Evaluating Opportunistic Multi-Channel MAC: Is Diversity Gain Worth the Pain?

Yang Liu, Mingyan Liu and Jing Deng

***Abstract*—We evaluate the performance of an opportunistic multi-channel medium access control protocol and compare it to that of the corresponding single-channel MAC (S-MAC) and a non-opportunistic multi-channel MAC (M-MAC). We do this in three different settings: (1) an ideal scenario where no control channel is used and no sensing delay is incurred, (2) a more realistic scheme where users compete for access on a control channel using random access, and (3) a scheme similar to (2) but with a time-division multiplexing (TDM) based access scheme on the control channel. Our analysis and numerical results show that in terms of delay performance, the random access and competition on the control channel, which typically occupy a fraction of the total bandwidth, almost always wipe out the channel diversity gain, a main motivation behind an opportunistic multi-channel MAC. On the other hand opportunistic access increases bandwidth utilization which reduces the system's total busy time. As a result it helps reduce power consumption in general. When TDM is employed on the control channel, the data sub-channel sensing delay becomes the main bottleneck to attaining better performance. In this case the performance of opportunistic multi-channel MAC gets closer to that of the single-channel MAC when the channel sensing overhead is substantially reduced.**

***Index Terms*—Opportunistic Spectrum Access (OSA), diversity gain, multichannel MAC, performance evaluation.**

I. Introduction

Recent advances in cognitive radio technologies have led to a number of dynamic multi-channel MAC schemes (see e.g., [9], [17]) that allow transmitters to dynamically switch between channels in search of good instantaneous channel condition. The fundamental idea is the exploitation of *multi-channel diversity* or *spectral diversity*: if a radio uses a single, fixed channel, then over time it sees the *average* condition of that channel and attains an average rate. In contrast, if a radio is allowed to always pick a channel with better instantaneous condition (e.g., higher instantaneous received SNR) from a set of channels, then over time it sees (potentially much) higher average rate, see e.g., [1], [3], [9], [12], [15]. This is an improvement over more traditional schemes, see e.g., channel splitting [4], [14], [18], [19], multi-channel CSMA/CD [13], or a multi-rate system [8], though it may come at the expense of delay as channel sensing takes time.

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While intuitively appealing, such an opportunistic approach [9] cannot easily avoid control overhead. Firstly, a control channel is typically needed for purposes including reservation (gaining the right to use one of the data channels), homing (finding an intended destination node on the control channel), and control information exchange (broadcasting information such as channel selection, completion of transmissions, etc.). Constrained by the same amount of total bandwidth, this may take resources away from data communication if it cannot be allocated separately. Even when this control channel is allocated separately from the data channels, it is likely to have limited bandwidth which ultimately has performance implications as we show later. Secondly, it takes resources to determine which channel has better instantaneous condition. Specifically, successive channel sensing costs energy as well as time, reducing the amount of time for data communication. Note that the use of a control channel exists in non-opportunistic multi-channel systems as well for coordination purposes, see e.g., [6], [11].

The above observations motivate us to examine whether there is indeed an advantage in using dynamic multi-channel MAC as it has been designed and proposed in the literature, and if so under what conditions. To achieve this goal, we perform the following sequence of comparisons of various versions of multi-channel MAC in terms of delay, throughput stability region and power consumption. We start by considering an idealized opportunistic multi-channel MAC (referred to as I-MAC), whereby a genie oversees channel access and has full information on the instantaneous conditions of all data sub-channels. It automatically assigns an arriving packet to the best channel among those currently available; here by "best" we mean the channel that can achieve the highest instantaneous rate. This allows us to eliminate the need for a control channel and fully use the bandwidth for data communication. This is compared to a single-channel MAC (S-MAC) and a non-opportunistic multi-channel MAC (M-MAC) under similarly ideal (collision-free) conditions. As expected, in this scenario I-MAC has a clear advantage over S-MAC and M-MAC due to the channel diversity gain under all comparison criteria.

We then consider a more realistic opportunistic multi-channel MAC (O-MAC), where users must compete for access to data sub-channels on a control channel first, using an RTS-CTS based random access scheme. Once a user gains access it performs channel sensing before choosing one; it then announces its selection on the control channel. We assume each user has two radios, with one dedicated to the control channel so that each user is able to accurately track channel usage. This is therefore a much more efficient use of resources than some studies have proposed, see e.g., [9], where the entire set of data channels must be reserved during channel sensing and data transmission. Our main finding is that this more realistic O-MAC significantly under-performs S-MAC under a similar random access setting in terms of their delay performance. There are two main reasons for this. One is that random access on the control sub-channel becomes a bottleneck as the control sub-channel is typically a very small fraction of the overall bandwidth. The second reason is the extra overhead caused by channel sensing, which decreases the completion rate (or the output) of the control channel. Similarly, the stability region under O-MAC is smaller. It does achieve more efficient power consumption as it takes advantage of good channel conditions.

These observations further lead us to consider a third scheme similar to O-MAC but with a TDM-type of access scheme on the control channel, called T-MAC. The intention is to separate the effect of random access from that of sensing overhead. Our finding is that while it does remove the random access on the control channel as a bottleneck, the sensing delay remains a significant obstacle. As a result, T-MAC continues to under-perform, though its performance gets closer to that of a similar TDM-based S-MAC in terms of delay, when channel sensing can be performed much faster than a regular RTS-CTS packet exchange.

The remainder of the paper is organized as follows. After presenting the system model in Section II, we detail the three sets of comparisons in Sections III, IV, and V, respectively. Related work is presented in Section VI and Section VII concludes the paper.

II. System Model and Preliminaries

A. System model

We consider a set of n active users within a single interference domain. The total amount of bandwidth available is B. Under a single-channel MAC (S-MAC), this is treated as a single aggregate channel for data transmission. Under a multi-channel MAC (M-MAC), the amount B is divided into a control channel of bandwidth B_c, and m equal data sub-channels each of bandwidth $B_d = (B - B_c)/m$ (in the ideal case $B_c = 0$; see Section III). For a single data sub-channel of bandwidth B_d, its maximum achievable rate is given by

$$R_d = B_d \cdot \log(1 + \text{SNR}) \ , \quad (1)$$

where *SNR* is the signal-to-noise ratio. We assume that the aggregated single channel has the same SNR as a data sub-channel (e.g., by assuming that users keep the bit error rate at the same level). The transmission rate of the aggregated single channel is thus given by

$$R = B \log(1 + \text{SNR}) = (B_c + m \cdot B_d) \log(1 + \text{SNR}). \quad (2)$$

Thus when $B_c = 0$, the service rate of the single channel is m times that of a data sub-channel. We will assume that these m data sub-channels are statistically identical, and that the dynamics of a channel is such that for a fixed-size packet, its total transmission time (or service time, including retransmissions) is given by an i.i.d. exponential random variable; more on this assumption is discussed in the next section. Variable service times model the fact that higher received SNR leads to higher data rate or lower bit error probability and thus shorter overall transmission time for successful reception. The assumptions of statistically identical sub-channels and exponential services times are simplifications to obtain tractability in the analysis; they however do not affect the qualitative conclusions we draw from the analysis (see also simulation results with non-exponential channels). We assume that in an opportunistic access setting, a user always picks the best from the set of currently available channels using channel sensing. This is again a simplification and an assumption in favor of the opportunistic MAC. In practice a user may not sense all available channels depending on the channel coherence time and the decision rules used [9]. Below we detail the sequence of schemes to be analyzed in subsequent sections.

I-MAC: This is an ideal multi-channel system that consists of m data sub-channels of equal bandwidth B/m. There is no control channel, and a genie is assumed to be present with full knowledge of the instantaneous channel states. Using this knowledge the genie assigns an arriving packet to the best data sub-channel among those currently available, incurring no sensing overhead. If all data sub-channels are busy then the packet is held in a FIFO queue; the head of the queue is assigned to the next available data sub-channel.

O-MAC: This is a more practical version of **I-MAC**, in that we no longer assume the presence of a genie. Instead there is a control sub-channel and access to the data sub-channels are gained through competition (random access) on this control sub-channel. A user that obtains the right to transmit on the control channel proceeds to perform channel sensing/probing over the data sub-channels. When this is completed the user releases the control sub-channel and all other unused data sub-channels, and starts data transmission over a selected data sub-channel, i.e., other users can now compete for the control channel. If no data sub-channels are available, a packet enters a "virtual queue" in the order of arrival and the head of the queue can access the next available data sub-channel. If this queue is full then the packet has to compete again on the control channel. The size of this virtual queue is tunable, and may be set to zero in which case a packet finding all data sub-channels busy immediately starts re-competing on the control channel.

T-MAC: This MAC is similar to O-MAC with the only difference that access on the control sub-channel is through a fixed TDMA schedule rather than through random access.

B. Assumptions

To fully exploit the diversity gain, each user is assumed to have two radio transceivers to enable parallel access, one for data transmission, the other dedicated to monitoring activities on the control channel. This assumption has an important implication. Since users can continuously monitor the control channel, a user is fully aware of which data sub-channel is currently in use for data transmission and by which user. This means that a user only needs to reserve the control channel for channel sensing/probing purposes but not for data transmission. What this means is that as soon as a user has completed channel sensing and selected a data sub-channel for transmission, it can release all unselected data sub-channels as well as the control channel. The next user gaining access to the control channel can proceed to its own channel sensing, knowing which data sub-channels are currently being used and thus avoiding them in its sensing and data transmission. This enables parallel access – multiple users can simultaneously perform data transmission over different sub-channels, while one other user may be engaged in channel sensing. The resulting system is thus much more efficient than that under a single-radio assumption, where a user gaining access of the control channel has to reserve the entire set of data sub-channels throughout the transmission process and cannot release them until its data transmission is completed. One version of such a system is the Multi-channel Opportunistic Auto-rate Medium access control protocol (MOAR) designed and analyzed in [15]. This dual-radio assumption is clearly in favor of the set of multi-channel MACs we study in this paper, as the second radio has no utility in a single-channel system. The intention is to study a system that fully realizes the spectral diversity gain.

There are in general two types of queuing models for IEEE 802.11 type of random access used in the literature for the purpose of characterizing performance such as delay and throughput. One is an $M/G/\cdot$ model with a general service time distribution and the other an $M/M/\cdot$ model with exponential service times. Both models assume Markovian arrival processes, i.e., Poisson. For instance, the exponential service time distribution assumption was used in [4], [6], [20]. This, however, was done without further verification. An asymptotic justification was given in [16], where it was shown that the service interval distribution in an 802.11 system converges to an exponential distribution when the number of nodes is sufficiently large. Note that the service model in an opportunistic multi-channel system is different from the above literature as in our case contention resolution occurs on the control channel and the data sub-channels are collision free. Therefore the uncertainty in service times primarily arises from the stochasticity of wireless channel condition. In this sense it is reasonable to adopt an exponential service time assumption, which is equivalent to assuming that each (re)transmission of the same packet succeeds or fails independent of the other (re)transmissions. Numerical results show that relaxing this assumption does not alter our qualitative conclusions.

Following this assumption, we will subsequently model our m-channel system as an $M/M/m/\cdot$ queue (or an $M/M/1$ queue for its single-channel counterpart). More specifically, we will assume that there is a queue of size $q, 0 \leq q$; it models the aggregate waiting capacity of all n users in the system. This may be thought of as a "virtual queue" that attempts to capture the effect of separate queueing by individual users at the MAC layer. In practice, when users compete (over the control channel or over the single aggregate channel) for access, an arriving packet finding the channel busy is kept in the MAC queue and re-attempted/retransmitted up to a maximum number of times. A virtual queue of size q is thus an approximation of the real system, while the tunable parameter q gives us the flexibility to adjust the model. As we show in the next section, the resulting $M/M/m/m+q$ model leads to a closed form characterization of the multi-channel system performance, and numerical results demonstrate that it is quite accurate.

III. Comparison Under Ideal Conditions

In this section we compare S-MAC, M-MAC and I-MAC under idealistic conditions. For I-MAC, we assume that a genie has full information on the data sub-channels and immediately assigns an arriving packet to an available channel. This requires no control channel and incurs zero sensing delay.

A. S-MAC: a single-channel MAC

Under an idealized setting, we will model the dynamics of S-MAC as an $M/M/1+q$ queue, where the aggregate arrival process is Poisson with rate λ, the mean service rate is $m\mu$ (μ will be taken as the mean service rate of a single data sub-channel in subsequent analysis), and q denotes the virtual queue size that models the fact that packets arriving to a busy channel are forced to wait as explained in the previous section. This parameter can be adjusted to model a finite queue or a no-queue situation. Denoting by π_i the steady state probability of having i packets in such a system, and by $\rho = \frac{\lambda}{m\mu}$ the utilization factor, basic queuing analysis suggests

$$\pi_{i+1} = \rho \cdot \pi_i, i = 0, 1, 2, ...q \ ; \sum_{i=0}^{1+q} \pi_i = \sum_{i=0}^{1+q} \rho^i \pi_0 = 1 \ . \quad (3)$$

The packet delay is given by D_s with s denoting "single-channel":

$$D_s = \frac{\sum_{i=0}^{1+q} i \cdot \pi_i}{\lambda(1 - \pi_{1+q})} \ , \quad (4)$$

where $\lambda(1 - \pi_{1+q})$ is the throughput of the system, also denoted as Th_s. The average power consumption of the system is given by

$$P_s = P \cdot B \cdot \sum_{i=1}^{1+q} 1 \cdot \pi_i = P \cdot B \cdot (1 - \pi_0) \quad (5)$$

where we have assumed all transmissions are at a constant power P.

B. M-MAC: a multi-channel, non-opportunistic MAC

We similarly model the non-opportunistic multi-channel MAC as an $M/M/m/m+q$ queue with an aggregate arrival rate of λ

and service rate μ per channel. Here we use the same waiting capacity for fair comparison with the single-channel system. Noting that the number in the queue is given by $\sum_{i=0}^{q} i\cdot\pi_{m+i}$, and denoting the packet delay by D_m, where m stands for "multi-channel", we have

$$D_m = \frac{1}{\mu} + \frac{\sum_{i=0}^{q} i\cdot\pi_{m+i}}{\lambda\cdot(1-\pi_{m+q})}, \tag{6}$$

where $\lambda\cdot(1-\pi_{m+q})$ is again the system throughput Th_m. Note that for simplicity we have reused the same notation π_i to denote the steady state probability in this system:

$$\pi_i = \begin{cases} \pi_0\frac{(m\rho)^i}{i!}, & i\le m \\ \pi_0\frac{m^m\rho^i}{m!}, & m<i\le m+q. \end{cases} \tag{7}$$

The average power consumption P_m is

$$P_m = P\cdot B_d\cdot\sum_{i=1}^{m+q}\min(i,m)\cdot\pi_i\ . \tag{8}$$

C. I-MAC: an opportunistic multi-channel MAC

Under the ideal assumption, an arriving packet is immediately assigned to the best sub-channel among all those currently available under I-MAC. A packet finding all sub-channels busy is put in the virtual queue under the $M/M/m/m+q$ model as previously done. However, since a packet is always assigned the "best" channel among all those available, we can no longer model the service rate of a single data sub-channel as a constant μ. Its characterization is much more complicated: a particular sub-channel's service rate is strictly speaking a function of the number of available sub-channels when this sub-channel was selected. In this sense the evolution of the system state, the number of packets in the system, is no longer Markovian.

To address this problem, we adopt the following approximation. We first characterize the *average per sub-channel service rate* under I-MAC, $\bar{\mu}$, and then use $\bar{\mu}$ as the service rate in a standard $M/M/m/m+q$ system. We proceed as follows. We have assumed that the service times of each sub-channel are i.i.d. exponential random variables with rate μ. Basic properties of exponential distribution suggest that the minimum of a collection of independent exponential random variables remains exponential with a rate given by the sum of individual rates. This immediately indicates that when there are k sub-channels busy (or $m-k$ available) and the best one is chosen, the service rate of the chosen sub-channel has a mean of $(m-k)\mu$. It follows that the average service rate of *any* chosen channel is given by:

$$\begin{aligned}\bar{\mu} &= \sum_{k=0}^{m-1}(m-k)\mu\cdot\Pr\{k \text{ channels are busy}\} \\ &= \sum_{k=0}^{m-1}(m-k)\mu\cdot\frac{\pi_k}{\sum_{i=0}^{m-1}\pi_i}\ . \end{aligned}\tag{9}$$

Here again we have reused the notation π_i to denote the steady state probability of having i packets in this system. Define the utilization factor for this $M/M/m/m+q$ model as $\rho = \frac{\lambda}{m\bar{\mu}}$. Combined with the steady-state distribution of $M/M/m/m+q$ given in (7), we can solve $\bar{\mu}$ and the steady-state distributions simultaneously through the set of fixed point equations consisting of (7) and (9). Define

$$F(\bar{\mu}) := \mu\cdot\sum_{k=0}^{m-1}\frac{\pi_k(\bar{\mu})}{\sum_{i=0}^{m-1}\pi_i(\bar{\mu})}\cdot(m-k)\ . \tag{10}$$

Lemma 3.1: $F(\bar{\mu})$ is an non-decreasing and concave function with respect to $\bar{\mu}$.

Proof: Proof can be found in Appendix A. ■

Using the above lemma and the fact $F(\mu)>\mu, F(m\mu)<m\mu$, we immediately obtain the following result.

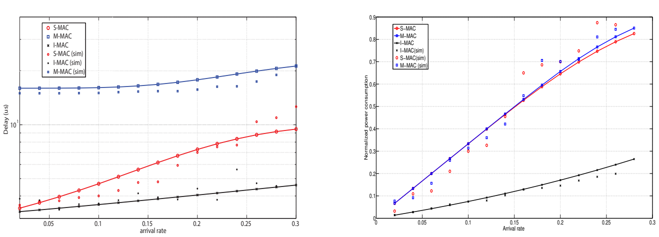
Theorem 3.2: There is a unique solution to $F(\bar{\mu})=\bar{\mu}$.

Once $\bar{\mu}$ is computed, the rest of the delay analysis is similar to Section III-B, from which we can derive the average packet delay D_I. The throughput and average power consumption are also similarly derived as follows.

$$Th_I = \lambda(1-\pi_{m+q}), P_I = PB_d\sum_{i=1}^{m+q}\min(i,m)\cdot\pi_i. \tag{11}$$

D. Performance comparison

We first compare the delay performance of the preceding MAC schemes, D_s, D_m, and D_I, respectively, as given by Equations (4) and (6). Firstly as $m\bar{\mu}\ge m\mu$, and D_m and D_I are derived from the same model, we have $D_m\ge D_I$. Intuitively, for $M/M/m/m+q$ queues the one with faster service rate experiences lower delay. Next we consider D_s and D_m. When traffic is light (λ small), the delay is dominated by the service rate, resulting in $D_s\le D_m$. Analytical and simulation results shown in Figure (1a) illustrate this. The following parameter values are used in the simulation: $m=5, q=5$[1], and data packet length $L_d=1024$ bits. The same parameter values are used for all simulations throughout the paper.



(a) Delay (b) Power consumption

Fig. 1: Comparing S-MAC, M-MAC and I-MAC

Next, numerical and simulation results on power consumption are presented in Figure (1b). It is observed that by opportunistically selecting channels, the I-MAC scheme greatly reduces

[1]It turns out that the length of the queue does not have significant impact on the result.

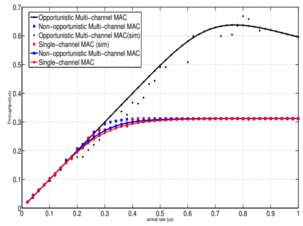


Fig. 2: Comparing S-MAC, M-MAC and I-MAC

the system's power consumption. This is quite intuitive as better use of the same amount of channel bandwidth increases the system service rate. This in turn allows packets to go through the system quickly, reducing the system's busy/working time. These results show quite clearly the benefit of exploiting multi-channel diversity in the absence of overhead.

IV. O-MAC: A More Realistic Opportunistic Multi-channel MAC

We next turn to a more practical setting, where a control sub-channel is allocated for the users to compete for access to the data sub-channels, and the competition is through an RTS-CTS based random access scheme. This setting is close to the protocol MOAR proposed in [9], [11], but has higher channel utilization due to the two-radio assumption as mentioned in the previous section and detailed below.

A. Analysis of O-MAC

O-MAC operates in the following steps. (1) Any user having packets to send first competes on the control channel for the right to access data sub-channels. This is performed through carrier sense, random back-off followed by RTS-CTS packet exchange, in a similar fashion as in IEEE 802.11. (2) Upon completion of an RTS-CTS exchange, the pair of users enters a sensing period, where they successively probe the set of currently available data sub-channels. Exactly how this is done is left unspecified; we will simply assume that certain channel sensing packets need to be exchanged between the pair (e.g., they can be RTS-CTS packets again), and ultimately they are able to select the sub-channel with the best current condition. If upon completion of the RTS-CTS exchange the pair finds all data sub-channels busy, then they immediately send an ACK on the control channel (i.e., step (3) below) and start competing for the control channel again. (3) Upon such a decision the pair sends an ACK on the control channel announcing its channel selection decision as well as the duration of occupancy. This serves the purpose of letting all other users accurately track the busy/available status of all data sub-channels. From this point on the reservation on the control channel and all available data sub-channels is released by the pair and other users can resume competing for access. (4) In the meantime, the pair returns to the sub-channel of their selection to perform data transmission. An example is illustrated in Figure (3).

Note that due to the two-radio assumption, a user can continue to monitor traffic on the control channel even when it is engaged in data transmission on a data sub-channel. Compared to MOAR, the biggest difference of the above approach is in step (3). Under MOAR, the control channel is not released until the pair has completed data transmission on a sub-channel. Therefore under MOAR the *entire* set of sub-channels are reserved by the pair of nodes for the entire duration of data transmission.

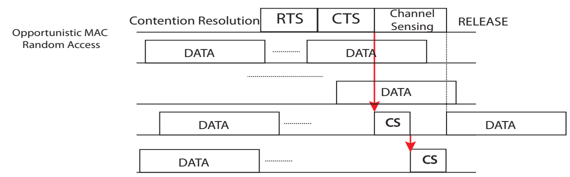


Fig. 3: O-MAC scheme: "cs" denotes channel sensing

To characterize the delay performance of O-MAC, we consider the following types of delays experienced by a user under O-MAC, also illustrated in Figure (4).

1) D_1: time between a packet arrival and the completion of the current RTS-CTS exchange (if any).
2) D_2: time between the start of a competition and the next channel sensing and data transmission. Note that the user initiating the competition may not be the same as the one gaining access and performing the sensing and transmission.
3) D_3: time used for channel sensing upon gaining access to the data sub-channels and provided the system is not blocking.
4) D_4: time for data transmission.

We have ignored the ACK to release the control channel as it is typically a much smaller packet. For simplicity, in the following derivation the above quantities are treated as averages or expectations. Our overall model is summarized in Figure (5), where "C" denotes the control channel and "D" the data sub-channels. As can be seen there is a portion of the traffic that will re-compete for the control channel upon either failure in RTS-CTS exchanges or upon finding all data sub-channels and the queue full.

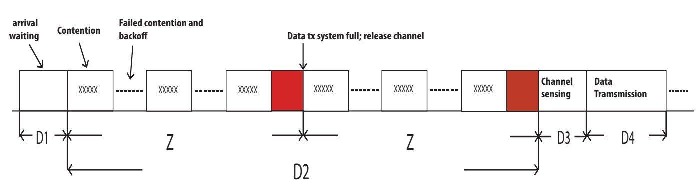


Fig. 4: Delay of O-MAC

We begin with the derivation of D_2. Denote by G the aggregate traffic arrival rate (including both new arrivals and retransmissions) on the control channel (per time unit). As illustrated in Figure (4), the duration D_2 consists of a sequence

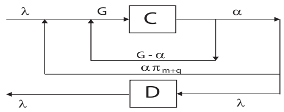


Fig. 5: Throughput/Arrival (λ), competition (G), and completion rates (α).

of periods each denoted by Z: it starts with a period of contention (of duration W), followed by either a successful or failed RTS-CTS exchange (of duration 2); in the case of a failure there is also an average back-off duration $1/\zeta$ before the next contention period. The expected length of Z may be computed as:

$$\begin{aligned} E[Z] &= E[M-1]E\{1/\zeta + W + 2\} + W + 2 \\ &= (e^{2G}-1)(1/\zeta + \frac{1}{Ge^{-2G}} - 1 + 2) + \frac{1}{Ge^{-2G}} - 1 + 2 \ , \end{aligned} \tag{12}$$

where $M = e^{2G}$ denotes the average number of contention periods within a single Z, and is obtained from the fact that M may be modeled as a geometric random variable with a success probability of e^{-2G} using known results on random access and queueing [2]. $W = \frac{1}{Ge^{-2G}} - 1$ is the length of the contention period similarly derived.

The Z period (or a successful RTS-CTS) may not lead to channel sensing if all data sub-channels are found to be busy and the queue full. When this happens, the users that succeeded in RTS-CTS must repeat the contention. We can therefore express D_2 as $D_2 = E[N] \cdot E[Z] = \frac{1}{1-\pi_{m+q}}E[Z]$; where N denotes the number of times it takes for an available data sub-channel to be found. Under our model, N is geometrically distributed with a success probability $1-\pi_{m+q}$ (the probability the system is non-blocking). Here again we reuse the notation π_i to denote the steady state distribution of this system. This steady state distribution is determined by the effective arrival rate α to the data sub-channels and is calculated as follows based the above discussion on when a successful RTS-CTS leads to data channel sensing/access:

$$\alpha = \frac{\pi_{m+q}}{W+2} + \frac{1-\pi_{m+q}}{W+2+D_3} \ . \tag{13}$$

We next turn to the derivation of D_3. Denote by L_s the size of sensing packets involved in sensing one channel; this could be the size of a pair of packets if channel probing involves the exchange of a pair of packets. In general these are smaller packets than the RTS-CTS pair, whose size is denoted by L_c. Denote the ratio of the two as $r_{cs} = L_s/L_c$, $0 < r_{cs} \leq 1$. Also denote the ratio between the average rate of the control channel and that of a data sub-channel as $r = R_c/R_d$. We shall normalize the time to transmit one pair of RTS/CTS on the control channel to 2 units[2]. D_3 is the expected amount of time a user spends on channel sensing provided the system is not blocked. Denote the event that the system is not blocked by $\mathcal{A}$, and consider the above quantity when the system is in state i (i.e., a total of i packets either being served or waiting in the queue) conditioned on this event, denoted by $D_3(i|\mathcal{A})$. Note that when $i = m-1, m, ...m+q$, no channel sensing delay is incurred either because there is only one data sub-channel available ($i = m-1$) or no channels are available ($i > m-1$). For $i = 0, 1, ..m-2$, there are exactly $m-i$ idle sub-channels to be sensed. The sensing delay (normalized to the same time unit) is thus given by:

$$D_3(i|\mathcal{A}) \quad = \mathcal{I}_{\{m-i\geq 2\}} \cdot (m-i)r_{sc} \cdot r \cdot 2 \cdot \frac{\pi_i}{1-\pi_{m+q}} \ , \tag{14}$$

where $\mathcal{I}_{\{m-i\geq 2\}}$ is the indicator function that takes value 1 when $m-i \geq 2$ is true and 0 otherwise. Finally, $D_3 = \sum_{i=0}^{m+q-1} D_3(i|\mathcal{A})$. D_3 is clearly a function of α (through π_i). This implies that (13) is a fixed point equation; the existence of a solution can be proved in a manner similar to Lemma 1. It's worth noting the two performance implications of channel sensing. Firstly, it incurs the additional delay D_3. A second and more subtle point is that since the control channel is reserved during the sensing phase (this is to prevent collision in channel sensing), this additional delay effectively reduces the average completion rate of RTS-CTS exchanges on the control channel.

We next calculate D_1, the average delay due to arriving during an on-going RTS/CTS exchange or channel sensing. Let Y be the random variable denoting the time till the completion of the current on-going RTS-CTS-sensing. Then D_1 is this amount plus the random back-off $1/\zeta$. As we have assumed the inter-arrival time of successful RTS/CTS reservation is exponentially distributed with parameter α, we have $f_Y(y) = \alpha e^{-\alpha y}$; thus we have

$$\begin{aligned} D_1 &= \int_0^{1+D_3} f_Y(y)(1/\zeta + y)dy = \int_0^{1+D_3} (y\alpha e^{-\alpha y} + \frac{\alpha}{\zeta}e^{-\alpha y})dy \\ &= \frac{1}{\alpha} + \frac{1}{\zeta} - (D_3 + 1 + \frac{1}{\alpha} + \frac{1}{\zeta})e^{-(D_3+1)\alpha} \end{aligned} \tag{15}$$

Finally, following the earlier queuing model (as in the ideal case) we have $D_4 = \frac{\sum_{j=0}^{m+q} j \cdot \pi_j}{\lambda}$. Here λ is the system throughput or the external arrival rate which is given by $\lambda = \alpha(1-\pi_{m+q})$.

We now have completely characterized the delay performance of O-MAC. The system throughput is given by $Th_o = \alpha \cdot (1-\pi_{m+q})$, noting that the access rate α is a result of competition on the control channel and using similar arguments as in the previous section. The power consumption P_o consists of two parts, $P_o = P_o^c + P_o^d$: the consumption on the control channel P_o^c and that on a data sub-channel P_o^d. As discussed in the ideal access case, P_o^d is given by $P_o^d = P \cdot B_d \cdot \sum_{i=1}^{m+q} \min(i,m) \cdot \pi_i$. We compute P_o^c by modeling the control channel as an $M/M/1$ queue with arrival rate $(2+D_3)\cdot\alpha$ and service rate 1 (using our normalization). Using standard result on the steady state distribution of an $M/M/1$ queue we have $P_o^c = P \cdot B_c \cdot (2+D_3)\alpha$.

[2]The actual quantity is unimportant as all other quantities will simply be scaled accordingly.

B. Performance comparison

For the O-MAC we have $D_o = \frac{L_c}{R_c}\{D_1 + D_2 + D_3 + D_4\}$. Applying results from Section IV-A, we have

$$D_o = \frac{L_c}{R_c}\{\underbrace{\frac{(e^{2G}-1)(1/\zeta + \frac{1}{Ge^{-2G}} - 1 + 2) + \frac{1}{Ge^{-2G}} - 1 + 2}{1-\pi_{m+q}}}_{D_2}$$

$$+ \underbrace{\sum_{i=0}^{m+q-1} D_3(i|\mathcal{A})}_{D_3} + \underbrace{\frac{\sum_{j=0}^{m+q} j\cdot\pi_j}{\alpha_m(1-\pi_{m+q})}}_{D_4}$$

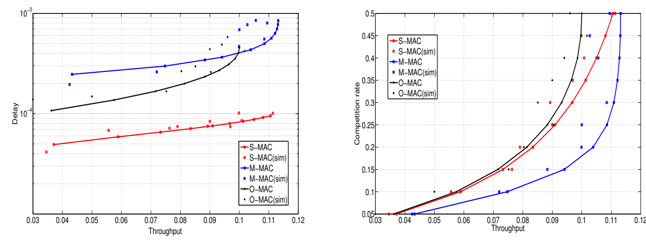
$$+ \underbrace{\frac{1}{\alpha_m} + \frac{1}{\zeta} - [D_3 + 1 + \frac{1}{\alpha_m} + \frac{1}{\zeta}]e^{-(D_3+1)\alpha_m}}_{D_1}\} \qquad (16)$$

The delay for single and multi-channel MACs under random access can be derived and calculated in the same way. Here S-MAC is a standard random access based single-channel MAC, while M-MAC employs a control channel and functions similarly as O-MAC except that M-MAC does not involve channel sensing, and following a successful reservation on the control channel a user selects randomly from the set of available data channels. Details are omitted for brevity.

$$D_s = \frac{L_c}{R}\{(e^{2G_1}-1)(1/\zeta + T_c + r_d) + 2 + 1/m\mu + 1/\alpha_s$$
$$+ 1/\zeta - [r_d + 1 + 1/\alpha_s + 1/\zeta]e^{-(r_d+1)\alpha_s}\} \qquad (17)$$
$$D_m = \frac{L_c}{R_c}\{\frac{(e^{2G_m}-1)(1/\zeta+2) + T_c}{1-\pi_{m+q}} + \frac{\sum_{j=0}^{m+q} j\cdot\pi_j}{\alpha_m(1-\pi_{m+q})} + 1/\alpha$$
$$+ 1/\zeta - [1 + 1/\alpha_m + 1/\zeta]e^{-\alpha_m}\}\ , \qquad (18)$$

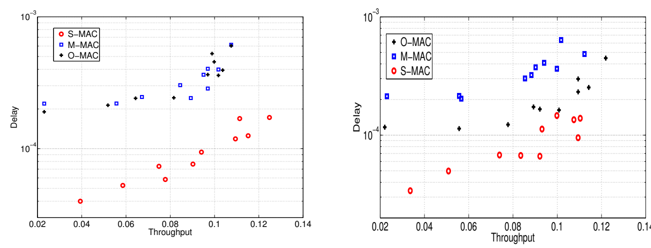
where $r_d = \frac{L_d}{L_c}$ is the ratio between data and control packet lengths. In the simulations the control and data packet lengths are set to $L_c = 48$ bits, $L_d = 1024$ bits, respectively. We also assume that channel sensing is performed using RTS-CTS packet exchanges, i.e., $r_{cs} = 1$. The overall channel data rate is 35 Mbps; the back-off parameter $1/\zeta$ is set to 37 time units. Similar parameters are also used later in Section V.

Figures (6a) shows the delay comparison results. We see that O-MAC significantly under-performs S-MAC. The are three contributing factors to this: the lower successful access rate over the control channel (due to lower bandwidth), the additional sensing delay (which blocks out the control channel from other users) that further reduces the access rate, and the longer data transmission time over a data sub-channel. O-MAC does improve upon M-MAC in terms of delay; it appears that the shorter transmission time (faster under O-MAC due to selecting better channels) more than compensates for the additional sensing delay which does not exist under M-MAC. However, we note that O-MAC has a smaller throughput region than M-MAC, i.e., it supports a smaller sets of arrival rates before delay starts to increase rapidly as shown in Figure (6a). This is further evidenced in Figure (6b), where we see that under the same external arrival rate O-MAC experiences a higher amount of competition (or total traffic on its control channel). An interesting side observation is that M-MAC has the lowest level of competition, lower than S-MAC; this is due to the fact that M-MAC enables parallel process among its data sub-channels, while under S-MAC at any given time only one user is allowed access to the system. This however is not sufficient to improve the delay performance of M-MAC due to its lower data transmission rate. Meanwhile in Figure (7a) and Figure (7b) we added the simulation results with Gaussian distributed and uniform distributed service time respectively; from which we can see the general qualitative results appear to be similar as with the exponential case.



(a) Delay (b) Competition rate (G) v.s. throughput (λ)

Fig. 6: Comparing S-MAC, M-MAC and O-MAC



(a) Gaussian distribution (b) Uniform distribution

Fig. 7: Comparison under non-exponential distributions

Given the above results, it is natural to ask whether by arbitrarily reducing the size of the control or sensing packets O-MAC can hope to outperform S-MAC. The next result suggests the answer is negative; the primary reason being that random access on the control channel significantly lowers the effective packet arrival rate to the data channels. Proof may be found in the appendix.

Theorem 4.1: We have $D_o \geq D_s$, and $D_m \geq D_s$ for arbitrarily large L_d/L_c. Furthermore, $D_o \geq D_s$ for arbitrarily small r_{cs}.

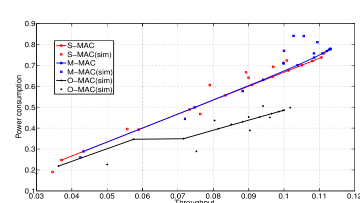


Fig. 8: Power consumption: S-MAC, M-MAC and O-MAC

V. T-MAC: A TDM-BASED MULTI-CHANNEL MAC

The observation made in Section IV, that the random access on the control channel poses a significant bottleneck to the system performance, motivates us to consider an alternative access scheme on the control channel, especially with emphasis on delay performance. In this section we will consider a TDM based access scheme on the control channel while keeping other features unchanged. Again we assume the external arrival rate on the control channel is given by λ (which is from all-together n users); also for simplicity, we assume all users have the same arrival rate (λ/n). If users have different arrival rates, then it can be modeled as a dynamic TDM system [10] with similar analysis. Again denote the overall competition rate as G (including retransmission).

A. TDM-based non-opportunistic M-MAC

There are two components to the delay in a TDM-based multi-channel MAC:

1) D_1: time between the arrival of a packet and when it gains right to transmit.
2) D_2: time for data transmission.

We normalize the time for transmitting one pair of control packets to be 2 and in this case $\mu = 1$ (for serving one control packet). For D_1, standard results on TDM yield the following delay on a single attempt: $E[T_1] = \frac{n}{\mu - 2G} = \frac{n}{1-2G}$. Meanwhile, with a retransmission probability π_{m+q} the expected number of transmission times N on control channel is given by $E[N] = 1/(1 - \pi_{m+q})$. Thus we have

$$D_1 = E[N] \cdot E[T_1] = \frac{n}{(1-2G)\cdot(1-\pi_{m+q})} \ . \tag{19}$$

Following earlier analysis we have $D_2 = \frac{\sum_{j=0}^{m+q} j \cdot \pi_j}{G(1-\pi_{m+q})}$ and $D_m = \frac{L_c}{R_c}\{D_1 + D_2\}$.

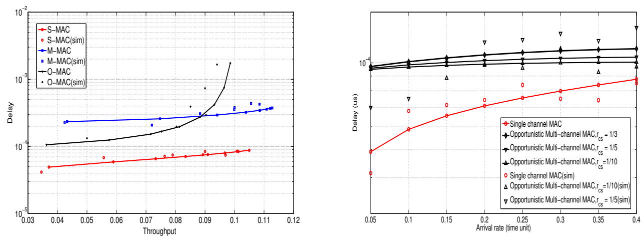
B. T-MAC: TDM-based O-MAC

The operation of T-MAC is as follows. A transmitter waits for its time slot, and then performs RTS/CTS exchange on the control channel with an intended receiver followed by channel sensing, which is then followed by announcing their channel selection, all within the same TDM time slot. Data communication is performed on the chosen channel (which may happen beyond the TDM time slot). As before we normalize the RTS-CTS exchange to 2. The expected delay till the completion of channel sensing is thus $2 + D_3$ (D_3 is the same as in the random access section.) The delay under T-MAC is derived similarly as for TDM-based M-MAC in Section V-A,

$$D_1 = \frac{n}{\{1-(2+D_3)\cdot G\}\cdot(1-\pi_{m+q})}, D_2 = \frac{\sum_{j=0}^{m+q} j \cdot \pi_j}{G(1-\pi_{m+q})} \tag{20}$$

And again $D_o = \frac{L_c}{R_c}\{D_1 + D_2\}$.

C. Performance Comparison



(a) $r_{cs} = 1$ (b) Different r_{cs}

Fig. 9: Delay performance comparison, TDM

We consider first the results shown in Figure (9a), where we have set $n = 5$ and $r_{cs} = 1$. We see that the two multi-channel schemes continue to under-perform their single-channel counterpart. T-MAC's advantage starts to emerge as we lower the sensing delay by using a smaller r_{cs}; this is shown in Figure (9b) as we repeat the same experiment with decreasing values of r_{cs}. This advantage of TDM is more pronounced at higher arrival rates, when the amount of collision increases under random access (for S-MAC). However, this advantage only allows T-MAC to approach the performance of S-MAC but not exceeding it.

VI. RELATED WORKS

For performance improvement consideration, researchers proposed to split single channel into multiple sub channels with one used as control channel and the others used as data channels. Related works on split channel can be found in [14], [18], [19]. In [4], Deng et al. analyze and evaluate the throughput performance for split-channel MAC schemes based on RTS/CTS dialogue and use pure ALOHA or CSMA contention resolution techniques. A queue model is proposed to characterize a close-form solution; and by using the model, the effects of randomness of the contention resolution periods can be captured. In [5], delay performance is further analyzed for split-channel MAC schemes. Following similar favor as above, delay performance can be clearly captured. The conclusion was that multi-channel MAC scheme would not improve either delay or throughput performance compared to single channel MAC. However, the analysis did not take into consideration of diversity gain.

To further exploit the frequency diversity brought in by multi-channel systems, opportunistic spectrum access(OSA) has been investigated. In general, system's performances may be enhanced by three kinds of diversity gains, i.e., multi-user, spatial and multi-channel. Scheduling works regarding opportunistic multi-channel multi-rate system can be found in [1], [3], [12]. In [9], [15], an opportunistic auto rate multi-channel MAC protocol MOAR is presented to exploit the frequency diversity for multi-channel multi-rate IEEE 802.11 enabled wireless ad hoc networks under CSMA/CA. Though this stopping rule driven opportunistic algorithm can bring in certain diversity gain, it does not support parallel access, i.e., the multi-user diversity.

VII. Conclusion

In this paper we analyzed the delay performance of opportunistic multi-channel MAC schemes compared to their non-opportunistic multi-channel and single-channel counterparts. Our general conclusion is that while there is significant channel diversity gain in opportunistic access, the overhead is also significant. It comes in two forms: the much slower rate of access on the control channel and the cost in channel sensing. Using a TDM based access scheme on the control channel can help remove the first bottleneck, but only when channel sensing can be done sufficiently fast. This is despite the fact that our analysis has generally assumed favorable conditions for the multi-channel MAC. We have ignored in our analysis inaccurate channel evaluation, channel quality change during packet reception, and packet reception error, only the last of which may benefit the opportunistic multi-channel MAC scheme as compared to single-channel MAC scheme. These results, while largely negative in nature, do provide guidelines for MAC design and raise the validity of proposals of dividing a single channel into multiple sub-channels to seek better system performance, as in the case of many ISM bands.

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Appendix A
Proof of Lemma 1

We first take the derivative of F with respect to ρ.

$$\frac{\partial F}{\partial \rho} = \mu \cdot \frac{m-k}{S_1^2}\{\sum_{k=1}^{m-1}\frac{(m\rho)^{k-1}}{(k-1)!}\cdot mS_1 - \sum_{k=0}^{m-1}\frac{(m\rho)^k}{k!}\cdot\frac{\partial S_1}{\partial \rho}\} \tag{21}$$

Here we denote $S_1 = \sum_{i=0}^{m-1}(m\rho)^i/i!$ and we have

$$-\frac{\partial S_1}{\partial \rho}\cdot S_1 + \frac{(m\rho)^{m-2}}{(m-2)!}m\rho\cdot S_1 \leq 0, \tag{22}$$

$$\rho\cdot\frac{\partial S_1}{\partial \rho}^2 - (m\rho)\frac{\partial S_1}{\partial \rho}\cdot S_1 \leq 0 \tag{23}$$

Combine with all above we have (details of derivation omitted here)$\frac{\partial F}{\partial \rho} \leq 0$. Since $\rho = \lambda/(m\bar{\mu})$, we proved the non-decreasing property of $F(\bar{\mu})$. To prove it is a concave function, we first introduce some previous results. Denote the Erlang-Loss Function as $\mathrm{EB}(\bar{\mu}) := \frac{(m\rho)^m/m!}{\sum_{i=0}^{m}(m\rho)^i/i!}$. According to [7], $\mathrm{EB}(\rho)$ is convex with respect to $\bar{\mu}$ and it is a strictly decreasing function. Re-order the formula for F we have $F(\bar{\mu}) = (-\rho)\frac{EB(\bar{\mu})}{1-EB(\bar{\mu})} + \text{const.}$. As $f(x) = \frac{x}{1-x}, x\in[0,1]$ is convex and non-decreasing, we have $\frac{EB(\bar{\mu})}{1-EB(\bar{\mu})}$ is also convex with respect to ρ. Also it is easy to verify that $\frac{EB(\bar{\mu})}{1-EB(\bar{\mu})}$ is non-increasing. Take the second order of F w.r.t $\bar{\mu}$ we have

$$\begin{aligned}\frac{\partial^2 F}{\partial^2 \bar{\mu}} &= \frac{-\partial^2\rho}{\partial^2\bar{\mu}}\cdot\frac{EB(\bar{\mu})}{1-EB(\bar{\mu})}\\ &\quad + 2\cdot\frac{-\partial\rho}{\partial\bar{\mu}}\cdot\frac{\partial\frac{EB(\bar{\mu})}{1-EB(\bar{\mu})}}{\partial\bar{\mu}} + (-\rho)\cdot\frac{\partial^2\frac{EB(\bar{\mu})}{1-EB(\bar{\mu})}}{\partial^2\bar{\mu}}\end{aligned} \tag{24}$$

Therefore we know $\frac{\partial^2 F}{\partial^2 \bar{\mu}} \leq 0$. Thus F is concave with respect to ρ. As we proved above,$F(\rho)$ is concave with respect to ρ. Thus $\frac{\partial^2 F}{\partial^2 \rho} \leq 0$; meanwhile we have $\frac{\partial F}{\partial \rho} \leq 0$. And

$$\frac{\partial^2 F(\bar{\mu})}{\partial^2 \bar{\mu}} = \frac{\partial^2 F}{\partial^2 \rho} \cdot (\frac{\partial \rho}{\partial \bar{\mu}})^2 + \frac{\partial F}{\partial \rho} \cdot (\frac{\partial \rho}{\partial \bar{\mu}})^2 \tag{25}$$

Obviously $\frac{\partial^2 F(\bar{\mu})}{\partial^2 \bar{\mu}} \leq 0$, i.e., $F(\bar{\mu})$ is a concave function over its solution set. ∎

Appendix B
Proof of Theorem 1

For the proof, we try to analyze the impacts from $D_1 \sim D_4$ and see how the individual terms compare between single channel and multi-channel and therefore reach our conclusions. As $r_d \to \infty$, it becomes the dominant term and we only need to consider the terms involve r_d. First we need to notice the difference of scaling factors between single channel and multi-channel system as $\frac{L_c}{R_c} = \frac{L_c}{R}\frac{r+m}{r} = \frac{L_c}{R}(1+\frac{m}{r})$. And denote G_1 and G_m as the arrival rate for single channel and multi-channel respectively. Remember for fair comparison, we have $\alpha_s = \frac{r}{r+m} \cdot \alpha_m$, i.e., under the same throughput level. Notice that when control packet size goes arbitrarily small, the need for control channel goes negligible; or mathematically $r = \mathrm{O}(1/r_d)$.

Now consider D_1. For single channel, $\frac{1}{m\mu} = r_d$ and $(r_d + 1)e^{-(r_d+1)\alpha_s} \approx 0$, thus $D_1 \leq \frac{1}{\alpha_s} + \frac{1}{\zeta}$ (which is a bounded term at the order of O(1) w.r.t. r_d) while for multi-channel $D_1 \approx 0$. Now consider channel competition, channel sensing and data transmission parts. As we have

$$\frac{r+m}{r}\frac{\sum_{j=0}^{m+q} j \cdot \pi_j}{\alpha_m(1-\pi_{m+q})} \geq \frac{r+m}{r}\frac{1}{\bar{\mu}} = (r+m)\frac{r_d}{\Phi} \tag{26}$$

Φ is the increase factor for sub data channel's transmission rate which is given by

$$1 \leq \Phi = \sum_{j=0}^{m-1}(m-j) \cdot \frac{\pi_j}{\sum_{i=0}^{m-1}\pi_i} \leq m \tag{27}$$

therefore $\frac{r+m}{r}\frac{\sum_{j=0}^{m+q} j\cdot\pi_j}{\alpha_m(1-\pi_{m+q})} \geq r_d + \frac{r}{m}r_d$. Now look at the $\frac{r}{m}\cdot r_d$ term in Equation (B) and we need to prove $\frac{r}{m}r_d \geq \{e^{2G_1} - 1\}r_d$, or equivalently $\frac{r}{m} \geq e^{2G_1} - 1$. From $\alpha_s = \frac{r}{r+m}\alpha_m$ we know $\frac{r_d \cdot G_1 e^{-2G_1}}{1+(r_d+1)\cdot G_1 e^{-2G_1}} = \alpha_m$. When r_d goes sufficiently large, $G_1 \to 0$; and $G_1 = \mathrm{O}(1/r_d)$ and $\frac{r}{m} \geq e^{2G_1} - 1$. Meanwhile when r_d gets large, other terms will become O(1) w.r.t. r_d which can be neglected. Proved. For the other claim, the arguments follows the same style and thus omitted due to limited space. ∎

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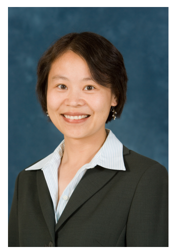

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