Properties Of Optimal Power And Admission Control For A Single User In A Time Varying Wireless Channel

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Abstract

In this paper we consider the problem of joint optimal power and admission control for a single user queue. The user may be in a finite set of channel fading states, each of which corresponds to a certain power-rate function. The user may choose a particular transmission power level and incur a cost that increases with the power. Packets arrive at the queue as a Poisson process with a constant rate. The user may choose a dropping probability for the incoming packet, which incurs a cost that increases with the dropping probability. Packets remaining in the queue also incur a holding cost. The goal is for the user to choose optimally its transmission power/rate and its admission rate so as to minimize the sum of the above costs. These costs model the tradeoff between increasing transmission power, increasing packet delay, and dropping a packet. In this paper we investigate a number of monotonicity properties of the optimal solution to the above problem. Specifically we prove the following results under an optimal strategy: (1) The output or transmission rate of a queue does not decrease and the acceptance rate does not increase as the queue size increases; (2) The output rate does not decrease and the acceptance rate does not increase as the time horizon (or steps to go) increases; (3) For a fixed transmission rate, we show that the acceptance rate does not increase as the system enters a worse fading state; (4) Under certain conditions on the arrival and maximum transmission rates, we show that the output rate does not increase as the system enters a worse fading state. These results provide good insight on the structure of the problem.

1 Introduction

As the use of wireless networks increases, the problem of optimally allocating available resources in order to enhance system efficiency becomes more challenging, due to many trade-offs involved in the design. On the one hand, it is desirable to adapt transmission power to channel variations to achieve better energy efficiency. On the other hand, different applications impose different quality of service requirements, e.g., packet loss and delay, on transmission strategies.

In this paper we study these trade-offs by imposing costs on the transmission power, queueing and rejecting packets in a finite buffer. Specifically, a user queue is sending packets in a time varying wireless channel. The fading state belongs to a finite set and is assumed to be known. Packets arrive at the queue at random with a constant rate. The queue controls the transmission power and admission probability, which in turn determine the transmission/departure/output rate and the effective admission/arrival rate of the queue, and it adapts these quantities according to the fading state and the number of packets already in the queue. The transmission cost is a non-decreasing and convex function of the transmission power used, and the dropping cost is a non-decreasing and convex function of the packet drop probability used. Queued packets also incur a holding cost which is a non-decreasing and convex function of the queue size. The objective is to control the transmission and admission rates as the channel state changes so as to minimize the total cost over a finite or infinite horizon.

The problem of optimal rate allocation has been extensively studied in the literature. For a good survey on control policies for queues see for example [1]. [2, 3] studied the problem of rate allocation in tandem queues where arrivals are not controlled, and proved several monotonicity results on the optimal policy, including that the service rate at each queue does not increase as a costumer leaves that queue. [4] studied the problem of optimal routing and server allocation for two queues and proved that the optimal policy is of the threshold type, under linear cost functions and uncontrolled arrivals. Time varying channel was not considered in these papers. [5, 6] considered the problem of power allocation in satellite networks with varying channel condition when the total power is fixed. While their policy does not necessarily minimize the queue size, they showed that it stabilizes the system whenever the system can be stabilized. Also relevant are studies on dynamic bandwidth allocation where a fixed amount of shared bandwidth is assigned to multiple users so as to optimize certain performance objectives, see for example [7, 8, 9, 10, 11]. In this paper we will instead focus on a single user with a fixed bandwidth, and investigate the joint control of transmission and admission rates.

Compared to prior work, the problem considered in this paper has the following distinct features: (1) We assume that both the output rate and input rate of the user queue are controllable. This assumption is essential in allowing us to consider systems with finite buffers. (2) We also consider a fading channel that is time varying. We derive a number of monotonicity properties of an optimal solution to the problem outlined above. In our proofs we extensively use the inductive method established and used in earlier works such as [4, 12, 13, 14, 15] to show that certain properties of the value function (cost to go) propagate with time.

The rest of the paper is organized as follows. In the next section we describe our system and formulate the problem. In Sections 3, 4 and 5 we show that an optimal policy has a monotone structure with respect to the queue size, the time horizon, and the fading state, respectively. In Section 6 we extend our results to the infinite horizon and Section 7 concludes the paper.

2 Problem Formulation

We consider a set of user queues transmitting to a single base station. Each queue has a fixed amount of dedicated bandwidth and it controls its transmission power and admission rate. The problem of optimal power and admission control can thus be considered for each user independent of the others. Subsequently for the rest of this paper we will limit our discussion to a single user.

We assume that the power can be controlled within a range $[0, \bar{P}]$. Using a power level $p \in [0, \bar{P}]$ incurs a cost $u(p)$, assumed to be convex and charged continuously over time. A channel fading state belongs to the finite set $\mathcal{S} = \{\prime, \infty, \in, \cdots, \mathcal{S}\}$. When in state s, if the user transmits with power level p, then a rate $\mu = R_s(p)$ can be achieved, and the completion time for sending a single packet is exponentially distributed with rate μ. $R_s(\cdot)$ is assumed to be concave, e.g., it can represent the Shannon capacity of the channel. Alternatively it may be viewed that the queue controls the transmission rate within the range $[0, \bar{\mu}_s]$, $\bar{\mu}_s = R_s(\bar{P})$, and selecting rate μ incurs a cost $h_s(\mu) = u(R_s^{-1}(\mu))$, where $R_s^{-1}(\cdot)$ is the inverse of $R_s(\cdot)$. By concavity of $R_s(\cdot)$ and convexity of $u(\cdot)$, it follows that $h_s(\mu)$ is a convex function of μ. Let $\bar{\mu} = \max_s\{\bar{\mu}_s\}$.

Packet arrival is assumed to be Poisson with a fixed rate $\bar{\lambda}$. The queue controls the drop probability $q \in [0, 1]$, which incurs a cost $v(q)$ that is charged continuously over time, regardless of an actual arrival or not [1]. The acceptance rate of the queue is thus $\lambda = (1 - q)\bar{\lambda}$. Alternatively, if we define function $g(\lambda) = v((\bar{\lambda} - \lambda)/\bar{\lambda})$, then this may be viewed as the cost of selecting an admission rate $\lambda \leq \bar{\lambda}$.

We assume $u(\cdot)$ and $v(\cdot)$ are non-decreasing and convex, and it follows that $h(\cdot)$ is non-decreasing and convex, and that $g(\cdot)$ is non-increasing and convex. We further assume that both g and h are continuous but not necessarily differentiable.

In addition, a holding cost $c(x(t))$ is also charged, where $x(t)$ denotes the number of packets in the queue at time t. $c(\cdot)$ is assumed to be non-decreasing and convex.

The objective is to find a policy π that minimizes the following discounted cost:

$$J_T^\pi = E^\pi \int_{t=0}^{T} e^{-\alpha t}[c(x(t)) + (u(p(t)) + v(q(t)))]dt. \tag{1}$$

We assume that when a user is in fading state s it remains in that state for an exponentially distributed amount of time with rate τ and then enters any other state r, $r \neq s$, with equal probability γ (independent of r and s). Note that $\gamma S = 1$. Let $\bar{\gamma} = \tau(1 + \gamma)$. If the system is observed at rate $\bar{\gamma}$ then at the next observation time it is equally likely for the queue to be in any of the fading states (i.e., the queue will be in state $s \in \mathcal{S}$ with probability $\frac{\gamma}{1+\gamma}$, with $(S+1)\frac{\gamma}{1+\gamma} = 1$).

The *state of the system* at time t is defined by the pair $(x(t), s(t))$, the queue size and the fading state, respectively. Note that the assumption that the arrival rate is Poisson, service completion time is exponential and the channel is memoryless results in the following property of the optimal policy: the optimal transmission and admission rates change their values only when the state of the system changes (see [2, 13, 16, 17]).

Using the uniformization method introduced in [18, 19, 20] we "observe" the system at *potential times when the system state changes*. This includes potential departure and arrival times, the times when the fading state changes and null events. Define $V_n(x, s)$ to be the n-stage minimum cost to go starting from state (x, s). Let T_n be the (random) time when the n-th event occurs, then $V_n(x, s)$ is the minimum over all policies of the cost function defined in (1) where T is replaced

[1] This assumption differs from some of the previous work that assumed a cost charged only upon dropping a packet.

by T_n. Letting $\Lambda = \bar{\mu} + \bar{\lambda} + \bar{\gamma}$ and $\beta = \frac{\Lambda}{\alpha+\Lambda}$, we then have

$$\begin{aligned} V_n(x,s) &= \min_{\pi} E^{\pi} \int_{t=0}^{T_n} e^{-\alpha t}[c(x(t)) + (u(p(t)) + v(q(t)))]dt \\ &= \frac{1}{\alpha+\Lambda} \min_{\pi} E^{\pi}[\sum_{k=0}^{n-1} \beta^k c(x(k)) + u(p(k)) + v(q(k))], \end{aligned} \tag{2}$$

where $x(k) = x(T_k^-)$. Without loss of generality we will assume that $\alpha + \Lambda = 1$, and the following dynamic program may be obtained (note that $\bar{\gamma}\frac{\gamma}{1+\gamma} = \tau\gamma$):

$$\begin{aligned} V_n(x,s) = c(x) + \{ &\min_{\mu\in[0,\bar{\mu}_s]} [h_s(\mu) + \mu V_{n-1}(x-1,s) + (\bar{\mu}-\mu)V_{n-1}(x,s)] \\ + &\min_{\lambda\in[0,\bar{\lambda}]} [g(\lambda) + \lambda V_{n-1}(x+1,s) + (\bar{\lambda}-\lambda)V_{n-1}(x,s)] + \bar{\gamma}\frac{\gamma}{1+\gamma}\sum_{r\in\mathcal{S}} V_{n-1}(x,r)\}, \end{aligned} \tag{3}$$

We will make the natural assumption that whenever the queue is empty the control policy chooses zero power level. To avoid confusion, in subsequent sections when there may be multiple minimizers in the above equation we will make the following assumption [2].

Assumption 1 *The control policy always chooses the highest transmission rate and the lowest admission rate that minimize the terms in (3).*

Note that as $c(\cdot)$, $u(\cdot)$ and $v(\cdot)$ (and thus $h(\cdot)$ and $g(\cdot)$) are applied continuously over time, strictly speaking these are the *rates* at which the cost is incurred rather than the cost itself (see the integration in (1)). However, as the control does not change between different system states, they are simply multiplied by the same constant (mean of discounted time between state transition) in the discrete version (see (2)). We will thus loosely refer to all these functions as cost functions.

The two functions $c(\cdot)$ and $v(\cdot)$ model the trade-off between holding a packet (increasing delay) and dropping the packet (increasing loss). For applications that are more delay sensitive, $c(x)$ is large compared to $v(q)$, so that the queue drops more packets in order to keep the queue size small. For applications that are more sensitive to packet losses, $c(x)$ is small compared to $v(q)$, so that more packets are kept in the buffer instead of being dropped.

Also note that the above model can accommodate both infinite and finite buffers. Let B be the buffer size. Then define $c(x) = \infty$ for all x such that $x > B$. Therefore the optimal policy for the queue would be to set $q = 1$ ($\lambda = 0$) when $x = B$.

Definition 1 *We say that state s is better than state r (in notation $r \prec s$) if we have $R_r(p) \leq R_s(p)$, $\forall p \in [0, \bar{P}]$.*

In the next few sections we derive a number of monotonicity properties of the optimal policy for the problem formulated above. Due to space limit, most of the detailed proofs are not provided in this paper. Interested readers may find them in [21].

[2]This assumption does not affect the monotonicity properties derived in this paper. It merely specifies an explicit strategy when there are multiple minimizers.

3 Monotonicity With Respect To Queue Size

In this section we show the for a fixed channel fading state, the optimal policy is monotone with respect to the queue size.

Definition 2 *We define $\mathcal{X}$ to be the region where the cost is finite, i.e. $\mathcal{X} = \{x|c(x) < \infty\}$. By convexity of $c(x)$ we have that if $x-1, x+1 \in \mathcal{X}$ then $x \in \mathcal{X}$.*

We say that a function $f : \mathcal{X} \times \mathcal{S} \to \mathbb{R}$ is convex on $\mathcal{X}$ if we have

$$2f(x+1,s) \leq f(x,s) + f(x+2,s), \qquad \forall s \in \mathcal{S}, \forall x \in \mathcal{X}. \tag{4}$$

For a function $f : \mathcal{X} \times \mathcal{S} \to \mathbb{R}_+$, we define the following transformations:

$$T_1(f(x,s)) = \min_{\mu \in [0,\bar{\mu}_s]} \{h_s(\mu) + \mu f(x-1,s) + (\bar{\mu} - \mu) f(x,s)\}; \tag{5}$$

$$T_2(f(x,s)) = \min_{\lambda \in [0,\bar{\lambda}]} \{g(\lambda) + \lambda f(x+1,s) + (\bar{\lambda} - \lambda) f(x,s)\}; \tag{6}$$

$$T_3(f(x,s)) = \bar{\gamma} \frac{\gamma}{1+\gamma} \sum_{r \in \mathcal{S}} f(x,r), \tag{7}$$

where it is understood that whenever $x-1 \notin \mathcal{X}$ and $x \in \mathcal{X}$, then $\mu = 0$ is the minimizer in (5) (queue empty), and whenever $x+1 \notin \mathcal{X}$ and $x \in \mathcal{X}$, then $\lambda = 0$ is the minimizer in (6) (queue full). We want to show the following results for a fixed channel fading state s: (1) The acceptance rate does not increase as the queue size increases; (2) The output rate does not decrease as the queue size increases (note that since in this case we are keeping the state fixed, if the rate is non-decreasing then the power will also be non-decreasing). By looking at Equation (3) we see that to establish these results, we first need to show that $V_n(x,s)$ is a convex function of x.

Lemma 1 *Suppose $f(x,s)$ is a convex function on $\mathcal{X}$. Then $T_1(f), T_2(f)$ and $T_3(f)$ are convex functions on $\mathcal{X}$.*

Theorem 1 *$V_n(x,s)$ is convex on $\mathcal{X}$ for all $n \geq 0$.*

Definition 3 *For a function f, define $f'_+(z)$ to be the derivative of the function from above at point z and define $f'_-(z)$ to be the derivative from below at point z.*

Theorem 2 *Suppose that for $0 \leq x < B$ the optimal policy in state (x,s) when there are $n+1$ steps to go ($n \geq 0$) is (λ_1, μ_1) and let the optimal policy in state $(x+1,s)$ when there are $n+1$ steps to go be (λ_2, μ_2). Then we have:*

$$\mu_2 \geq \mu_1 \quad \textit{and} \quad \lambda_2 \leq \lambda_1 \tag{8}$$

Proof: Using the optimality of (λ_1, μ_1) in state (x,s) and the fact that h is convex and non-decreasing, and that g is convex and non-increasing, we have (by considering the first two minimizers in Equation (3) respectively):

$$\begin{aligned} V_n(x,s) - V_n(x-1,s) &\geq h'_{s-}(\mu_1) \ \text{ for } \ x > 0, \\ V_n(x+1,s) - V_n(x,s) &\geq -g'_+(\lambda_1), \end{aligned}$$

where the inequality is due to the fact that functions f and g are not necessarily differentiable. By convexity of V_n, along with the above inequalities and Assumption 1, we obtain
$V_n(x+1,s) - V_n(x,s) \geq V_n(x,s) - V_n(x-1,s) \geq h'_{s-}(\mu_1), \rightarrow \mu_2 \geq \mu_1, \quad 0 < x < B,$
$V_n(x+2,s) - V_n(x+1,s) \geq V_n(x+1,s) - V_n(x,s) \geq -g'_{+}(\lambda_1), \rightarrow \lambda_2 \leq \lambda_1, \quad 0 \leq x < B-1.$
Also note that when $x = 0$, $\mu_1 = 0$, and when $x = B-1$, $\lambda_2 = 0$. Therefore for $0 \leq x < B$, we have $\lambda_2 \leq \lambda_1$ and $\mu_2 \geq \mu_1$. ∎

4 Monotonicity With Respect To Time Horizon

In this section we show that for a fixed channel fading state, the optimal policy is monotone with respect to time horizon.

Lemma 2 *Suppose that functions f_n and f_{n+1}, both $\mathcal{X} \times \mathcal{S} \rightarrow \mathbb{R}_+$, satisfy*

$$f_{n+1}(x-1,s) + f_n(x,s) \leq f_n(x-1,s) + f_{n+1}(x,s). \tag{9}$$

Then we have for $k \in \{1,2,3\}$,

$$T_k(f_{n+1}(x-1,s)) + T_k(f_n(x,s)) \leq T_k(f_n(x-1,s)) + T_k(f_{n+1}(x,s)). \tag{10}$$

Theorem 3 *For all $n \geq 0$ we have:*

$$V_n(x,s) - V_n(x-1,s) \leq V_{n+1}(x,s) - V_{n+1}(x-1,s). \tag{11}$$

Proof: We prove the theorem by induction. Note that the statement is true for $n = 0$. Assume that the statement holds for n and we want to show that it also holds for $n+1$. Note that:
$V_{n+1}(x,s) = c(x) + T_1(V_n(x,s)) + T_2(V_n(x,s)) + T_3(V_n(x,s)),$
$V_{n+2}(x,s) = c(x) + T_1(V_{n+1}(x,s)) + T_2(V_{n+1}(x,s)) + T_3(V_{n+1}(x,s)).$
Using Lemma 2 we have
$T_1(V_{n+1}(x-1,s)) + T_1(V_n(x,s)) \leq T_1(V_n(x-1,s)) + T_1(V_{n+1}(x,s)),$
$T_2(V_{n+1}(x-1,s)) + T_2(V_n(x,s)) \leq T_2(V_n(x-1,s)) + T_2(V_{n+1}(x,s)),$
$T_3(V_{n+1}(x-1,s)) + T_3(V_n(x,s)) \leq T_3(V_n(x-1,s)) + T_3(V_{n+1}(x,s)).$
Adding the above inequalities and rearranging, we get

$$V_{n+1}(x,s) - V_{n+1}(x-1,s) \leq V_{n+2}(x,s) - V_{n+2}(x-1,s), \tag{12}$$

completing the induction. ∎

Theorem 4 *Suppose the optimal policy in state (x,s) when there are $n+1$ steps to go is (λ_1, μ_1) and let the optimal policy for state (x,s) when there are $n+2$ steps to go be (λ_2, μ_2). Then we have $\lambda_2 \leq \lambda_1$ and $\mu_2 \geq \mu_1$.*

The proof uses inequality (11) proved in Theorem 3 and taking the same steps used in the proof of Theorem 2.

5 Monotonicity With Respect To Channel State

5.1 Monotonicity of Admission Control - Fixed Transmission Power

In this subsection we assume that the transmission power is fixed at some P_c, so for any state s we have $\mu(x,s) = \mu_s = R_s(P_c)$. For states r and s such that $r \prec s$, we have $\mu_r \leq \mu_s$. With this assumption we show below that the admission probability does not increase if the state becomes worse. By examining Equation (3) and using the same method used in Theorem 2 we see that it suffices to prove that the following inequality holds for all $n \geq 0$ and for any pair $r \prec s$:

$$V_n(x+1,s) - V_n(x,s) \leq V_n(x+1,r) - V_n(x,r). \tag{13}$$

Lemma 3 *Let $f : \mathcal{X} \times \mathcal{S} \to \mathbb{R}_+$ be a function that satisfies (13) with V_n replaced by f and let $\hat{f} = T_2(f)$. Then $\hat{f}$ also satisfies (13).*

Lemma 4 *Let $f : \mathcal{X} \times \mathcal{S} \to \mathbb{R}_+$ be a function that satisfies (13) with V_n replaced by f and define $\hat{f}$ as follows:*

$$\hat{f}(x,s) = u(P_c) + \mu_s f(x-1,s) + (\bar{\mu} - \mu_s) f(x,s), \qquad \forall s \in \mathcal{S}.$$

Then $\hat{f}$ also satisfies (13).

Theorem 5 *Assume fixed transmission power. Suppose we have $n+1$ steps to go. Let λ_1 be the optimal arrival rate when the state is (x,s) and let λ_2 be the optimal arrival rate when the state is (x,r) and $r \prec s$. Then we have $\lambda_2 \leq \lambda_1$.*

It suffices to show that (13) holds. Note that $V_0(x,s) = 0$ satisfies (13). Using Lemmas 3 and 4 and inducting on n, we have that $V_n(x,s)$ satisfies (13).

5.2 Monotonicity of Transmission Rate Control

In this subsection we show that the optimal transmission rate is monotone with respect to the channel fading state under certain conditions on the cost functions. We will assume that the maximum achievable rate in all fading states is the same, instead of assuming a fixed maximum power. Note that when $c(x)$ is bounded in $\mathcal{X}$ we have the relation $V_n(x,s) - V_n(x-1,s) \leq \frac{\max_x\{c(x)-c(x-1)\}}{1-\beta}$ for all n. Therefore by adding a linear section with slope greater than $\max\{\frac{\max_x\{c(x)-c(x-1)\}}{1-\beta}, \max_\mu\{h'_{s-}(\mu)\}\}$ to the cost function $h_s(\mu)$, the problem of fixed maximum power can be transformed to the problem of fixed maximum rate (note that this section of rates is never optimal). Also note results in previous sections hold under this alternative assumption. We are interested in the following property concerning the monotonicity of the optimal policy with respect to the fading state.
Property P-1: Consider $n+1$ steps to go. Let μ_1 be the optimal transmission rate when the state is (x,s) and let μ_2 be the optimal transmission rate when the state is (x,r). The value function $V_n(x,s)$ satisfies property **P-1** if for $r \prec s$ we have $\mu_2 \leq \mu_1$.

For simplicity of subsequent expressions, let $a_s(\mu) = \frac{h_s(\mu)}{\mu}$ for $\mu > 0$. In what follows we identify conditions on the cost functions and show that these conditions are sufficient for the optimal policy of our problem to have Property **P-1**.

Condition 1 *For all $r \prec s$, $a_r(\mu) - a_s(\mu) \le h'_{r+}(\mu) - h'_{s+}(\mu)$ for all μ.*

Condition 2 *For all $r \prec s$, $a_r(\mu) - a_s(\mu)$ is a non-decreasing function of μ.*

Condition 3 *There exist $\nu_1, \nu_2 > 0$, $\frac{1}{\nu_1} + \frac{1}{\nu_2} = 1$, such that $\bar{\mu} \le \frac{L}{\nu_1(1+L)}$ and $\bar{\lambda} \le \frac{M}{\nu_2(1+M)}$, where $L = \min_{r \prec s} L_{s,r}$, $M = \min_{r \prec s} M_{s,r}$, and*

$$L_{s,r} = \inf_{\mu_2 \le \mu_1} \frac{a_r(\mu_1) - a_s(\mu_1)}{[(a_r(\mu_1) - a_s(\mu_1)) - (a_r(\mu_2) - a_s(\mu_2))]} \tag{14}$$

$$M_{s,r} = \inf_{\mu_2 \ge \mu_1} \frac{a_r(\mu_1) - a_s(\mu_1)}{a_r(\mu_2) - a_s(\mu_2)} \tag{15}$$

Lemma 5 *Let μ^*, μ^{**} be the optimal transmission rates at states (x, s) and (x, r), respectively, when there are n steps to go. Then under Condition 1, $V_{n-1}(x, s)$ satisfies property* **P-1**, *i.e., $\mu^* \le \mu^{**}$ if for any state $r \prec s$ and any $x \ge 1$ we have*

$$V_{n-1}(x, r) - V_{n-1}(x-1, r) - a_r(\mu^*) \le V_{n-1}(x, s) - V_{n-1}(x-1, s) - a_s(\mu^*). \tag{16}$$

Lemma 6 *Suppose that V_n is such that*

$$V_n(x, r) - V_n(x-1, r) - a_r(\mu_1) \le V_n(x, s) - V_n(x-1, s) - a_s(\mu_1), \tag{17}$$

where μ_1 is the optimal transmission rate in state (x, s) when there are $n+1$ steps to go. Let $\hat{f} = T_1(V_n)$ and $\mu^* \ge \mu_1$. Then under Conditions 2 and 3, $\hat{f}$ satisfies:

$$\hat{f}(x, r) - \hat{f}(x-1, r) - \frac{a_r(\mu^*)}{\nu_1} \le \hat{f}(x, s) - \hat{f}(x-1, s) - \frac{a_s(\mu^*)}{\nu_1}.$$

Lemma 7 *Suppose V_n is such that*

$$V_n(x, r) - V_n(x-1, r) - a_r(\mu_1) \le V_n(x, s) - V_n(x-1, s) - a_s(\mu_1), \tag{18}$$

where μ_1 is the optimal transmission rate in state (x, s) when there are $n+1$ steps to go. Let $\hat{f} = T_2(f)$. Then under Conditions 2 and 3, $\hat{f}$ satisfies the following:

$$\hat{f}(x, r) - \hat{f}(x-1, r) - \frac{a_r(\mu_1)}{\nu_2} \le \hat{f}(x, s) - \hat{f}(x-1, s) - \frac{a_s(\mu_1)}{\nu_2}.$$

Theorem 6 *Under Conditions 1-3, value function $V_n(x, s)$ satisfies property* **P-1**.

6 Infinite Horizon

Recall that we have $V_n(x, s) = \min_\pi E^\pi[\sum_{k=0}^{n-1} \beta^k c(x(k)) + u(p(k)) + v(q(k))]$. In order to study the properties of the optimal policy as $n \to \infty$ we define $V_\infty(x, s)$ to be $V_\infty(x, s) = \inf_\pi E^\pi[\lim_{n\to\infty} \sum_{k=0}^{n-1} \beta^k c(x(k)) + u(p(k)) + v(q(k))]$. The proof of following theorem can be found in [22], Chapter 5.4.

Theorem 7 *If $c(x) \ge 0$, then we have $V_\infty(x, s) = \lim_{n\to\infty} V_n(x, s)$.*

Using Theorem 7, we can extend the results proved for the finite horizon to the case of an infinite horizon as stated in the following corollary. Note that in this case we can limit ourselves to the set of stationary policies (see [23]).

Corollary 1 *Consider an infinite horizon.*

(a) Suppose for $0 \leq x < B$ *the optimal policy in state* (x, s) *and* $(x+1, s)$ *is* (λ_1, μ_1) *and* (λ_2, μ_2)*, respectively. Then we have* $\lambda_1 \geq \lambda_2$ *and* $\mu_1 \leq \mu_2$*.*

(b) Fix the transmission power. Let λ_1 *and* λ_2 *be the optimum arrival rates when the state is* (x, s) *and* (x, r)*, respectively. Then if* $r \prec s$*, we have* $\lambda_1 \geq \lambda_2$*.*

(c) Under Assumptions 1-3, value function $V_\infty(x, s)$ *satisfies property* **P-1***.*

7 Conclusion

In this paper we considered the problem of optimal power allocation and admission control for a single user with time varying fading channel. The objective is to minimize the total cost over a finite or an infinite horizon. We showed that an optimal policy has a number of monotonicity properties with respect to the queue size, the time horizon, and the channel fading state.

We would also like to see how the acceptance rate changes for different states if the output rate is controllable. In addition, results in Section 5.B may hold under weaker conditions than the ones stated here; this is part of our current research.

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