]m/s. Results were averaged over 30 scenarios with 50 nodes. Such decay has a significant effect on when one can start collecting data, thus this solution still requires extreme care on the part of the researcher.

Navidi and Camp independently and concurrently developed a method for constructing a stationary process for the random waypoint model [16]. This paper extends and generalizes that result to several classes of mobility models.

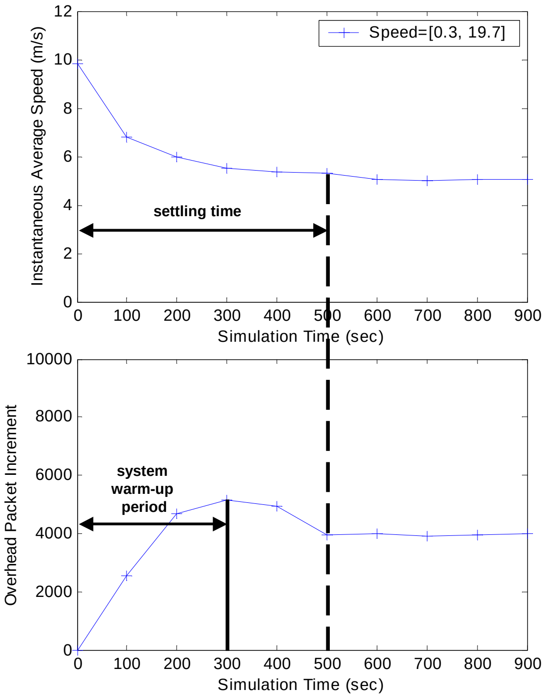


Figure 1: Illustration of the incomplete solution to the average node speed decay problem in random waypoint model. Settling time caused by mobility model may be longer than a system warm-up period.

3. MOBILITY MODELS AND STEADY-STATE SPEED DISTRIBUTIONS

This section classifies general random mobility models according to how the random elements of a model are chosen. It then derives the steady-state speed distribution for different classes and explore their properties.

The basic random elements underlying any random mobility model include speed, distance, angle, destination, and travel time. A particular model typically selects two or more of these elements according to some probability distribution that determines a *trip*. Usually the selection of these elements is independent for a single trip, independent for successive trips of a single node, and—for entity mobility models—the selection of these elements for different nodes is also independent. A notable exception is by Bettstetter [4] where accelerations and decelerations are added to a selected nodal speed. We will not consider this case, and instead focus on models that employ a single fixed speed per trip. The difference between different mobility models thus mainly lies in which of these random elements to choose, and what probability distributions to use for each choice.

Table 1 contains a few constructions of random mobility models that choose two of the following four: speed, distance, time, and destination. These are mostly existing models. For example, the combination of `(uniform speed, uniform destination)` represents the random waypoint model [14] or the vector model [11]; the combination of `(uniform speed, uniform distance)` corresponds to the modified random direction model [20]; and `(uniform speed, exponential distance)` corresponds to the model

Table 1: Classification of random mobility models based on the combinations of various factors and distributions.

Speed	*Distance*	*Time*	*Destination*
Uniform Normal	Uniform Exponential		
		Uniform Exponential	
			Uniform

that Ko and Vaidya [15] used for the simulation of the Location-Aided Routing (LAR) protocol.

We have left out the element *angle* from the above table. This is because the choice of angle only affects the spatial properties of node distribution. In this paper, we focus solely on the temporal properties of node speed, i.e., the node mobility over time, and thus will not consider angle. Note that from the speed point of view, choosing a destination has the same effect as choosing a distance. This is because a destination determines the travel distance, in addition to determining the spatial distribution. Consequently, for the rest of our discussion we will concentrate on random mobility models that are based on selecting two of the three elements—speed, time, and distance—for a trip. More specifically, since there are only two degrees of freedom among these three elements and speed is almost always directly specified, we will only consider models that are based on the selection `(speed, time)` and `(speed, distance)`.

In the next three subsections, we will explore the steady-state speed distribution or steady-state mobility property of these classes of random mobility models. In particular, we will study the general case where the selections of the elements may or may not be independent, the case where the selections of speed and time are independent, and the case where speed and distance are independent.

3.1 General case: dependence unknown

We first consider the general case where the dependence of the random elements are unknown. All three elements speed, time, and distance, denoted by random variables V, S and R, respectively, are chosen from random distributions. They will be assumed to be within finite minimum and maximum values, denoted by $V_{min}, V_{max}, S_{min}, S_{max}, R_{min}$, and R_{max}, respectively, regardless of whether they are directly specified or indirectly derived. In assuming so we are also implicitly assuming that the minimum speed, V_{min}, is strictly positive since otherwise the maximum travel time S_{max} can be unbounded. This also guarantees a positive steady state average node speed [21].

If pauses are added between successive trips, a mobility model can be viewed as an alternating renewal process that has two independent renewal processes: a move process and a pause process [8]. In fact, the pause process can be equally viewed as a move process only with a different distribution of speed and travel time, (i.e., zero-speed with probability one and pause time independent of speed). In other words, the pause process has a dirac delta function, $\delta(v)$, as its probability density function (pdf) of speed. We will denote by P and V_P the pause time and pause speed, respectively.

When (speed, time) are chosen, The cumulative distribution function (cdf) of the steady-state speed, V_{ss}, can be obtained as follows. Let $\mathcal{A}_{move}$ denote the set of speeds for the move process, i.e., $\mathcal{A}_{move} = \{v : V_{min} \leq v \leq V_{max}\}$, and let $\mathcal{A}_{pause}$ denote the set of speeds for the pause process, i.e., $\mathcal{A}_{pause} = \{v : v = 0\}$. Let $\mathcal{A} = (\mathcal{A}_{move} \cup \mathcal{A}_{pause})$. Assume v, v', and v'' are such that $v \in \mathcal{A}$, $v' \in \mathcal{A}_{move}$, and $v'' \in \mathcal{A}_{pause}$. Note that $\int_{\mathcal{A}_{pause}} \delta(v)dv = \int_{0-}^{0+} \delta(v)dv = 1$. Using the fact that pause time and pause speed are independent, we have

$$\begin{aligned}
&F_{V_{ss}}(v) = P(V_{ss} \leq v)\\
&= \text{fraction of time speed falls below } v\\
&= \frac{\iint_{v' \leq v} s f_{S,V}(s,v')dsdv' + \iint_{v'' \leq v} p f_{P,V_p}(p,v'')dpdv''}{\iint_{S,V} s f_{S,V}(s,v')dsdv' + \iint_{P,V_p} p f_{P,V_p}(p,v'')dpdv''}\\
&= \frac{\iint_{v' \leq v} s f_{S,V}(s,v')dsdv' + E[P]\int_{v'' \leq v} \delta(v'')dv''}{\iint_{S,V} s f_{S,V}(s,v')dsdv' + E[P]} \qquad (1)
\end{aligned}$$

where $f_{S,V}(s,v')$ for $v' \in \mathcal{A}_{move}$ and $f_{P,V_p}(p,v'')$ for $v'' \in \mathcal{A}_{pause}$ are the joint pdfs of (travel time, travel speed) and (pause time, pause speed), respectively. $E[P]$ is the expectation of pause time. Note that pause speed is zero and thus $f_{V_p}(v) = \delta(v)$. It is also clear that $f_V(v) = 0$ for $v \in \mathcal{A}_{pause}$ and $f_{V_p}(v) = 0$ for $v \in \mathcal{A}_{move}$.

Similarly, when (speed, distance) are chosen, the steady-state cdf of V_{ss} is

$$\begin{aligned}
&F_{V_{ss}}(v) = P(V_{ss} \leq v)\\
&= \text{fraction of time speed falls below } v\\
&= \frac{\iint_{v' \leq v} \frac{r}{v'} f_{R,V}(r,v')drdv' + \iint_{v'' \leq v} p f_{P,V_p}(p,v'')dpdv''}{\iint_{R,V} \frac{r}{v'} f_{R,V}(r,v')drdv' + \iint_{P,V_p} p f_{P,V_p}(p,v'')dpdv''}\\
&= \frac{\iint_{v' \leq v} \frac{r}{v'} f_{R,V}(r,v')drdv' + E[P]\int_{v'' \leq v} \delta(v'')dv''}{\iint_{R,V} \frac{r}{v'} f_{R,V}(r,v')drdv' + E[P]} \qquad (2)
\end{aligned}$$

where $f_{R,V}(r,v')$ for $v' \in \mathcal{A}_{move}$ is the joint pdf of travel distance and travel speed.

We can differentiate Eqns.(1) and (2) with respect to v to obtain the pdf of steady-state speed $f_{V_{ss}}(v)$. The expectation of the steady-state speed is then

$$E[V_{ss}] = \int_V v f_{V_{ss}}(v)\ dv.$$

Alternatively, the expected steady-state speed can be obtained through time average. Suppose that a pause time is inserted right after each trip, as if one move and one pause together constitute the n^{th} trip for all n. Then, the long-term time average of node speed with non-zero pause time is

$$\begin{aligned}
\bar{V} &= \lim_{t \longrightarrow \infty} \frac{1}{t} \int_0^t v(\tau)d\tau,\\
&= \lim_{t \longrightarrow \infty} \frac{\sum_{n=1}^{N(t)} V_n S_n}{t}\\
&= \lim_{t \longrightarrow \infty} \frac{\sum_{n=1}^{N(t)} R_n}{\sum_{n=1}^{N(t)} (S_n + P_n)}\\
&= \lim_{t \longrightarrow \infty} \frac{\frac{1}{N(t)} \sum_{n=1}^{N(T)} R_n}{\frac{1}{N(t)} \sum_{n=1}^{N(t)} (S_n + P_n)}\\
&= \frac{E[R]}{E[S] + E[P]}, \qquad (3)
\end{aligned}$$

where $N(t)$ is the total number of trips taken up to time t, including the last one which may be incomplete. R_n, S_n, and P_n are the travel distance, travel time, and pause time of the n^{th} trip, respectively. Note that $\{R_n\}$, $\{S_n\}$, and $\{P_n\}$ are finite, and are independent and identically distributed (iid) random sequences. Thus, the average of iid sequences converges to the ensemble average as $t \to \infty$ by the strong law of large numbers.

Before we proceed to discuss the speed decay phenomenon, it is necessary to define the initial average speed, or the expectation of initial speed as a reference for comparison. For example, if there is no pause or if there is pause but a node always starts from a move state, then the initial average speed, denoted by $E[V_{init}]$, is simply $E[V] = \int_V v f_V(v)dv$. If a node always starts from a pause state, then it is trivial that $E[V_{init}] = 0$. A more interesting case is where a node starts in either state with a certain probability. From the point of view of using a mobility model for simulation, it is only reasonable to assume that these are exactly the probabilities that a node is found to be in either states when the mobility model reaches equilibrium, denoted by P_{move} and P_{pause}, respectively. Then the initial average speed is simply

$$\begin{aligned}
E[V_{init}] &= E[V]P_{move} + 0 \cdot P_{pause}\\
&= E[V]P_{move}. \qquad (4)
\end{aligned}$$

We will consider only this case in the following discussions.

3.2 Speed, time: independent

Since speed and time for each trip are selected independently, the joint distribution of speed V and time S is simply the product of individual distributions, i.e., $f_{S,V}(s,v) = f_S(s)f_V(v)$. Thus Eqn.(1) reduces to the following:

$$\begin{aligned}
&F_{V_{ss}}(v) = P(V_{ss} \leq v)\\
&= \frac{\iint_{v' \leq v} s f_{S,V}(s,v')dsdv' + E[P]\int_{v'' \leq v} \delta(v'')dv''}{\iint_{S,V} s f_{S,V}(s,v')dsdv' + E[P]}\\
&= \frac{\int_{V_{min}}^{v}\int_{S_{min}}^{S_{max}} s f_S(s) f_V(v')dsdv' + E[P]\int_{v'' \leq v}\delta(v'')dv''}{\int_{V_{min}}^{V_{max}}\int_{S_{min}}^{S_{max}} s f_S(s) f_V(v')\ dsdv' + E[P]}\\
&= \frac{E[S]\int_{V_{min}}^{v} f_V(v')\ dv' + E[P]\int_{v'' \leq v} \delta(v'')dv''}{E[S] + E[P]}.
\end{aligned}$$

Then the probability that a node is in a pause state is

$$\begin{aligned}
P_{pause} &= F_{V_{SS}}(v \in \mathcal{A}_{pause})\\
&= \frac{E[P]}{E[S] + E[P]}, \qquad (5)
\end{aligned}$$

and the probability that a node is in a move state is

$$\begin{aligned}
P_{move} &= 1 - P_{pause}\\
&= \frac{E[S]}{E[S] + E[P]}. \qquad (6)
\end{aligned}$$

Since $f_V(v) = 0$ for $v \in \mathcal{A}_{pause}$ and $\delta(v) = 0$ for $v \in \mathcal{A}_{move}$, the pdf of the steady-state speed V_{ss} is

$$f_{V_{ss}}(v) = \begin{cases} \frac{E[S]f_V(v)}{E[S]+E[P]} = f_V(v)\ P_{move}, & v \in \mathcal{A}_{move}\\ \frac{E[P]\delta(v)}{E[S]+E[P]} = \delta(v)\ P_{pause}, & v \in \mathcal{A}_{pause}. \end{cases} \qquad (7)$$

This pdf indicates that a node either moves at a certain speed selected from the pdf $f_V(v)$ with probability P_{move} or pauses with probability P_{pause}. From Eqn.(7), the expectation of steady-state node speed is

$$E[V_{ss}] = \frac{E[S]E[V]}{E[S]+E[P]}, \tag{8}$$

which indicates that there is no speed decay, because $E[V_{ss}]$ is the same as the initial average speed $E[V_{init}] = E[V]P_{move}$ in Eqn.(4).

If there is no pause (i.e., $E[P] = 0$), the pdf of the steady-state speed simply reduces to

$$f_{V_{ss}}(v) = f_V(v), \quad v \in \mathcal{A}_{move}. \tag{9}$$

We see in this case the steady-state speed distribution is identical to the initial speed distribution. It is also trivially true that

$$E[V_{ss}] = E[V] = E[V_{init}]. \tag{10}$$

The intuition and significance of this result will become clearer in the next subsection, in comparison with the case that speed and distance are chosen independently.

3.3 Speed, distance: independent

Choosing distance and choosing destination are equivalent if we are not concerned with the spatial properties of a model. We will thus limit ourselves to the discussion of distance, although our conclusion applies to models that choose explicit destinations as well.

Since speed and distance are independent, the joint distribution of speed V and distance R is simply the product of individual distributions, i.e., $f_{R,V}(r,v) = f_R(r)f_V(v)$. Under the independence assumption, we have from Eqn.(2)

$$\begin{aligned} &F_{V_{ss}}(v) = P(V_{ss} \le v) \\ &= \frac{\iint_{v' \le v} \frac{r}{v'} f_{R,V}(r,v') dr dv' + E[P] \int_{v'' \le v} \delta(v'') dv''}{\iint_{R,V} \frac{r}{v'} f_{R,V}(r,v') dr dv' + E[P]} \\ &= \frac{E[R] \int_{V_{min}}^{v} \frac{1}{v'} f_V(v')\, dv' + E[P] \int_{v'' \le v} \delta(v'') dv''}{E[R]E[\frac{1}{V}] + E[P]}. \end{aligned}$$

In the same manner as in Section 3.2, the probability that a node is in a pause state is

$$\begin{aligned} P_{pause} &= F_{V_{SS}}(v \in \mathcal{A}_{pause}) \\ &= \frac{E[P]}{E[R]E[\frac{1}{V}] + E[P]}, \end{aligned} \tag{11}$$

the probability that a node is in a move state is

$$\begin{aligned} P_{move} &= 1 - P_{move} \\ &= \frac{E[R]E[\frac{1}{V}]}{E[R]E[\frac{1}{V}] + E[P]}, \end{aligned} \tag{12}$$

and the pdf of the steady-state speed V_{ss} is

$$f_{V_{ss}}(v) = \begin{cases} \frac{E[R]\frac{1}{v} f_V(v)}{E[R]E[\frac{1}{V}]+E[P]} = \frac{\frac{1}{v} f_V(v)}{E[\frac{1}{V}]}\ P_{move}, \\ \qquad v \in \mathcal{A}_{move} \\ \frac{E[P]\delta(v)}{E[R]E[\frac{1}{V}]+E[P]} = \delta(v)\ P_{pause}, \\ \qquad v \in \mathcal{A}_{pause} \end{cases} . \tag{13}$$

The steady-state pdf in Eqn.(13) is interpreted in exactly the same way as in Eqn.(7): a node moves at a certain speed according to the pdf $\frac{\frac{1}{v} f_V(v)}{E[\frac{1}{V}]}$ with probability P_{move}, or pauses with probability P_{pause}.

From the pdf in Eqn.(13), the expectation of steady-state speed is

$$E[V_{ss}] = \frac{E[R]}{E[R]E[\frac{1}{V}] + E[P]}. \tag{14}$$

If pause time is set to zero, Eqns.(13) and (14) also reduce to

$$f_{V_{ss}}(v) = \frac{\frac{1}{v} f_V(v)}{E[\frac{1}{V}]}, \quad v \in \mathcal{A}_{move} \tag{15}$$

and

$$E[V_{ss}] = \frac{1}{E[\frac{1}{V}]}. \tag{16}$$

It can be shown that the steady-state average $E[V_{ss}]$ is always less than or equal to the initial average $E[V_{init}]$ regardless of pause time. Let us consider zero pause time first. In this case, $E[V_{init}] = E[V]$ and $E[V_{ss}] = \frac{1}{E[\frac{1}{V}]}$. Applying Jensen's inequality [9] that if a function $g(X)$ is convex, $g(E[X]) \le E[g(X)]$, we have if $g(V) = \frac{1}{V}$,

$$\frac{1}{E[V]} \le E[\frac{1}{V}] \Longrightarrow \frac{1}{E[\frac{1}{V}]} \le E[V]. \tag{17}$$

where the equality holds only when $V_{min} = V_{max}$.

Now suppose that there are non-zero pause times. In this case, from Eqn.(4)

$$\begin{aligned} E[V_{init}] &= E[V]P_{move} \\ &= E[V]\frac{E[R]E[\frac{1}{V}]}{E[R]E[\frac{1}{V}] + E[P]} \end{aligned}$$

and from Eqn.(14)

$$E[V_{ss}] = \frac{E[R]}{E[R]E[\frac{1}{V}] + E[P]}.$$

Since from Eqn.(17) $1 \le E[V]E[\frac{1}{V}]$,

$$\begin{aligned} E[V_{ss}] &= \frac{E[R]}{E[R]E[\frac{1}{V}] + E[P]} \\ &\le \frac{E[V]E[\frac{1}{V}]E[R]}{E[R]E[\frac{1}{V}] + E[P]} = E[V_{init}]. \end{aligned} \tag{18}$$

Thus from Eqns.(17) and (18), $E[V_{ss}] \le E[V_{init}]$. This means that the average node speed always decays with time no matter what pause time is—even with zero pause times—unless the node speed is constant.

The above results are recapitulated as follows: (i) the steady-state speed distribution is always different from the initial speed distribution; (ii) the steady-state distribution and expectation of node speed are completely characterized by the initial speed distribution $f_V(v)$, average distance $E[R]$, and average pause time $E[P]$, which can be computed from the given distributions of distance and pause time; (iii) if pause time is set to zero, the steady-state speed distribution is determined only by the node speed distribution, and not by how distances/destinations are chosen; finally (iv) the steady-state average node speed is always lower than the initial average speed regardless of pause time. This means that if distance/destination is chosen independently of speed, there will always be speed decay. (iii) and (iv) further indicate that if pause time is set to zero, models that only differ in distance/destination selection are essentially indistinguishable in terms of their speed properties.

An intuitive explanation for (iv) is that when speed and distance are chosen independently, a lower speed results in a longer trip. Note that the steady-state speed is weighted by travel time, thus is always lower than the initial average speed. To see this more clearly, consider the following intermediate result from Eqn.(3):

$$E[V_{ss}] = \bar{V} = \lim_{t \longrightarrow \infty} \frac{\sum_{n=1}^{N(t)} V_n S_n}{t}.$$

Recall that V_n and S_n are node speed and travel time of the n^{th} trip, respectively. Here, low speed V_n's are more likely to be weighted by large S_n's, which leads to a lower long-term average node speed. This explanation also applies to the case in Section 3.2. There since speed and time are chosen independently, speed V_n is not correlated with travel time S_n, and thus low speed V_n's are not weighted by large S_n's.

Alternatively, (iv) can be explained using the properties of *harmonic mean* of renewal speed, where the steady-state average speed, with zero pause times, can be viewed as the average rate in the system performance measure [2]. Consider again the following intermediate result from Eqn.(3) with zero pause time,

$$\begin{aligned} E[V_{ss}] = \bar{V} &= \lim_{t \longrightarrow \infty} \frac{\sum_{n=1}^{N(t)} R_n}{\sum_{n=1}^{N(t)} S_n} \\ &= \lim_{t \longrightarrow \infty} \frac{\sum_{n=1}^{N(t)} R_n}{\sum_{n=1}^{N(t)} \frac{R_n}{V_n}} \end{aligned} \quad (19)$$

where R_n is the travel distance of the n^{th} trip. Eqn.(19) is known as the *weighted harmonic mean* since each $\frac{1}{V_n}$ is weighted by distance R_n. However, due to the strong law of large numbers and the speed-distance independence assumption here, Eqn.(19) reduces to the *(unweighted) harmonic mean* as follows.

$$\begin{aligned} E[V_{ss}] = \bar{V} &= \lim_{t \longrightarrow \infty} \frac{\sum_{n=1}^{N(t)} R_n}{\sum_{n=1}^{N(t)} \frac{R_n}{V_n}} \\ &= \frac{E[R]}{E[\frac{R}{V}]} \\ &= \frac{E[R]}{E[R]E[\frac{1}{V}]} = \frac{1}{E[\frac{1}{V}]} \end{aligned}$$

which is the same as Eqn.(16). Therefore if speed and distance are independent, the steady-state node speed with zero pause times is the harmonic mean of renewal speed, and is more weighted by lower speeds by the properties of harmonic mean.

Eqns.(13) and (14) are very general results. They hold regardless of the distributions of speed, distance, and pause time used. They show that the average node speed of an *arbitrary* mobility model starts from an initial value, decays over time, and then settles to a certain steady-state value, as long as the speed and the distance are chosen independently. Moreover, if pause time is set to zero, the steady-state average is identical under all models where the speed is chosen from the same distribution, regardless of the distribution of distance.

An interesting question is then, what happens when speed and distance are chosen dependently? For example, a model that gives higher probability to higher speeds when the distance chosen is larger. Judging from our results in the first part of this section, there does not seem to be a unified answer. Based on our intuitive explanation of speed decay, one can certainly hope to reduce speed decay by correlating the two. In Section 4 we will show an example (Case 4) where travel time is correlated with travel speed. In this particular example, speed decay exists. However, one can conceivably construct a joint distribution of speed/time or speed/distance so that the resulting average speed process is stationary.

4. CALCULATING STEADY-STATE AVERAGE SPEED

The previous section showed how one can calculate the steady-state speed distribution and the steady-state average speed given a random mobility model. Specifically, Eqns.(1), (2), and (3) in a general case, Eqns.(7) and (8) in a speed-time-independent case, and Eqns.(13) and (14) in a speed-distance-independent case can be used to derive speed distributions and expectations. In this section, we use these results to derive the steady-state speed distribution for a few special cases from Table 1. We show that such derivations can be much simplified, given the independence property. In addition, such derivation is an important step toward constructing a completely stationary, decay-free mobility model as we will show in the next section.

4.1 Case 1: uniform speed, uniform destination, independent

This case is equivalent to the random waypoint model [14]. Let us consider the simple zero-pause-time case first. Applying Eqns.(15) and (16), we have

$$\begin{aligned} f_{V_{ss}}(v) &= \frac{\frac{1}{v} f_V(v)}{E[\frac{1}{V}]} \\ &= \frac{\frac{1}{v} \frac{1}{V_{max}-V_{min}}}{\int_{V_{min}}^{V_{max}} \frac{1}{v'} \frac{1}{V_{max}-V_{min}} \, dv'} \\ &= \frac{1}{v \ln\left(\frac{V_{max}}{V_{min}}\right)}, \quad v \in \mathcal{A}_{move} \end{aligned} \quad (20)$$

and

$$\begin{aligned} E[V_{ss}] &= \frac{1}{\int_{V_{min}}^{V_{max}} \frac{1}{v} \frac{1}{V_{max}-V_{min}} dv} \\ &= \frac{V_{max} - V_{min}}{\ln\left(\frac{V_{max}}{V_{min}}\right)} \end{aligned}$$

which is the same result obtained in [21], but in a much simpler way since we avoided the calculation of distance distribution and therefore did not have to make any assumptions about U.

Now let us consider the case of random pause time chosen from a uniform distribution $f_P(p) = \frac{1}{P_{max}}$ for $0 \le p \le P_{max}$. Using Eqns.(13) and (20),

$$f_{V_{ss}}(v) = \begin{cases} \frac{\frac{1}{v} f_V(v)}{E[\frac{1}{V}]} \; P_{move} \\ \quad = \frac{1}{v \ln(V_{max}/V_{min})} \; P_{move}, & v \in \mathcal{A}_{move} \\ \delta(v) \; P_{pause}, & v \in \mathcal{A}_{pause} \end{cases}$$

where

$$P_{pause} = \frac{\frac{P_{max}}{2}}{E[R]\frac{\ln(V_{max}/V_{min})}{V_{max}-V_{min}} + \frac{P_{max}}{2}}$$

and

$$P_{move} = \frac{E[R]\frac{\ln(V_{max}/V_{min})}{V_{max}-V_{min}}}{E[R]\frac{\ln(V_{max}/V_{min})}{V_{max}-V_{min}} + \frac{P_{max}}{2}}.$$

From Eqn.(14), the expectation of steady-state speed is

$$\begin{aligned} E[V_{ss}] &= \frac{E[R]}{E[R]E[\frac{1}{V}] + E[P]} \\ &= \frac{E[R]}{E[R]\frac{\ln(V_{max}/V_{min})}{V_{max}-V_{min}} + \frac{P_{max}}{2}} \end{aligned}$$

where $E[R]$ is the expectation of travel distance in a rectangle and can be computed by integrating over the rectangle. More specifically, suppose that two points (x_1, y_1) and (x_2, y_2) are in a rectangle of $X \times Y$ and that $\mathsf{X} = |x_1 - x_2|$ and $\mathsf{Y} = |y_1 - y_2|$. Then $E[R]$ in this rectangle of $X \times Y$ can be found to be

$$\begin{aligned} E[R] &= \int_0^Y \int_0^X \sqrt{\mathsf{X}^2 + \mathsf{Y}^2} f_{\mathsf{X},\mathsf{Y}}(x,y) d\mathsf{X} d\mathsf{Y} \\ &= 4\left\{ \frac{X^2}{12Y}\left[\frac{\sin\phi}{2\cos^2\phi} + \frac{1}{2}\ln\left(\sec\phi + \frac{Y}{X}\right)\right] \right. \\ &\quad \left. + \frac{X^3}{60Y^2}\left(1 - \frac{1}{\cos^3\phi}\right)\right\} \\ &\quad + 4\left\{ \frac{Y^2}{12X}\left[\frac{\cos\phi}{2\sin^2\phi} - \frac{1}{2}\ln\left(\csc\phi - \frac{X}{Y}\right)\right] \right. \\ &\quad \left. + \frac{Y^3}{60X^2}\left(1 - \frac{1}{\sin^3\phi}\right)\right\} \end{aligned}$$

where $\phi = \arctan\left(\frac{Y}{X}\right)$.

4.2 Case 2: uniform speed, uniform distance, independent

This is a model that chooses a destination by selecting an angle from 0 to 2π and a distance from 0 to R_{max}. As mentioned in the previous section, if pause time is zero, the steady-state speed distribution and its expectation are not affected by how distance/destination is determined. Thus the steady-state speed property in this case is identical to that in the first case, given that the uniform speed distributions in both cases are identical. Therefore, we immediately have

$$f_{V_{ss}}(v) = \frac{1}{v\ln\left(\frac{V_{max}}{V_{min}}\right)}, \quad v \in \mathcal{A}_{move}$$

and

$$E[V_{ss}] = \frac{V_{max} - V_{min}}{\ln\left(\frac{V_{max}}{V_{min}}\right)}.$$

If pause is not zero, the steady-state speed distribution and its expectation can be computed by applying Eqns.(13) and (14), resulting in the same distribution and expectation as in Case 1, only with $E[R] = \frac{R_{max}}{2}$. The modified random direction model [20] could be cast as an instance of this case.

4.3 Case 3: normal speed, uniform distance, independent

Here we consider a "clipped" normal distribution (i.e., one that is distributed between finite maximum and minimum values) with the initial pdf

$$f_V(v) = \frac{1}{\kappa\sqrt{2\pi\sigma^2}}\, e^{-\frac{(v-M)^2}{2\sigma^2}} \quad (V_{min} \le v \le V_{max}),$$

where $\kappa = \int_{V_{min}}^{V_{max}} \frac{1}{\sqrt{2\pi\sigma^2}}\, e^{-\frac{(v-M)^2}{2\sigma^2}}\, dv$ is the normalizing constant for the clipped normal distribution, $M = \frac{V_{max}+V_{min}}{2}$, and σ is the standard deviation of the normal distribution.

Applying Eqns.(15) and (16) to this clipped normal distribution with a zero pause time first, we have for $v \in \mathcal{A}_{move}$

$$\begin{aligned} f_{V_{ss}}(v) &= \frac{\frac{1}{v} f_V(v)}{E[\frac{1}{V}]} \\ &= \frac{\frac{1}{v}\frac{1}{\kappa\sqrt{2\pi\sigma^2}}\, e^{-\frac{(v-M)^2}{2\sigma^2}}}{\int_{V_{min}}^{V_{max}} \frac{1}{v'}\frac{1}{\kappa\sqrt{2\pi\sigma^2}}\, e^{-\frac{(v'-M)^2}{2\sigma^2}}\, dv'} \\ &= \frac{\frac{1}{v}\, e^{-\frac{(v-M)^2}{2\sigma^2}}}{\int_{V_{min}}^{V_{max}} \frac{1}{v'}\, e^{-\frac{(v'-M)^2}{2\sigma^2}}\, dv'} \end{aligned} \quad (21)$$

and

$$\begin{aligned} E[V_{ss}] &= \frac{1}{E[\frac{1}{V}]} \\ &= \frac{1}{\int_{V_{min}}^{V_{max}} \frac{1}{v}\frac{1}{\kappa\sqrt{2\pi\sigma^2}} e^{-\frac{(v-M)^2}{2\sigma^2}}\, dv}. \end{aligned} \quad (22)$$

In this case, numerical integration is required to calculate the exact value. For example, if we use V_{min} = 1m/s, V_{max} = 19m/s, and $\sigma = \frac{1}{4}(V_{max} - V_{min})$, Eqn.(22) results in $E[V_{ss}]$ = 7.7m/s. This will be verified by simulation in Section 6.

If pause time is not zero, the steady-state speed distribution and its expectation become a little more complicated. Using Eqn.(21) and applying it to Eqn.(13),

$$f_{V_{ss}}(v) = \begin{cases} \dfrac{\frac{1}{v}\, e^{-\frac{(v-M)^2}{2\sigma^2}}}{\int_{V_{min}}^{V_{max}} \frac{1}{v'} e^{-\frac{(v'-M)^2}{2\sigma^2}} dv'}\, P_{move}, & v \in \mathcal{A}_{move} \\ \delta(v)\; P_{pause}, & v \in \mathcal{A}_{pause} \end{cases}$$

where $P_{pause} = \dfrac{\frac{P_{max}}{2}}{\frac{R_{max}}{2}\int_{V_{min}}^{V_{max}} \frac{1}{v}\frac{1}{\kappa\sqrt{2\pi\sigma^2}}\, e^{-\frac{(v-M)^2}{2\sigma^2}}\, dv + \frac{P_{max}}{2}}$ and

$$P_{move} = \frac{\frac{R_{max}}{2}\int_{V_{min}}^{V_{max}} \frac{1}{v}\frac{1}{\kappa\sqrt{2\pi\sigma^2}}\, e^{-\frac{(v-M)^2}{2\sigma^2}}\, dv}{\frac{R_{max}}{2}\int_{V_{min}}^{V_{max}} \frac{1}{v}\frac{1}{\kappa\sqrt{2\pi\sigma^2}}\, e^{-\frac{(v-M)^2}{2\sigma^2}}\, dv + \frac{P_{max}}{2}}.$$

The expectation of steady-state speed is

$$\begin{aligned} E[V_{ss}] &= \frac{E[R]}{E[R]E[\frac{1}{V}] + E[P]} \\ &= \frac{\frac{R_{max}}{2}}{\frac{R_{max}}{2}\int_{V_{min}}^{V_{max}} \frac{1}{v}\frac{1}{\kappa\sqrt{2\pi\sigma^2}} e^{-\frac{(v-M)^2}{2\sigma^2}}\, dv + \frac{P_{max}}{2}}. \end{aligned} \quad (23)$$

Again by numerically computing with V_{min} = 1m/s, V_{max} = 19m/s, $\sigma = \frac{1}{4}(V_{max} - V_{min})$, R_{max} being the diagonal of 1500m × 500m rectangle, and P_{max} = 60sec, Eqn.(23) gives 6.0m/s as will be also shown in Section 6.

4.4 Case 4: uniform speed, exponential time, correlated

In this case, we consider a "bounded" exponential time distribution that is distributed between finite maximum and minimum values and that is correlated with the choice of speed as follows:

$$f_{S|V}(s|v_0) = \frac{\lambda e^{-\lambda s}}{\kappa} \quad (0 \le s \le \frac{R_{max}}{v_0}) \ , \qquad (24)$$

where $\kappa = \int_0^{\frac{R_{max}}{v_0}} \lambda e^{-\lambda s}\ ds$ is the normalizing constant, R_{max} is the maximum distance, and v_0 is the speed selected from a uniform pdf. Note that the pdf of time in Eqn.(24) is conditioned on a speed v_0 which is determined before travel time S is selected. Thus λ and κ change from trip to trip, depending on the different v_0 value selected. For simplicity, we will let $\lambda = \alpha \frac{v_0}{R_{max}}$ for a given v_0 and some fixed constant $\alpha > 0$. Then κ reduces to $\kappa = (1 - e^{-\alpha})$, which makes all κ-related computation much simpler. Such a model generates shorter travel time when the travel speed selected is larger, since the maximum time is bounded by $\frac{R_{max}}{v_0}$.

Now we can compute the steady-state distribution and expectation of node speed by applying the general equation which is Eqn.(1). As before, first suppose that pause time is zero.

$$\begin{aligned} & P(V_{ss} \le v) \\ &= \frac{\iint_{v' \le v} s f_{S,V}(s,v')\ ds dv'}{\iint_{S,V} s f_{S,V}(s,v')\ ds dv'} \\ &= \frac{\int_{V_{min}}^{v} \int_0^{\frac{R_{max}}{v'}} s \frac{\lambda e^{-\lambda s}}{\kappa} \frac{1}{V_{max}-V_{min}}\ ds dv'}{\int_{V_{min}}^{V_{max}} \int_0^{\frac{R_{max}}{v'}} s \frac{\lambda e^{-\lambda s}}{\kappa} \frac{1}{V_{max}-V_{min}}\ ds dv'} \\ &= \frac{\int_{V_{min}}^{v} \int_0^{\frac{R_{max}}{v'}} s\lambda e^{-\lambda s}\ ds dv'}{\int_{V_{min}}^{V_{max}} \int_0^{\frac{R_{max}}{v'}} s\lambda e^{-\lambda s}\ ds dv'} \\ &= \frac{\int_{V_{min}}^{v} \frac{R_{max}}{v'} \left(\frac{1}{\alpha}(1 - e^{-\alpha}) - e^{-\alpha}\right)\ dv'}{R_{max}\left(\frac{1}{\alpha}(1 - e^{-\alpha}) - e^{-\alpha}\right) \ln\left(\frac{V_{max}}{V_{min}}\right)} \\ &= \frac{\int_{V_{min}}^{v} \frac{1}{v'}\ dv'}{\ln\left(\frac{V_{max}}{V_{min}}\right)}. \end{aligned} \qquad (25)$$

By differentiating Eqn.(25) with respect to v, we can obtain the steady-state pdf of speed

$$f_{V_{ss}}(v) = \frac{1}{v \ln\left(\frac{V_{max}}{V_{min}}\right)} \quad (V_{min} \le v \le V_{max}) \ ,$$

and the expected steady-state speed is

$$E[V_{ss}] = \frac{V_{max} - V_{min}}{\ln\left(\frac{V_{max}}{V_{min}}\right)}.$$

The fact that these results turn out to be exactly the same as those in Cases 1 and 2 seems to be an interesting coincidence. Again as we will show in Section 6 this model exhibits speed decay as well. Note that unlike the speed-distance-independent case, in general correlated speed and time or distance selection does not necessarily lead to decay.

Now suppose that pause time is not zero. We have

$$\begin{aligned} & F_{V_{ss}}(v) = P(V_{ss} \le v) \\ &= \frac{\iint_{v' \le v} s f_{S,V}(s,v') ds dv' + E[P] \int_{v'' \le v} \delta(v'') dv''}{\iint_{S,V} s f_{S,V}(s,v') ds dv' + E[P]} \\ &= \frac{\int_{V_{min}}^{v} \int_0^{\frac{R_{max}}{v'}} \frac{s\lambda e^{-\lambda s}}{\kappa(V_{max}-V_{min})} ds dv' + E[P] \int_{v'' \le v} \delta(v'') dv''}{\int_{V_{min}}^{V_{max}} \int_0^{\frac{R_{max}}{v'}} \frac{s\lambda e^{-\lambda s}}{\kappa(V_{max}-V_{min})}\ ds dv' + E[P]} \\ &= \frac{\int_{V_{min}}^{v} \frac{R_{max}(1-e^{-\alpha}-\alpha e^{-\alpha})}{v'\alpha(1-e^{-\alpha})(V_{max}-V_{min})} dv' + E[P] \int_{v'' \le v} \delta(v'') dv''}{\frac{R_{max}(1-e^{-\alpha}-\alpha e^{-\alpha})}{\alpha(1-e^{-\alpha})(V_{max}-V_{min})} \ln(\frac{V_{max}}{V_{min}}) + E[P]}. \end{aligned}$$

From the cdf above, the probabilities that a node is pausing or moving are

$$\begin{aligned} P_{pause} &= F_{V_{SS}}(v \in \mathcal{A}_{pause}) \\ &= \frac{\frac{P_{max}}{2}}{\frac{R_{max}(1-e^{-\alpha}-\alpha e^{-\alpha})}{\alpha(1-e^{-\alpha})(V_{max}-V_{min})} \ln(\frac{V_{max}}{V_{min}}) + \frac{P_{max}}{2}}, \end{aligned}$$

and

$$\begin{aligned} P_{move} &= 1 - P_{move} \\ &= \frac{\frac{R_{max}(1-e^{-\alpha}-\alpha e^{-\alpha})}{\alpha(1-e^{-\alpha})(V_{max}-V_{min})} \ln(\frac{V_{max}}{V_{min}})}{\frac{R_{max}(1-e^{-\alpha}-\alpha e^{-\alpha})}{\alpha(1-e^{-\alpha})(V_{max}-V_{min})} \ln(\frac{V_{max}}{V_{min}}) + \frac{P_{max}}{2}}. \end{aligned}$$

Thus the steady-state pdf of speed is

$$f_{V_{ss}}(v) = \begin{cases} \frac{1}{v \ln(V_{max}/V_{min})}\ P_{move}, & v \in \mathcal{A}_{move} \\ \delta(v)\ P_{pause}, & v \in \mathcal{A}_{pause} \end{cases}$$

and the expectation of steady-state speed is

$$E[V_{ss}] = \frac{\frac{R_{max}(1-e^{-\alpha}-\alpha e^{-\alpha})}{\alpha(1-e^{-\alpha})}}{\frac{R_{max}(1-e^{-\alpha}-\alpha e^{-\alpha})}{\alpha(1-e^{-\alpha})(V_{max}-V_{min})} \ln(\frac{V_{max}}{V_{min}}) + E[P]}.$$

Numerical computation with V_{min} = 1m/s, V_{max} = 19m/s, R_{max} being the diagonal of 1500m $\times$ 500m rectangle, P_{max} = 60sec, and $\alpha = 4$, results in $E[V_{ss}]$ = 4.1m/s which will be shown in Section 6.

5. ELIMINATING DECAY

As we have seen so far, random mobility models based on independent speed and travel distance/destination exhibit speed decay. The duration of this speed decay depends on the actual distribution from which speeds are chosen. As has been shown for the case of the random waypoint model, the smaller the minimum speed, the longer the speed decay [21]. The extreme case is the zero minimum speed where the steady-state average speed is zero and the decay is infinite.

Speed decay is undesirable since the performance of a mobile system is highly dependent on the speed of its constituents, and simulation results collected before the average speed of the system settles to a stable level will not be reliable. Methods of reducing such a negative effect have been suggested and used in the literature [13]. One way is to reduce the range of allowed speed by setting the maximum speed and minimum speed to be within a certain percentage of a set value, e.g., $\pm$10% of 15 miles per hour [6]. This significantly reduces both the magnitude and the duration of speed decay, but also heavily limits the variation of nodal speed within the same experiment. Having a wide range of speeds may be highly desirable in some scenarios. Another method is to warm up the simulation by discarding a certain portion of the initial data, or simply to run the simulation long enough and collect results averaged over time so that the effect of the initial decay is diluted. The problem with this method is that it is not always clear how much one should discard or how long is indeed long enough. If we do not warm up enough, then the effect of speed decay still exists; on the other hand, discarding too much results in waste. In order to do this appropriately, we may need to pre-run the mobility model which adds inconvenience and resources required for simulation studies. In short, none of these methods *eliminate* the speed decay inherent to such mobility models in a fundamental way.

In Section 3, we presented a method of deriving the steady-state speed distribution. Can we start the mobility model directly from the steady state now that we know the steady-state distribution? In other words, is it possible for us to construct a stationary process that is free of the transient speed decay period?

It is important to note is that this does not mean we can use the distribution derived in Eqn.(13) for the selection of node speed throughout simulation. We restate the same equation here, assuming a zero pause time for now:

$$f_{V_{ss}}(v) = \frac{f_V(v)}{\text{constant}} \left(\frac{1}{v}\right)$$

The above result essentially indicates that the steady-state speed distribution $f_{V_{ss}}(v)$ is always different from any non-trivial distribution $f_V(v)$ from which node speeds are chosen, since $f_{V_{ss}}(v)$ is a time-weighted version of $f_V(v)$. Consequently, if we use $f_{V_{ss}}(v)$ to select node speed throughout the simulation, then again there will be speed decay and the steady-state speed distribution will be $f_{V'_{ss}}(v)$, which is a time-weighted version of $f_{V_{ss}}(v)$. Therefore, any such model that uses a single speed distribution for node speed is subject to speed decay. Thus we should be looking for a *composite* random mobility model in order to remove the speed decay.

The discrepancy between the initial average speed and the steady-state average speed is due to the fact that the initial speeds are not weighted by travel times, while subsequent trips are; a second speed cannot be chosen until the first trip is completed. This naturally points to applying the steady-state distribution to the first trip in order to randomize first trip. This is analogous to the construction of an equilibrium renewal process found in renewal theory [8].

The same argument applies to the initial pause time selection, since the sequence of pause times indeed constitutes a simple renewal process. The difference, however, is that we are only concerned with the duration of the pause time since the speed is always zero. By the same argument, the initial pause time should be selected according to the steady-state pdf of pause times, which is known to be (the limiting distribution of forward recurrence time using renewal theory)

$$f_{P_{ss}}(p) = \frac{1 - F_P(p)}{E[P]} \tag{26}$$

where $F_P(p)$ is the cdf of pause time. Thus if a node starts from a pause state, the pause time should be selected from the pdf $f_{P_{ss}}(p)$ in Eqn.(26).

To summarize, we construct a composite stationary random mobility model as follows:

1. Determine whether a node starts from a move state or a pause state, with probability P_{move} and P_{pause}, respectively. These are calculated using methods shown in Section 3.

2. If a node starts from a move state, use $f_{V_{ss}}(v \in \mathcal{A}_{move})$ to select the travel speed.

3. If a node starts from a pause state, use $f_{P_{ss}}(p)$ to choose the pause time.

4. After the first trip (either move or pause) of a node, use $f_V(v)$ and $f_P(p)$ to select all subsequent travel speeds and pause times, respectively.

Technically, there are other ways to construct a stationary process by modifying the initial part of the mobility model. For example, if pause is not inserted between successive trips, we could randomize the starting *time* of nodes' first trips, which may seem more intuitive. This method involves the derivation of the distribution of the initial starting time. This may or may not be desirable depending on the mobile system being simulated in that a significant portion of the network may not be moving for some period of time.

On the other hand, modification through the steady-state speed distribution provides an indirect but very effective way of eliminating speed decay and producing a stationary process. We emphasize that the above composition methodology can be applied to *any* random mobility models that choose speed and distance/destination independently and that employ a single speed distribution, to obtain a decay-free sound random mobility model. The effectiveness of this methodology is demonstrated in the next section.

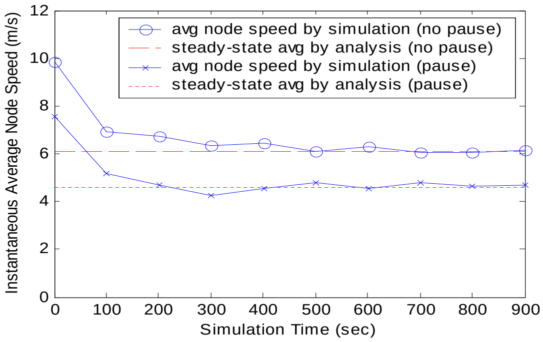
6. SIMULATION RESULTS

In this section we show via simulation the evolution of instantaneous average node speed over time for the four mobility models studied in the Section 4. This instantaneous average speed $\bar{v}(t)$ is defined as

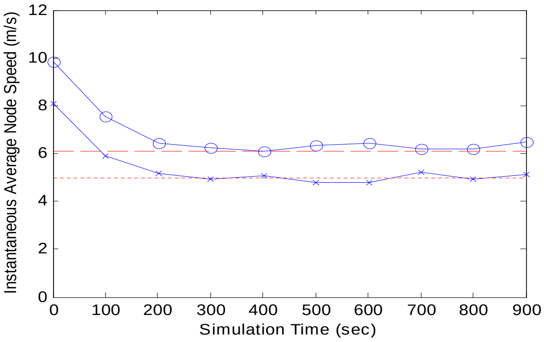
$$\bar{v}(t) = \frac{\sum_{i=1}^{N} v_i(t)}{N},$$

where N is the total number of nodes in the simulation scenario and $v_i(t)$ is the speed of node i at time t. Fig.2 depicts the behavior of each of the original mobility models, while Fig.3 shows the behavior of the composite models. As described in Section 5, each node in these composite models chooses either its speed or pause time only for the first trip from the computed steady-state distributions of speed and pause time, respectively, depending on the probabilities P_{move} and P_{pause}. Thereafter, each node alternately chooses its speed and pause time from the original distributions. Each graph also plots the steady-state average node speed predicted by analysis.

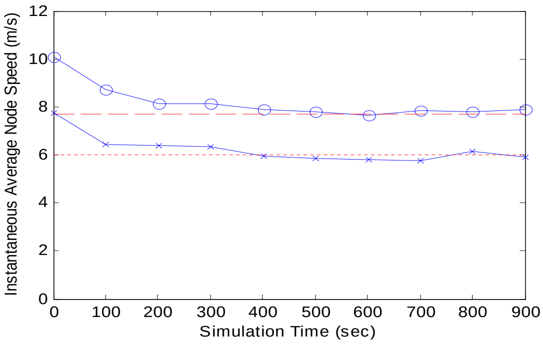
In this set of simulation results, each curve is the average over 10 different scenarios/random seeds. Each scenario contains 50 mobile nodes moving independently in a movement space of 1500m × 500m, according to the specified mobility model. The speed range for all scenarios is from 1m/s to 19m/s, which results in an initial average node speed of 10m/s with zero pause time. When non-zero pause is applied, pause time is randomly selected from the uniform distribution from 0 to 60sec.



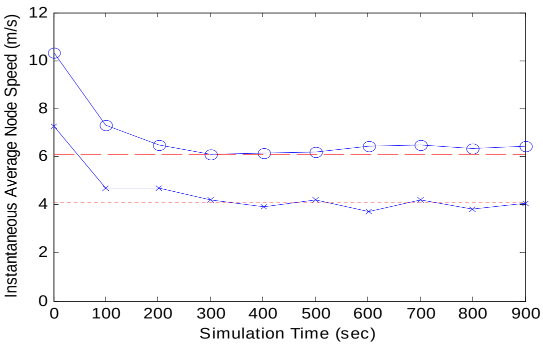
(a) Speed = uniform, destination = uniform (i.e., random waypoint model)



(b) Speed = uniform, distance = uniform

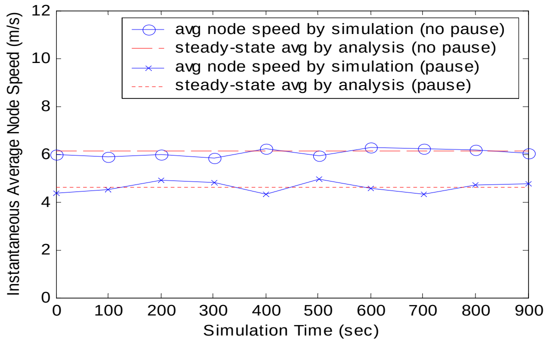


(c) Speed = clipped normal, distance = uniform

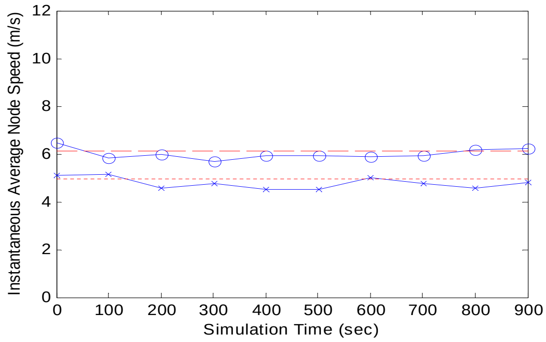


(d) Speed = uniform, time = bounded exponential

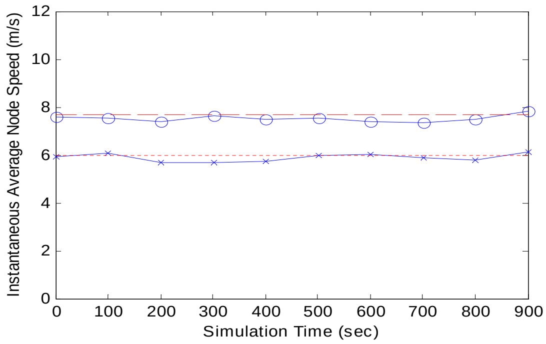
Figure 2: Average speed decays in four models examined in Section 4 with and without pause. Speed = [1,19]m/s. Pause = [0,60]sec



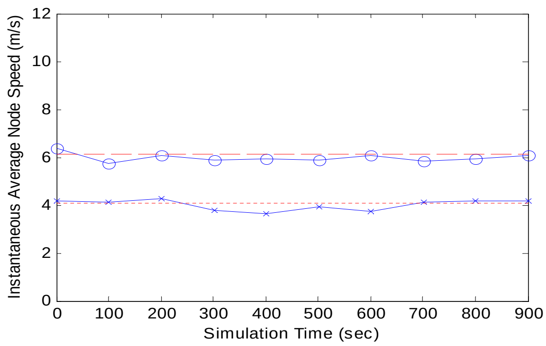
(a) Speed = uniform, destination = uniform (i.e., random waypoint model)



(b) Speed = uniform, distance = uniform



(c) Speed = clipped normal, distance = uniform



(d) Speed = uniform, time = bounded exponential

Figure 3: No speed decays by using a steady-state pdf for the first trip. Speed = [1,19]m/s. Pause = [0,60]sec

As shown in Fig.2, speed decay exists in all four cases. In the first three cases, the speed and distance/destination are chosen independently. In the fourth case, the speed and travel time are correlated. We see from Fig.3 that the constructed composite models successfully eliminated such decay in all cases. Thus this construction methodology is effective regardless of the dependency between travel speed and distance or time, so long as the steady-state speed distribution can be characterized. As expected, the unmodified models converge to the predicted values, while the composite models start and remain there. Such composite models greatly simplify the evaluation process in a simulation study.

7. CONCLUSION

Simulation has become an indispensable tool in the construction and evaluation of mobile systems. By using mobility models that describe constituent movement, one can explore large systems, producing repeatable results for comparison between alternatives. As simulation is becoming not only a qualitative tool but also a quantitative one, having sound mobility models that are suited for simulation studies is critical.

This paper examined a range of random mobility models that are based on the selection of node speed, travel distance or destination, or travel time from random probability distributions. The vast majority of these models—including all those that select node speed and distance independently—exhibit speed decay, where average node speed decreases over time before reaching a steady-state value. Such decay provides an unsound basis for simulation studies that collect results averaged over time, complicating the experimental process.

This decay is easily explained with a general analytical framework, and this paper demonstrates how to apply this framework to a number of practical mobility models. Furthermore, this framework allows one to transform any given mobility model into a stationary one, by choosing initial speeds from the steady-state distribution, and subsequent speeds from the original speed distribution. This constructive method, confirmed through evaluation, provides sound models for simulation, eliminating variations in average nodal speed.

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