Opting out of Incentive Mechanisms: A Study of Security as a Non-Excludable Public Good

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***Abstract*—In a network of interdependent users, the expenditure in security measures by an entity affects not only herself, but also other users interacting with her. As a result, users' efforts towards security can be viewed as a public good, the optimal provision of which in a system of self-interested entities requires the design of appropriate incentives through an external mechanism. In this paper, we propose the notion of exit equilibrium to study users' voluntary participation in such incentive mechanisms. We show a fundamental result that, due to the non-excludable nature of security, there exists no reliable mechanism which can incentivize socially optimal investments, while ensuring voluntary participation and maintaining a weakly balanced budget, for all instances of security games. To further illustrate this result, we analyze the performance of two well-known incentive mechanisms, namely the Pivotal (VCG) and Externality mechanisms, in security games. We illustrate how, given a mechanism, stable coalitions of participating users may emerge, leading to an improved, yet sub-optimal security status. We further extend the impossibility result to risk-averse users, and discuss its implications on the viability of using cyber-insurance contracts to improve the state of cyber-security.**

***Index Terms*—Security games, Voluntary participation, Mechanism design, Exit equilibrium, Cyber-insurance**

I. Introduction

DESPITE advances in cyber-defense technologies, cyber-attacks on organizations across all sectors remain rampant; examples of high-profile attacks in 2015 include those on Anthem Insurance, Ashley Madison, and the US Government Office of Personnel Management [32]. From an economic perspective, it has been argued that this sub-optimal security status is due to lack of incentives by organizations to properly adopt existing software and best practices to improve their state of security [1], [9], [16]. The importance of incentivizing improved investments in security measures in this landscape is two-fold: while such expenditure helps entities protect their assets against security threats, by association it also benefits other interacting users, as an investing entity is less likely to be infected and used as a source of future attacks. In other words, a user's expenditure in security in an interconnected system provides *positive externalities* to other users. Consequently, the provision of security is studied as a problem of *public good* provision. Formally, a public good is defined as a non-rivalrous commodity; i.e., its use by a user does not reduce its availability to other users [21]. In particular, when users are rational, the strategic decision making process leading to security investment decisions is studied as a *security game*.

It is well-known that in the absence of regulation, the provision of public goods is in general inefficient [21]. To eliminate this inefficiency, the literature on security games has proposed regulating mechanisms for improving the level of cyber security to its socially desirable levels. Examples of existing mechanisms in the literature include introducing subsidies and fines based on security investments [10], [15], assessing rebates and penalties based on security outcomes [10], imposing a level of due care and establishing liability rules [15], [34], and premium-discriminating in cyber-insurance contracts [12], [19], [24]. Similar to the aforementioned mechanisms, our focus in the current paper is on the use of monetary payments/rewards to incentivize *socially optimal* security behavior, i.e., those minimizing the collective cost of security.

Tax-based mechanisms for incentivizing socially desirable behavior are generally required to satisfy two constraints, namely maintaining a *weakly balanced budget* and ensuring *voluntary participation* by all users. Weak budget balance is a requirement on the taxes collected/distributed by the mechanism; it ensures that the designer can redistribute users' payments as rewards, and ideally either retain a surplus as profit or at least not sustain losses. If the condition is not satisfied, the designer would need to spend (a potentially large amount of) external resources to run a proposed mechanism; such resources are not necessarily available to a designer. The voluntary participation requirement on the other hand ensures that each user prefers the outcome attained from participating in the mechanism, to what she could attain had she unilaterally opted out instead. The importance of ensuring voluntary participation lies in the fact that, in general, the mechanism designer either lacks the authority, or is reluctant, to mandate user cooperation. For example, cyber-insurance has been commonly considered as a method for incentivizing improved security decisions by users. Nevertheless, mandating the adoption of cyber-insurance is currently less than desirable: the market is not yet mature, there is insufficient actuarial data available to the insurers to properly insure against all losses, and mandating may not be "fair" as users face vastly different risks, and therefore some may not benefit from insurance. More importantly, mandating user cooperation must be accompanied with credible audits and sanctions for it to be effective, which incur additional costs. Ineffective or inaccurate audits can lead to erroneous sanctions which may reduce social welfare. Hence, it is of great interest to design

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mechanisms that naturally ensure voluntary participation by users.

A user's decision when contemplating participation in an incentive mechanism is dependent not only on the structure of the induced game, but also on the options available when staying out. The latter is what sets the study of incentive mechanisms for security games (as well as other *non-excludable* goods) apart from *excludable* public good problems.[1] To highlight this difference, note that due to the non-excludable nature of security, a user who opts out can continue benefiting from the spill-over of improved investments by those participating in the mechanism. Similarly, the security decisions of this outlier continue to affect the security outcome of the participating users. This is in contrast to excludable goods, in which an outlier neither benefits from, nor influences, the public good produced by the participating users. As a result, tax-based mechanisms, such as the Externality mechanism (e.g. [31]) and the Pivotal mechanism (e.g. [28]), can be designed so as to incentivize the socially optimal provision of an excludable good, guarantee voluntary participation, and maintain weak budget balance.

Given this distinction, to enable the design and study of similar incentive mechanisms for security games (and non-excludable public goods in general), we introduce the notion of *exit equilibrium*; this new notion captures the different nature of outside options available to users given non-excludability. In particular, at the exit equilibrium, a user unilaterally opts out of the proposed incentive mechanism, and best-responds to the remaining users who continue participating (these users are also best-responding to the outlier's action). A mechanism ensures voluntary participation if each user prefers the outcome attained in the socially optimal solution to that she can attain under her exit equilibrium.

In this paper, we first show a fundamental result that, with non-excludable goods (unlike excludable goods), there is no tax-based mechanism that can achieve social optimality, voluntary participation, and weak budget balance simultaneously in all instances of the game, i.e., without further information about the network structure (i.e., the graph of users' interdependencies) and users' preferences (i.e., the realization of their utility functions).

We then elaborate on this result by studying two well-known tax-based incentive mechanisms, namely the Pivotal and Externality mechanisms. We show that the Pivotal mechanism can *only* guarantee social optimality and voluntary participation, while the Externality mechanism can *only* ensure social optimality and budget balance. We examine the performance of these mechanisms in the specific class of *weighted effort* games. This interdependence model is of particular interest as it can capture varying degrees and possible asymmetries in the influence of users' security decisions on one another. We evaluate the effects of: (i) increasing users' self-dependence, and (ii) the presence of a single dominant user, on the performance of the two mechanisms. Through our analysis, we identify two classes of users: *main investors*, who receive a reward in return for improving their investment levels (from which they themselves and other users benefit), and *free-riders*, who pay a tax to benefit from a more secure environment. We highlight how users from *either* class may decide to opt out of the mechanism. In addition, our analysis leads to the identification of restricted problem environments in which, given the additional information on the network structure and user preferences, existing mechanisms can lead to a simultaneous guarantee on social optimality, weak budget balance, and voluntary participation. One such instance emerges when exit equilibria are *less beneficial* to the outliers (e.g., require a free-rider to become a main investor); another instance is when users coordinate to *exchange favors*, increasing their investments in return for increased effort by others.

We further extend our impossibility result by considering several variations of the main model. First, we generalize the notion of exit equilibrium, allowing multiple users to opt out of the mechanism. We show that stable coalitions of participating users may emerge under this extended exit equilibrium notion, leading to an improved (yet sub-optimal) security status for the system. However, we show that there still exist problem environments in which no mechanism can lead to the formation of any coalitions, so that the only possible outcome is the Nash equilibrium (i.e, the state of anarchy).

As a second extension, we show that the impossibility result continues to hold even if users are risk-averse (as opposed to risk-neutral users in the basic model). This finding has important implications in the design of cyber-insurance contracts that attempt to incentivize the adoption of better security practices by the insured, as proposed both in theory [18], [19], [27], as well as in practice, e.g., by the Department of Homeland Security as a method for incentivizing better cyber risk management [6].

Finally, we discuss the idea of bundling the role of the mechanism designer and a security vendor. The intent is to allow the vendor to leverage the additional revenue from the increased sale of security products to cover the deficit generated through the incentive mechanism. We show that it is possible to extend the space of positive instances in which social optimality, weak budget balance, and voluntary participation can be simultaneously achieved; nevertheless, the impossibility result continues to hold in general.

Main contributions: Our contributions in this paper are summarized as follows:

• We propose the notion of exit equilibrium to describe strategic users' outside options in mechanisms for incentivizing the adoption of optimal security practices. Our work hence formalizes the study of voluntary participation in security games, in which the assumption of compulsory compliance is commonly adopted.

• We show the fundamental impossibility of simultaneously guaranteeing social optimality, voluntary participation, and

[1] We note that according to an alternative definition, see e.g. [33, Chapter 23], all public goods are assumed non-excludable, while excludable non-rivalrous goods are referred to as *club goods*. However, it is more common in the literature, especially in the engineering application literature, to make the coarser distinction of public vs. private goods based on rivalry alone. Examples include Samuelson's seminal work on public goods [30], the definition of Mas-Colell, Whinston, and Green [21, Chapter 11.C], and [31] in this type of literature. We adopt this coarser categorization, and further distinguish based on excludability when needed.

weak budget balance in all instances of security games. By comparing this finding to existing possibility results (see Section V), our work highlights the crucial effect of users' outside options on the design of any mechanism for improving users' security behavior. Our insights are also applicable to other problems concerning the provision of non-excludable public goods over social and economic networks (Section V).

• We extend this impossibility result to risk-averse users. This finding highlights the difficulty of using cyber-insurance as an incentive for the adoption of better security practices, which has been widely proposed in both theory and practice as a tool for reaching a socially desirable cyber-security state.

• By finding restricted families of positive instances, we identify features of an environment that can affect the performance of incentive mechanisms for security games. We further analyze the role of a security vendor in extending the space of positive instances. These findings can guide the selection of a mechanism given additional information about the problem environment.

A preliminary version of this work has appeared in [25], where we first introduced the notion of exit equilibrium. In addition to a better exposition of our study of weighted effort games (specifically inclusion of the technical analysis in the appendix), we extend [25] in several directions. First, we generalize the notion of exit equilibria by allowing for deviations by multiple users, and studying the emergence of stable coalitions. Moreover, we study the role of a security vendor in extending the space of positive instances attainable in security games. Finally, we extend the impossibility result to risk-averse users.

Paper organization: The rest of this paper is organized as follows. We present the model for security games, as well as our general impossibility result, in Section II. Section III illustrates this result and identifies restricted families of positive instances by analyzing the Pivotal and Externality mechanisms, and applying them to weighted total effort models. We present several extensions of our impossibility result in Section IV, followed by related work in Section V. Section VI concludes the paper.

II. Security games

A. Model

Consider a network or system of N inter-connected and interdependent users. These users can be the operators of computers on a network, different divisions within a larger organization, or various sectors of an economy. Each user has an initial wealth W_i, and is subject to suffering a loss of $0 < L_i \leq W_i$ if her security is compromised. To decrease the probability of a successful attack, each user can choose a level of *effort* or *investment* in security $x_i \in \mathbb{R}_{\geq 0}$. User i incurs a cost of $h_i(x_i)$ when exerting effort x_i, where $h_i(\cdot) : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is referred to as the *cost function*.

We assume that the effort x_i not only protects the user herself, but further benefits other users in the system. This is because a better protected user generates positive externalities to (some or all) other users by decreasing the probability of contagious infections or attacks using inter-connections. Denote the vector of all users' efforts by $\mathbf{x} := \{x_1, \ldots, x_N\}$. The probability of a successful attack on user i, at a vector of efforts $\mathbf{x}$, is determined by the *risk function* $f_i(\mathbf{x}) : \mathbb{R}^N_{\geq 0} \to [0, 1]$.

The utility of user i is therefore given by:

$$u_i(\mathbf{x}) = W_i - L_i f_i(\mathbf{x}) - h_i(x_i) \ . \tag{1}$$

We refer to the full information, one shot game among the N rational users, choosing actions $x_i \geq 0$, with utility functions given in (1), as the *security game*. We make the following assumptions on the risk and cost functions.

Assumption 1: For all users i, $f_i(\cdot)$ is continuous, differentiable, decreasing (strictly decreasing in x_i), and strictly convex, in all arguments x_j.

Intuitively, the decreasing nature of this function in arguments $x_j, j \neq i$, models the positive externality of users' security decisions on one another. The convexity on the other hand implies that the effectiveness of security measures in preventing attacks (or the marginal benefit) is overall decreasing, as none of the available security measures can prevent all possible attacks.

Assumption 2: For all users i, $h_i(\cdot)$ is continuous, differentiable, strictly increasing, and convex.

Intuitively, the assumption of convexity entails that while implementing basic security measures is relatively cheap (e.g. limiting administrative privileges), protection may become increasingly costly as its effectiveness increases (e.g. implementing intrusion detection systems).

B. Improving investments: social optimality and exit equilibria

Security games have been extensively studied, see [16], [20] for surveys. The most commonly studied aspect of these games is their *Nash equilibrium*, i.e, the vector of investments in security that emerge when each user chooses an optimal level of effort accounting for her costs and benefits, while also best-responding to the investments of other users. Formally, at a Nash equilibrium $\tilde{\mathbf{x}}$, for all users i:

$$\tilde{x}_i = \arg\max_{x \geq 0} \ u_i(x, \tilde{\mathbf{x}}_{-i}) \ .$$

Nevertheless, it is well known in the economics literature that, as users do not account for the externality of their decisions on one another, this effort profile is sub-optimal from a social welfare perspective. Formally, a *socially optimal* solution profile $\mathbf{x}^*$ is given by:

$$\mathbf{x}^* = \arg\max_{\mathbf{x} \succeq 0} \ \sum_{j=1}^{N} u_j(\mathbf{x}) \ . \tag{2}$$

Existing literature has proposed several methods for inducing users to exert socially optimal efforts, by either mandating or incentivizing such actions [16]. In this paper, we are interested in incentive mechanisms that use appropriately designed monetary taxation/rewards to implement the socially optimal effort profile. Formally, the mechanism assesses a tax t_i to each participating user i; this tax may be positive, negative, or zero, indicating payments, rewards, or no transaction, respectively. We assume that users' utilities are quasi-linear, i.e., linear in the tax term (see Section IV-B for the extension of our results to risk-averse users). Therefore, the *total utility* of a user i when she is assigned a tax t_i is given by:

$$\mathrm{v}_i(\mathbf{x}, t_i) := u_i(\mathbf{x}) - t_i \ , \tag{3}$$

where the tax t_i can in general be a function of the security investment profile $\mathbf{x}$.

In addition to inducing socially optimal behavior, the mechanism designer selecting the tax terms t_i wishes to satisfy two goals. On one hand, the designer attempts to ensure that $\sum_j t_j \geq 0$.[2] This requirement, known as *weak budget balance (WBB)*, ensures that the implementation of the mechanism does not require spending (a potentially large amount of) external resources by the designer.

More importantly, for a successful implementation of an incentive mechanism, the designer has to ensure that users' *voluntary participation (VP)* constraints are satisfied, as the designer generally lacks the authority to enforce cooperation. Note the deliberate choice of the term voluntary participation as opposed to the commonly studied *individual rationality (IR)* constraint. Individual rationality often requires a user to prefer participation in a proposed mechanism to an outcome in which she opts out and receives no allocation of the (excludable) good at all. In contrast, we will define voluntary participation constraints as those ensuring that a user prefers implementing the socially optimal outcome while being assigned a tax t_i, to the outcome attained had she unilaterally opted out, but in which she could still benefit from the (non-excludable) public good. Such distinction is important with non-excludable public goods such as security, as a user can still benefit from the externalities generated by the participating users, and also potentially continue contributing to the production of the public good, even when opting out herself. This is in contrast to games with excludable public goods, where voluntary participation and individual rationality become equivalent. In other words, voluntary participation for excludable goods requires a user to prefer the outcome attained by participation to opting out and receiving a zero allocation of the good; in the latter case, the user's utility is a constant independent of the final outcome of the game, and often normalized to zero. This is however not the case for non-excludable goods.

Therefore, to study users' voluntary participation for non-excludable goods, we propose the concept of *exit equilibrium (EE)*: consider an *outlier*, who is contemplating unilaterally opting out of a proposed incentive mechanism. By the assumption of full information, the remaining participating users, who are choosing a welfare maximizing solution for their $(N-1)$-user system, will have the ability to predict the best-response of the outlier to their collective action, and thus choose their investments accordingly. As a result, the equilibrium investment profile when user i opts out is itself a Nash equilibrium (for the game between this outlier and the $N-1$ participating users). Formally, when user i is the outlier, the exit equilibrium $\hat{\mathbf{x}}^{\mathbf{i}}$ is given by:

$$\begin{aligned}\hat{\mathbf{x}}^i_{-i} &= \arg\max_{\mathbf{x}_{-i}\succeq 0} \ \sum_{j\neq i} u_j(\mathbf{x}_{-i}, \hat{x}^i_i) \ ,\\ \hat{x}^i_i &= \arg\max_{x_i\geq 0} \quad u_i(\hat{\mathbf{x}}^i_{-i}, x_i) \ .\end{aligned} \tag{4}$$

We would like to highlight two important considerations in studying exit equilibria. First, note that the study of exit equilibria to understand users' unilateral deviations from socially optimal investment profiles is similar to the study of users' deviation from Nash equilibria: neither concept precludes the possibility that coalitions of deviating users can break the equilibrium. Nevertheless, it is necessary (although indeed not sufficient) for a mechanism to be resilient against unilateral exit strategies in order to incentivize cooperation. We therefore only focus on unilateral exit strategies in this paper. In fact, as shown in Section II-C, there exists no incentive mechanism that can incentivize social optimal efforts, while guaranteeing weak budget balance and voluntary participation, even against unilateral deviations, much less against higher order coalitions. Secondly, we also note that the proposed exit equilibrium is *only* an equilibrium under the assumption that the $N-1$ remaining users are cooperating in the mechanism; it is itself not necessarily stable as any of the remaining users may also prefer to opt out. In Section IV-A, we extend the definition of exit equilibrium to allow for multiple outliers, and present instances in which the resulting exit equilibrium yields a stable coalition of participating users, as well as instances in which no stable exit equilibrium (other than the degenerate case of Nash equilibrium) exists.

C. An impossibility result

In this section, we prove the following result.

Proposition 1: There exists no tax-based incentive mechanism which can implement the socially optimal solution, while guaranteeing weak budget balance and voluntary participation simultaneously, in all instances of security games.

Proof: The proof is through a counter-example: we consider security games with a family of risk functions that approximate the *weakest link* risks $f_i(\mathbf{x}) = \exp(-\min_j x_j)$ [16], [34].[3] In particular, we use the approximation $\min_j x_j \approx -\frac{1}{\gamma}\log\sum_j \exp(-\gamma x_j)$, where the accuracy of the approximation is increasing in the constant $\gamma > 0$. User i's utility function is given by:

$$u_i(\mathbf{x}) = W - L(\sum_{j=1}^N \exp(-\gamma x_j))^{1/\gamma} - cx_i \ ,$$

where investment cost functions $h_i(\cdot)$ are assumed to be linear, and users are homogeneous, with the same initial wealth W, loss L, and unit investment cost c.

In this game, the socially optimal investment profile $\mathbf{x}^*$ can be found by solving the first order conditions of (2), which are given by:

$$N\exp(-\gamma x^*_i)(\sum_{j=1}^N \exp(-\gamma x^*_j))^{\frac{1}{\gamma}-1} = \frac{c}{L} \ , \forall i.$$

[2] At least at equilibrium, but ideally, both on and off equilibrium.

[3] We refer the interested reader to [25] for an alternative counter-example in which we fix the network structure to a star topology.

By symmetry, all users will be exerting the same socially optimal level of effort:

$$x_i^* = \frac{1}{\gamma} \ln \frac{N}{(\frac{c}{L})^\gamma} \ , \forall i \ .$$

Next, assume a user i unilaterally opts out of the mechanism, while the remaining users continue participating. The exit equilibrium profile $\hat{\mathbf{x}}^i$ can be determined using the first order conditions on (4), leading to:

$$(N-1)\exp(-\gamma \hat{x}_j^i)(\sum_{k\neq i} \exp(-\gamma \hat{x}_k^i) + \exp(-\gamma \hat{x}_i^i))^{\frac{1}{\gamma}-1} = \frac{c}{L} \ ,$$
$$\exp(-\gamma \hat{x}_i^i)(\sum_{k\neq i} \exp(-\gamma \hat{x}_k^i) + \exp(-\gamma \hat{x}_i^i))^{\frac{1}{\gamma}-1} = \frac{c}{L} \ .$$

Solving the above, we get:

$$\hat{x}_i^i = \frac{1}{\gamma} \ln \frac{2^{1-\gamma}}{(\frac{c}{L})^\gamma}$$
$$\hat{x}_j^i = \frac{1}{\gamma} \ln \frac{(N-1)2^{1-\gamma}}{(\frac{c}{L})^\gamma} \ , \forall j \neq i \ .$$

Assume some tax-based incentive mechanism $\mathcal{M}$ is proposed in this game. We can use the socially optimal investment profile and the exit equilibria to analyze users' participation incentives in $\mathcal{M}$, as well as the budget balance conditions. Denote by t_i^* the tax assigned to user i by $\mathcal{M}$.

A user i's utilities when participating and staying out are given by:

$$\begin{aligned} v_i(\mathbf{x}^*, t_i^*) &= W - L(N\exp(-\gamma x_i^*))^{\frac{1}{\gamma}} - c x_i^* - t_i^* \\ &= W - c(1 + x_i^*) - t_i^* \ . \\ u_i(\hat{\mathbf{x}}^i) &= W - L(\exp(-\gamma \hat{x}_i^i) + (N-1)\exp(-\gamma \hat{x}_j^i))^{\frac{1}{\gamma}} - c\hat{x}_i^i \\ &= W - c(2 + \hat{x}_i^i) \ . \end{aligned}$$

The voluntary participation condition for a user i will hold if and only if $v_i(\mathbf{x}^*, t_i^*) \geq u_i(\hat{\mathbf{x}}^i)$, which reduces to:

$$c(1 + x_i^*) + t_i^* \leq c(2 + \hat{x}_i^i) \Leftrightarrow t_i^* \leq c(1 + \frac{1}{\gamma} \ln \frac{2^{1-\gamma}}{N}) \ . \quad (5)$$

On the other hand, for weak budget balance to hold, we need $\sum_i t_i^* \geq 0$. Nevertheless, by (5), we have:

$$\sum_i t_i^* \leq cN(1 + \frac{1}{\gamma} \ln \frac{2^{1-\gamma}}{N}) \ .$$

It is easy to see that given γ and for any $N > e^\gamma 2^{1-\gamma}$, the above sum will always be negative, indicating a budget deficit for a general mechanism $\mathcal{M}$, regardless of how taxes are determined. ■

Intuition: note that the lack of any mechanism $\mathcal{M}$ occurs only when there is a sufficient number of players (given a finite γ). This is because with a sufficient number of participating users, the externality available to an outlier is high enough to dissuade her from participating. It is also interesting to point out that outside this region (i.e., $N \leq e^\gamma 2^{1-\gamma}$, the number of users sufficiently small), we in fact have a positive instance, in which the Externality mechanism introduced in Section III-A can guarantee social optimality, budget balance, and voluntary participation.

We close this section by noting that our impossibility result on a simultaneous guarantee of social optimality, voluntary participation, and weak budget balance, is demonstrated through a counter-example. In other words, we have shown that without prior knowledge of the graph structure or users' preferences, it is not possible for a designer to propose a *reliable* mechanism; that is, one which can promise to achieve social optimality, voluntary participation, and weak budget balance, regardless of the realizations of utilities. This should be contrasted with environments with the same utility functions and information constraints, but excludable public goods, in which there exist reliable mechanisms to guarantee all three properties simultaneously; see Section V. Nevertheless, as also suggested by the above counter-example, it may still be possible to design reliable mechanisms for non-excludable public goods under a restricted problem space. With this in mind, we next analyze the class of weighted effort models, and aim to identify such positive instances, as well as the intuition behind the existence of each instance.

III. A TALE OF TWO MECHANISMS: ANALYSIS OF EXISTING INCENTIVE SCHEMES

In light of the impossibility result of Section II-C on a simultaneous guarantee of social optimality, weak budget balance, and voluntary participation, in this section we set out to better understand the performance of existing incentive mechanisms in security games, and identify features of the problem environment that affect the properties attainable through given mechanisms. We further find positive instances (in a restricted utility space) for which these existing mechanisms can guarantee all three properties. Specifically, we analyze the performance of the Pivotal and Externality mechanisms within the restricted class of weighted effort security games.

A. The Pivotal and Externality mechanisms

Throughout this section, we will be studying the performance of two well-known tax-based incentive mechanisms, namely the Pivotal (VCG) and Externality mechanisms. We chose these mechanisms as they have been shown to simultaneously guarantee the achievement of social optimality, weak budget balance, and voluntary participation, in games of provision of *excludable* public goods. Our goal is hence to illustrate their inefficiencies in the provision of *non-excludable* public goods.

1) The Pivotal Mechanism: Groves mechanisms [21], [28], also commonly known as Vickery-Clarke-Groves (VCG) mechanisms, refer to a family of mechanisms in which, through the appropriate design of taxes for users with quasi-linear utilities, a mechanism designer can incentivize users to reveal their true preferences in dominant strategies, thus implementing the socially optimal solution. One particular instance of these mechanisms, the *Pivotal* mechanism, has been shown to further satisfy the participation constraints and achieve weak budget balance in many private and public good games [5], [11], [28]; however, as shown below this is not necessarily the case in security games. The taxes in the Pivotal mechanism for security games are given by:

$$t_i^P = \sum_{j\neq i} u_j(\hat{\mathbf{x}}_{-i}^i, \hat{x}_i^i) - \sum_{j\neq i} u_j(\mathbf{x}_{-i}^*, x_i^*) \ , \tag{6}$$

where $u_i(\mathbf{x})$ is user i's utility function, $\mathbf{x}^* = (\mathbf{x}_{-i}^*, x_i^*)$ is the socially optimal solution, and $\hat{\mathbf{x}}^i = (\hat{\mathbf{x}}_{-i}^i, \hat{x}_i^i)$ is the exit equilibrium under user i's unilateral deviation. In the online appendix [23], following existing proofs in [28], we show that the taxes in (6) incentivize users' voluntary participation and attain the socially optimal solution. However, in light of the negative result of Section II-C, these taxes may generate a budget deficit for the designer in security games.

2) The Externality Mechanism: we next introduce the Externality mechanism adapted from the work of Hurwicz in [13]. A main design goal of this mechanism is to guarantee a complete redistribution of taxes, i.e., strong budget balance. This mechanism has been adapted in [31], where it is shown to achieve social optimality, guarantee participation, and maintain a balanced budget, in allocation of power in cellular networks (an excludable public good). However, this is again not the case in security games. The tax terms t_i^E at the equilibrium of the Externality mechanism in security games are given by:

$$t_i^E(\mathbf{x}^*) = -\sum_{j=1}^{N} x_j^* L_i \frac{\partial f_i}{\partial x_j}(\mathbf{x}^*) - x_i^* \frac{\partial h_i}{\partial x_i}(x_i^*) \ . \tag{7}$$

The interpretation is that by implementing this mechanism, each user i will be financing part of user $j \neq i$'s reimbursement. According to (7), this amount is proportional to the positive externality of j's investment on user i's utility. In the online appendix [23], following existing proofs in [13], we show that the taxes in (7) attain the socially optimal solution and lead to a (strongly) balanced budget. However, in light of the negative result of Section II-C, they may fail to satisfy users' voluntary participation constraints in security games.

B. Choice of the risk function

The gap between the Nash equilibrium and the socially optimal investment profile of a security game, as well as users' participation incentives and possible budget imbalances, are dependent on the specifics of users' utility functions defined in (1). In particular, the risk function $f_i(\cdot)$ can model the types of connection and extent of interdependencies among users. Examples of existing security interdependence models include the *total effort*, *weakest link*, and *best shot* models considered in the seminal work of Varian [34], as well as the *weakest target* models studied in [9], the *effective investment* and *bad traffic* models in [14], and the *linear influence network* games in [22]. Here, we take the special case of *weighted effort* models, with exponential risks and linear investment cost functions. Formally, the total utility of user i is given by:

$$\mathrm{v}_i(\mathbf{x}, t_i) = W_i - L_i \exp(-\sum_{j=1}^{N} a_{ij} x_j) - c_i x_i - t_i \ . \tag{8}$$

For simplicity, we assume $W_i = W$, $L_i = L = 1$, and $c_i = c$, for all i. The coefficients $a_{ij} \geq 0$ determine the dependence of user i's risk on user j's action. Consequently, user i's risk is dependent on a weighted sum of all users' efforts. We define the *dependence matrix* containing these coefficients as:

$$A := \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1N} \\ a_{21} & a_{22} & \cdots & a_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ a_{N1} & a_{N2} & \cdots & a_{NN} \end{pmatrix} .$$

We isolate the effects of self-dependence and interdependence by focusing on the following two sub-classes of this model:

1. Varying users' self-dependence:

$$A = \begin{pmatrix} a & 1 & \cdots & 1 \\ 1 & a & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \cdots & a \end{pmatrix} , \tag{9}$$

for both $a > 1$ and $a < 1$.

2. Making all users increasingly dependent on a single user:

$$A = \begin{pmatrix} a & 1 & \cdots & 1 \\ a & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ a & 1 & \cdots & 1 \end{pmatrix} , \tag{10}$$

for $a > 1$.

C. Effects of self-dependence

Consider a network of N users, with the dependence matrix given by (9), and total utility functions: [4]

$$\mathrm{v}_i(\mathbf{x}, t_i) = W - \exp(-a x_i - \sum_{j\neq i} x_j) - c x_i - t_i \ .$$

The following theorem characterizes the possible exit equilibria of this game under different parameter conditions, as well as whether the voluntary participation conditions are satisfied under the Externality mechanism, and whether the Pivotal mechanism can operate without a budget deficit. The results are summarized in Table I.

Theorem 1: For the weighted effort security game described by the dependence matrix (9):

(i) There exist five possible exit equilibria (summarized in Table I) depending on the values of the number of players N, self-dependence a, and cost of investment c. In particular, note the multiplicity of exit equilibria under the parameter conditions of the last row in Table I: either γ, ω, or ζ may be realized in such instances.

(ii) Either of the Externality or Pivotal mechanisms can guarantee social optimality, voluntary participation, and weak budget balance, if and only if the realized exit equilibrium is ω or ζ of Table I.

The proof, including formal derivations of the exit equilibria, as well as the analysis of the Pivotal and Externality mechanisms under each exit equilibrium, is presented in Appendix A. Based on this analysis, we make the following observations:

Coordinating on the least beneficial exit equilibrium for the outlier: from Table I, we observe that if selection

[4] We assume $c < a$, so as to ensure the existence of non-zero equilibria, i.e., at least one user exerts non-zero effort at any equilibrium of the game.

TABLE I
CAN SO, VP, AND WBB, HOLD SIMULTANEOUSLY? - EFFECT OF SELF-DEPENDENCE

Parameter Conditions	Exit Equilibrium	VP in Externality	WBB in Pivotal
$a > 1$, with N and c s.t. $(1 + \frac{N-2}{a})^{N-1} > (\frac{a}{c})^{a-1}$	**CASE** $\alpha : \hat{x}_i^i = 0, \hat{x}_j^i > 0$	No	No
$a > 1$, with N and c s.t. $(1 + \frac{N-2}{a})^{N-1} < (\frac{a}{c})^{a-1}$	**CASE** $\beta : \hat{x}_i^i > 0, \hat{x}_j^i > 0$	No	No
$a < 1$, with N and c s.t. $(1 + \frac{N-2}{a})^{a} > (\frac{a}{c})^{1-a}$	**CASE** $\gamma : \hat{x}_i^i = 0, \hat{x}_j^i > 0$	No	No
$a < 1$, with N and c s.t. $(1 + \frac{N-2}{a})^{a} < (\frac{a}{c})^{1-a}$	**CASE** $\gamma : \hat{x}_i^i = 0, \hat{x}_j^i > 0$ **CASE** $\omega : \hat{x}_i^i > 0, \hat{x}_j^i = 0$ **CASE** $\zeta : \hat{x}_i^i > 0, \hat{x}_j^i > 0$	Yes (iff ω or ζ is realized)	Yes (iff ω or ζ is realized)

among multiple exit equilibria is possible, the Pivotal and Externality mechanism can simultaneously guarantee social optimality, voluntary participation, and weak budget balance under the less beneficial exit equilibrium. More specifically, we observe that for the parameter conditions in the last row of Table I, there are three possible exit equilibria, each of them leading to a different outcome for the outlier. In Case γ, the outlier becomes a free-rider, while the participants exert effort. In Case ω, an outlier becomes the main investor, while all participants free-ride. In Case ζ, the outlier (as well as the participants) continues exerting effort. We refer to equilibria such as those in ω and ζ as less beneficial equilibria: these require a free-rider to become an investor when leaving the mechanism, or require an investor to continue exerting effort when out (although possibly at a lower level). From Table I, we observe that the Pivotal and Externality mechanisms can simultaneously guarantee all desired properties if the exit equilibrium is one of less beneficial Cases ω or ζ. In other words, the less beneficial equilibria decrease the interest of users to opt out to their exit equilibrium, leading to positive instances for the incentive mechanisms.

An exchange of favors: it is also interesting to highlight another feature of the positive instances of Cases ω and ζ of Table I: as users are mainly dependent on others' investments under these parameter conditions ($a < 1$), the incentive mechanisms can facilitate coordination among them, so that each increase their investments in return for improved investments by others.

D. *Effects of a dominant user*

Consider a collection of N users, with dependence matrix given by (10), and total utility functions:

$$\mathrm{v}_i(\mathbf{x}, t_i) = W - \exp(-ax_1 - \sum_{j=2}^{N} x_j) - cx_i - t_i \ ,$$

where $c < 1 < a$, and user 1 is the dominant user. In Appendix B, we show that in a socially optimal profile, as well as for exit equilibria of non-dominant users, only user 1 will be exerting effort. When the dominant user opts out of the mechanism, however, she may become either a main investor or free-rider, depending on the problem parameters.

The following theorem characterizes the possible exit equilibria and parameter conditions for which each is possible, as well as the performance of both mechanisms. The results are summarized in Table II.

Theorem 2: For the weighted effort security game described by the dependence matrix (10):

(i) There exist two possible exit equilibria (summarized in Table II) depending on the values of the number of players N and dependence on the dominant user a.
(ii) None of the Externality or Pivotal mechanisms can guarantee social optimality, voluntary participation, and weak budget balance, regardless of the exit equilibrium.

The proof is presented in Appendix B. We make the following observation based on this analysis:

Either main investors or free-riders may opt out: Through our analysis, we show that the voluntary participation conditions of non-dominant users in the Externality mechanism are never satisfied: these users can avoid paying taxes to the dominant user, while others pay her to increase her investment. More interesting however, is the fact that the voluntary participation conditions for the main investor may also fail to hold. This is because when user 1's exit equilibrium does not require her to exert effort, and the externality generated by her is small (i.e.; small a), the collected taxes are not enough to persuade this dominant user to increase her effort level. Furthermore, we observe that although the Pivotal mechanism needs to give out a smaller reward to the dominant user as a increases, it still fails to avoid a deficit due to the small willingness of free-riders to pay the taxes required to cover this reward.

IV. DISCUSSION

A. *Extending exit equilibria: finding stable coalitions*

As mentioned in Section II, the definition of exit equilibrium considers unilateral deviations of users from mechanisms incentivizing socially optimal efforts, assuming all remaining users continue participating in the mechanism; the motivation is that it is a necessary (although not sufficient) condition for the users not to have incentives to unilaterally deviate if the socially optimal outcome is to be incentivized while maintaining voluntary participation and weak budget balance. In this section, we extend the definition of exit equilibrium, allowing E users to exit the mechanism while the remaining $N-E$ users continue participating. Specifically, when a subset of users $\mathcal{E} \subset \mathcal{N}$ exit the proposed incentive mechanism, the resulting exit equilibrium, $\hat{\mathbf{x}}^{\mathcal{E}} := (\hat{\mathbf{x}}^{\mathcal{E}}_{\mathcal{E}}, \hat{\mathbf{x}}^{\mathcal{E}}_{\mathcal{N}-\mathcal{E}})$, is given by:

TABLE II
CAN SO, VP, AND WBB, HOLD SIMULTANEOUSLY? - SINGLE DOMINANT USER

Parameter Conditions	Exit Equilibrium	VP in Externality	WBB in Pivotal
$a < N-1$	**CASE** α : $\hat{x}_1^1 = 0, \hat{x}_j^1 > 0, \forall j \neq 1,\ \hat{x}_1^i > 0, \hat{x}_j^i = 0, \forall i, j \neq 1$	No	No
$a > N-1$	**CASE** β : $\hat{x}_1^1 > 0, \hat{x}_j^1 = 0, \forall j \neq 1,\ \hat{x}_1^i > 0, \hat{x}_j^i = 0, \forall i, j \neq 1$	No	No

$$\hat{\mathbf{x}}^{\mathcal{E}}_{\mathcal{N}-\mathcal{E}} = \arg\max_{\mathbf{x}\succeq 0} \sum_{j\notin\mathcal{E}} u_j(\mathbf{x}, \hat{\mathbf{x}}^{\mathcal{E}}_{\mathcal{E}}) \ ,$$
$$\hat{x}^{\mathcal{E}}_k = \arg\max_{x\geq 0} u_k(x, \hat{\mathbf{x}}^{\mathcal{E}}_{\mathcal{E}-\{k\}}, \hat{\mathbf{x}}^{\mathcal{E}}_{\mathcal{N}-\mathcal{E}}) \ , \forall k \in \mathcal{E}. \quad (11)$$

1) A stable coalition: Through an illustrative example, we identify stable coalitions under this extended definition of exit equilibrium. In particular, similar to the counter-example in Section II-C, we consider (approximations) of the same weakest link risk function used earlier, i.e, users' utility functions are given by:

$$u_i(\mathbf{x}) = W - L(\sum_{j=1}^{N} \exp(-\gamma x_j))^{1/\gamma} - cx_i \ .$$

First, using the first order conditions on (2), and by symmetry, all users will be exerting the same socially optimal level of effort:

$$x_i^* = \frac{1}{\gamma}\ln\frac{N}{(\frac{c}{L})^\gamma} \ , \forall i \ .$$

Consider the subset of E users $\mathcal{E} \subset \mathcal{N}$ who exit the proposed incentive mechanism. The exit equilibrium under the deviation of these users can be derived using the first order conditions on (11), given by:

$$(N-E)\exp(-\gamma\hat{x}^{\mathcal{E}}_k)(\sum_{j\notin\mathcal{E}}\exp(-\gamma\hat{x}^{\mathcal{E}}_j) + \sum_{j\in\mathcal{E}}\exp(-\gamma\hat{x}^{\mathcal{E}}_j))^{\frac{1}{\gamma}-1} = \frac{c}{L} \ , \ \forall k \notin \mathcal{E} \ ,$$
$$\exp(-\gamma\hat{x}^{\mathcal{E}}_k)(\sum_{j\notin\mathcal{E}}\exp(-\gamma\hat{x}^{\mathcal{E}}_j) + \sum_{j\in\mathcal{E}}\exp(-\gamma\hat{x}^{\mathcal{E}}_j))^{\frac{1}{\gamma}-1} = \frac{c}{L} \ , \ \forall k \in \mathcal{E} \ .$$

Solving the above, the exit equilibrium $\hat{\mathbf{x}}^{\mathcal{E}}$ is given by:

$$\hat{x}^{\mathcal{E}}_k = \frac{1}{\gamma}\ln\frac{(N-E)(E+1)^{1-\gamma}}{(\frac{c}{L})^\gamma}, \qquad \forall k \notin \mathcal{E} \ ,$$
$$\hat{x}^{\mathcal{E}}_k = \frac{1}{\gamma}\ln\frac{(E+1)^{1-\gamma}}{(\frac{c}{L})^\gamma}, \qquad \forall k \in \mathcal{E} \ .$$

Now, assume a user $i \notin \mathcal{E}$ is considering exiting the mechanism as well. Again, using the first conditions on (11), and following similar steps as the above, this time for the subset of users $\mathcal{E}\cup\{i\}$ exiting the mechanism, the exit equilibrium $\hat{\mathbf{x}}^{\mathcal{E}\cup\{i\}}$ will be given by:

$$\hat{x}^{\mathcal{E}\cup\{i\}}_k = \frac{1}{\gamma}\ln\frac{(N-E-1)(E+2)^{1-\gamma}}{(\frac{c}{L})^\gamma}, \quad \forall k \notin \mathcal{E}\cup\{i\} \ ,$$
$$\hat{x}^{\mathcal{E}\cup\{i\}}_k = \frac{1}{\gamma}\ln\frac{(E+2)^{1-\gamma}}{(\frac{c}{L})^\gamma}, \qquad \forall k \in \mathcal{E}\cup\{i\} \ .$$

For a stable coalition of $N-E$ users to form, we need to find the smallest set $\mathcal{E}$, such that $u_i(\hat{\mathbf{x}}^{\mathcal{E}\cup\{i\}}) \leq v_i(\hat{\mathbf{x}}^{\mathcal{E}}, t_i)$, where t_i is the tax assigned to a participating user i in some proposed incentive mechanism. Substituting for the exit equilibria derived above, user i's utilities when participating and staying out are given by:

$$v_i(\hat{\mathbf{x}}^{\mathcal{E}}, t_i) = W - c(E+1+\hat{x}^{\mathcal{E}}_i) - t_i \ ,$$
$$u_i(\hat{\mathbf{x}}^{\mathcal{E}\cup\{i\}}) = W - c(E+2+\hat{x}^{\mathcal{E}\cup\{i\}}) \ .$$

The voluntary participation condition therefore simplifies to:

$$t_i \leq c\left(1 + \frac{1}{\gamma}\ln\frac{1}{N-E}(\frac{E+2}{E+1})^{1-\gamma}\right) \ , \forall i \notin \mathcal{E} \ .$$

Let E^* be the smallest number for which $N \leq E + e^\gamma(\frac{E+2}{E+1})^{1-\gamma}$ holds (note that we always have $E^* < N$). Given E^*, the Externality mechanism of Section III-A can lead to a stable coalition of size $M = N - E^*$ implementing the socially optimal solution in their M-user system, with the remainder E^* users not participating.[5]

It is also interesting to mention that the condition attained for having a stable coalition of all users (by setting $E=0$) coincides with the positive instance of Section II-C: if $N < e^\gamma 2^{1-\gamma}$, the Externality mechanism can achieve the socially optimal solution, while guaranteeing voluntary participation and budget balance.

2) A negative example:: We close this section by noting that a stable coalition does not necessarily emerge in all problem environments. In particular, consider the following family of total effort games:

$$u_i(\mathbf{x}) = W - \exp(-\sum_{j=1}^{N} x_j) - c_i x_i \ .$$

Users are indexed such that $c_1 < c_2 < \ldots < c_N$. Also, assume $c_1 < \frac{c_2}{N-1}$. Consider a set of E users, $\mathcal{E}\subset\mathcal{N}$ exiting the mechanism. The resulting exit equilibrium, $\hat{\mathbf{x}}^{\mathcal{E}}$, depends primarily on user 1's participation choice:

$$\text{If user } 1 \in \mathcal{E}: \quad \hat{x}^{\mathcal{E}}_1 = \ln\frac{1}{c_1}, \qquad \hat{x}^{\mathcal{E}}_1 = 0, \ \forall k \neq 1 \ ,$$
$$\text{If user } 1 \notin \mathcal{E}: \quad \hat{x}^{\mathcal{E}}_1 = \ln\frac{N-E}{c_1}, \quad \hat{x}^{\mathcal{E}}_1 = 0, \ \forall k \neq 1 \ .$$

First, note that once user 1 has already opted out, participation or opting out yields equivalent utilities for users $k \neq 1$. Therefore, any possible coalition has to include user 1.

Next, for any user to remain in a stable coalition $\mathcal{N}-\mathcal{E}$, we need to have $u_i(\hat{\mathbf{x}}^{\mathcal{E}\cup\{i\}}) \leq u_i(\hat{\mathbf{x}}^{\mathcal{E}}) - t_i$, where t_i is the tax

[5] The taxes assigned to the M participating users can be found using (7), and will in fact be zero at equilibrium (due to symmetry of the players). Also, the resulting effort profile will be an improved, yet sub-optimal solution for the N-user system.

assigned by the incentive mechanism to a participating user i. These conditions simplify to:

$$t_1 \leq c_1(1 - \tfrac{1}{N-E} - \ln(N-E)),$$
$$t_k \leq \tfrac{c_1}{(N-E)(N-E-1)}, \ \forall k \notin \mathcal{E}, k \neq 1.$$

For the mechanism to maintain weak budget balance, we need:

$$\sum_{i \notin \mathcal{E}} t_i \leq c_1(1 - \ln(N-E)) \ .$$

However, the above is always negative for $N \geq 3$, indicating that there exists no mechanism that can sustain such coalitions while maintaing weak budget balance. Also, note that the outcome for a coalition with $N = 2$ (user 1 and some user $k \neq 1$) will be equivalent to the Nash equilibrium. We therefore conclude that, regardless of the design of the mechanism, there exists no stable coalition in this family of total effort games.

B. Risk-averse users and cyber-insurance contracts

In this section, we extend the impossibility result of Section II-C to risk-averse users. Considering risk-averse users is of particular interest in studying the design of cyber-insurance contracts. Cyber-insurance has been widely proposed as a method for incentivizing the adoption of better security practices by users through strategies such as premium discrimination, see e.g. [12], [19]. Following the majority of the existing literature, we consider a monopolist cyber-insurer (e.g. the government). We assume the insurer is interested in improving the state of cyber-security to its socially optimal levels (e.g. as required or directed by the government) through appropriately designed insurance contracts.[6] The weak budget balance assumption ensures positive profits for this insurer, while voluntary participation models voluntary purchase of insurance from this provider.

1) CRRA functions for modeling risk aversion: Consider N interdependent users, with initial wealth W_i and loss L_i, each choosing an effort x_i. The cost of investment x_i is given by $h_i(x_i)$, with the probability of a successful attack given by $f_i(\mathbf{x})$. The utility function of user i is therefore:

$$u_i(\mathbf{x}) = f_i(\mathbf{x})U_i(W_i - L_i) + (1 - f_i(\mathbf{x}))U_i(W_i) - h_i(x_i) \ .$$

In general, for risk-averse users, the function $U_i(\cdot)$ is a concave function. Here, we model risk-aversion using CRRA (constant relative risk aversion) utility functions [21]:

$$U(c) = \begin{cases} \frac{1}{1-\theta}c^{1-\theta}, & \text{for } \theta > 0, \theta \neq 1 \ , \\ \ln c, & \text{for } \theta = 1 \ . \end{cases}$$

Assume users have the option of purchasing insurance contracts, specifying a *premium* ρ_i and an *indemnification payment* (coverage) level I_i. When insurance is purchased, the utility of user i will be given by:

$$\begin{aligned} v_i(\mathbf{x}, \rho_i, I_i) =& f_i(\mathbf{x})U_i(W_i - \rho_i - L_i + I_i) \\ &+ (1 - f_i(\mathbf{x}))U_i(W_i - \rho_i) - h_i(x_i) \ . \end{aligned}$$

[6] The assumption of a monopolist cyber-insurer is in fact indispensable for this analysis. This is because the competition among multiple cyber-insurers will inevitably lead to contracts that incentivize sub-optimal investments by the users [27].

2) A negative result: Similar to Section II-C, we again consider the (approximations) of weakest link risk functions $f_i(\mathbf{x}) = (\sum_{j=1}^N \exp(-\gamma x_j))^{1/\gamma}$, and set $W_i = W$, $L_i = L$, $h_i(x_i) = cx_i$, for all i.

In this game, the socially optimal investment profile $\mathbf{x}^*$ is determined using (2), leading to:

$$N \exp(-\gamma x_i^*)(\sum_{j=1}^N \exp(-\gamma x_j^*))^{\frac{1}{\gamma}-1} = \tfrac{c}{U(W)-U(W-L)} \ .$$

By symmetry, all users will be exerting the same socially optimal level of effort:

$$x_i^* = \ln \frac{N^{1/\gamma}(U(W) - U(W-L))}{c} \ , \forall i \ .$$

The utility of users under this outcome, while also purchasing the optimal insurance contract, is given by:

$$\begin{aligned} u_i(\mathbf{x}^*) =& \tfrac{-c}{U(W)-U(W-L)}(U(W-\rho) - U(W-L-\rho+I)) \\ &+ U(W-\rho) - c\ln\frac{N^{1/\gamma}(U(W) - U(W-L))}{c} \ . \end{aligned}$$

The exit equilibrium profile $\hat{\mathbf{x}}^i$ can be determined using the first order conditions on (4), leading to:

$$\begin{aligned} &(N-1)\exp(-\gamma\hat{x}_j^i)(\sum_{k\neq i}\exp(-\gamma\hat{x}_k^i) + \exp(-\gamma\hat{x}_i^i))^{\frac{1}{\gamma}-1} \\ &\qquad = \tfrac{c}{U(W)-U(W-L)} \ , \\ &\exp(-\gamma\hat{x}_i^i)(\sum_{k\neq i}\exp(-\gamma\hat{x}_k^i) + \exp(-\gamma\hat{x}_i^i))^{\frac{1}{\gamma}-1} \\ &\qquad = \tfrac{c}{U(W)-U(W-L)} \ . \end{aligned}$$

Solving the above, we get:

$$\begin{aligned} \hat{x}_i^i &= \ln\frac{2^{1/\gamma-1}(U(W) - U(W-L))}{c} \\ \hat{x}_j^i &= \ln\frac{(N-1)^{1/\gamma}2^{1/\gamma-1}(U(W) - U(W-L))}{c} \ , \forall j \neq i \ . \end{aligned}$$

The utility of the outlier i under the exit equilibrium is given by:

$$u_i(\hat{\mathbf{x}}^{\mathbf{i}}) = -2c + U(W) - c\ln\frac{2^{1/\gamma-1}(U(W) - U(W-L))}{c} \ .$$

We now proceed to the analysis of insurance contracts. First note that the insurer has the following total profit:

$$\begin{aligned} P^* :=& \sum_k \rho_k - \sum_k I_k f_k(\mathbf{x}^*) \\ =& N\rho - N\tfrac{c}{U(W)-U(W-L)}I \geq 0 \ . \end{aligned} \tag{12}$$

where, $\rho_k = \rho$ and $I_k = I$ for all users due to symmetry.

The voluntary participation condition for a user i to purchase insurance is given by:

$$\begin{aligned} &\tfrac{-c}{U(W)-U(W-L)}(U(W-\rho) - U(W-L-\rho+I)) \\ &\quad + U(W-\rho) - c\ln\frac{N^{1/\gamma}(U(W) - U(W-L))}{c} \\ &\qquad \geq -2c + U(W) - c\ln\frac{2^{1/\gamma-1}(U(W) - U(W-L))}{c} \ . \end{aligned}$$

Define the following:

$$K_1 := \frac{c}{U(W) - U(W-L)}, \quad K_2 := c(2 + \frac{1}{\gamma}\ln\frac{2^{1-\gamma}}{N}) \ .$$

Then, the voluntary participation conditions can be re-written as:

$$\begin{aligned} &U(W) - U(W-\rho) \leq \\ &\quad K_2 - K_1(U(W-\rho) - U(W-L-\rho+I)) \ . \end{aligned} \quad (13)$$

Can the insurer attain social optimality, voluntary participation, and weak budget balance in this game? Take equations (12) and (13) together. First, for all inequalities (13), relax the requirement by assuming $L = I$, that is, users are offered full coverage. Note that if this inequality fails for $L = I$, it will certainly fail for any $0 \leq I < L$. Also, for the inequality in (12), assume $I = 0$. Again, if (12) fails for $I = 0$, it will certainly fail for all $0 < I \leq L$. We show that the set of relaxed inequalities are inconsistent, and consequently, by the above argument, the original conditions in (12) and (13) can not be satisfied simultaneously either. We are therefore looking to find the premiums ρ, such that:

$$U(W) - U(W-\rho) \leq K_2 \ , \quad \rho \geq 0 \ .$$

Take any function in the CRRA family, $U(c) = \frac{1}{1-\theta}c^{1-\theta}$. The above conditions simplify to:

$$\rho \leq W - (W^{1-\theta} - (1-\theta)c(2 + \frac{1}{\gamma}\ln\frac{2^{1-\gamma}}{N}))^{\frac{1}{1-\theta}} \ , \quad \rho \geq 0 \ .$$

Fix the approximation parameter of the weakest link risk function, $\gamma > 0$, and that of the CRRA risk aversion function to a $\theta < 1$. We observe that, if $2 + \frac{1}{\gamma}\ln\frac{2^{1-\gamma}}{N} \leq 0$, then $\rho < 0$, and the second inequality (on the insurer's profit) cannot be satisfied. Therefore, if the number of users is such that $N > e^{2\gamma}2^{1-\gamma}$, it is impossible to design insurance contracts that guarantee social optimality, voluntary participation, and weak budget balance.

We conclude that unless additional information on users' preferences or the network structure is available, it is in general not possible to design insurance contracts that can result in a socially desirable state of security, are voluntarily purchased by the users, and can generate revenue for the cyber-insurer.

C. The role of a security software vendor

Given the potential budget deficit of the Pivotal mechanism when achieving social optimality and voluntary participation in some instances of security games, in this section, we consider the availability of additional external resources/payments to the designer of the Pivotal mechanism. In particular, we consider a security product vendor entering the game as the mechanism designer. The idea of bundling security product vendors and mechanism designers (more specifically, cyber-insurers) has been studied in [17], [26]. The authors in [17] propose the idea of a provider investing in increasing the security of widely used software products, leading to a decrease in monoculture risks. The focus of [26] on the other hand is on the security product pricing problem as a method for generating additional revenue for the cyber-insurer. Similarly, we consider the effects of such bundling on the performance of the Pivotal mechanism.

Specifically, we allow the vendor to leverage the profit from the additional sales in cyber-security products resulting from the requirements of improved security imposed on users, to cover the deficit and generate additional profit. Through an illustrative example, we show that this modification can lead to an expansion of the space of positive instances, but nevertheless, that this profit is not necessarily enough to cover the budget deficit in all instances of the game.

Formally, consider the total effort security game with exponential risks and linear investment costs, with uniform W and $L = 1$. The utility functions of users in this game are given by:

$$u_i(\mathbf{x}) = W - \exp(-\sum_j x_j) + c_i x_i \ .$$

Users are indexed such that $c_1 < c_2 < \ldots < c_N$. Also, assume $c_1 < \frac{c_2}{N-1}$. The socially optimal and exit equilibria are given by:

$$\begin{aligned} &\text{SO: } x_1^* = \ln\frac{N}{c_1}, x_j^* = 0, \forall j \neq 1 \ , \\ &\text{EE, } j \neq 1\text{: } \hat{x}_1^j = \ln\frac{N-1}{c_1}, \hat{x}_k^j = 0, \forall k \neq 1 \ , \\ &\text{EE, } j = 1\text{: } \hat{x}_1^1 = \ln\frac{1}{c_1}, \hat{x}_k^1 = 0, \forall k \neq 1 \ . \end{aligned}$$

We consider the Pivotal mechanism, as the taxes in it guarantee voluntary participation, and we can thus focus on budget balance issues. These taxes are given by:

$$\begin{aligned} t_j^P &= c_1(-\frac{1}{N} + \ln\frac{N}{N-1}), \forall j \neq 1 \ , \\ t_1^P &= -c_1\frac{(N-1)^2}{N} \quad . \end{aligned}$$

The sum of all taxes will be given by:

$$T^P := \sum_i t_i^P = c_1(N-1)(-1 + \ln\frac{N}{N-1}) \ .$$

The above is negative for all N, indicating a budget deficit for any number of users in the absence of external resources.

Alternatively, assume the Pivotal mechanism is implemented by a security product vendor. For simplicity, we assume that the marginal cost of production of security products is negligible for the vendor. Therefore, by the introduction of the Pivotal mechanism, the vendor makes the following additional profit from the increased security adoption (compared to the Nash equilibrium):

$$\Delta P := \sum_i c_i x_i^* - \sum_i c_i \tilde{x}_i = c_1 \ln\frac{N}{c_1} - c_1 \ln\frac{1}{c_1} = c_1 \ln N \ .$$

Considering the vendor's profit, the total budget following the introduction of the Pivotal mechanism is given by:

$$T^P + \Delta P = c_1(N-1)(-1 + \ln\frac{N}{N-1}) + c_1 \ln N \ .$$

The above is positive if and only if $N = 2$. We conclude that the space of positive instances has expanded (albeit slightly) once the profit of additional product sales enters the market.

In other words, with 2 users, a security vendor can introduce taxes that achieves the socially optimal levels of security, are voluntarily adopted by the users, and generate positive revenue for the designer/vendor. Note also that the budget deficit continues to hold for $N \geq 3$.

V. Related Work

A. *Existing possibility and impossibility results*

The presented impossibility result is different from those in the existing literature, in either the selected equilibrium solution concept, the set of properties the mechanism is required to satisfy, or the space of utility functions. For example, the Myerson and Satterthwaite result [28] (stronger version of Hurwicz's impossibility on dominant strategy implementation) establishes impossibility of Bayesian Nash implementation with optimality, individual rationality, and strong budget balance when users have quasi-linear utilities; our result differs in (1) solution concept (we are considering full Nash implementation), and (2) by only requiring a weaker condition of "weak" budget balance (thus making it a stronger impossibility result in this sense).

The most closely related impossibility result to our work is that of [29], which also studies impossibility results in the provision of non-excludable public goods. Our adoption of the term voluntary participation as opposed to individual rationality is similar to this work. However, our paper differs from [29] in two main aspects. First, in terms of users' preferences, [29] studies Cobb-Douglas utilities, whereas we consider quasi-linear utilities, as well as risk averse users with CRRA utilities. More importantly, [29] considers the production of a single (non-excludable) good with constant return to scale technology. As a result, although outliers benefit from the spill-over of the produced good, they no longer contribute to its provision. This is in contrast to the goods studied herein, e.g., the security of an outlier can still affect those of the participants. The notion of exit equilibrium is introduced to fully capture this distinction.

The current work should also be viewed in conjunction with existing possibility results, notably [5], [11], [28], [31], which consider the provision of excludable public goods (i.e., zero outside options) for users with the same utility functions and under the same informational constraints as the current work. As a result, they show that the Externality and Pivotal mechanisms simultaneously guarantee social optimality, voluntary participation, and weak budget balance. Therefore, the goal of our work is not solely to prove the impossibility of the design, but to highlight the important distinction users' outside options make in the choice of a mechanism.

B. *Public good provision games*

The problem of incentivizing optimal security investments in an interconnected system is one example of problems concerning the provision of non-excludable public goods in social and economic networks. Other examples include creation of new parks or libraries at neighborhood level in cities [2], reducing pollution by neighboring towns [7], or spread of innovation and research in industry [4]. We summarize some of the work most relevant to the current paper.

The authors in [4] introduce a network model of public goods, equivalent to a total effort game with linear investment costs and a general interdependence graph, and study several features of the Nash equilibria of this model. In particular, they show that these games always have a *specialized* Nash equilibrium, i.e., one in which users are either specialists exerting full effort (equivalent to main investors in our terminology), or free-riders. Similarly, in Section III-D, we show that roles of main investor/free rider can be identified in socially optimal and exit equilibrium outcomes as well. The work in [2] studies existence, uniqueness, and closed form of the Nash equilibrium, in a class of games for which best-responses are linear in other players' actions. The aforementioned work differ from the current paper in that they focus on the Nash equilibrium of the games, whereas we study the mechanism design problem, therefore analyzing socially optimal investments, and proposing exit equilibria. The work of [7] is also relevant to our work, as it studies Pareto efficient outcomes in the provision of non-excludable public goods, and establishes a connection between efforts at a Lindahl outcome and the eigenvalue centrality vector of a suitably defined benefits matrix. Our work in this paper is on the study of voluntary participation in such environments, which [7] mentions as a direction of future work.

In the context of security games, our findings in Section IV-B are most related to the study of cyber-insurance contracts. Cyber-insurance has been widely proposed as both a method for mitigating cyber-risks, and as an incentive mechanism for internalizing the externalities of security investments, see e.g. [3], [8], [12], [18], [19], [27]. In particular, [12], [19], [27] have shown that by engaging in premium discrimination, a monopolistic profit-neutral cyber-insurer can induce socially optimal security investments in an interdependent systems where security decisions are binary (i.e., invest or not). However, participation in these studies is assumed to be mandatory, e.g., users are enforced through policy mandates to purchase insurance. Our findings show that this assumption is indispensable, i.e., it is not in general possible to design non-compulsory insurance contracts that induce socially optimal behavior and generate profits for the cyber-insurer.

Finally, our work is also related to [14], [22]. The weighted effort risk model studied in Section III-B is a generalization of the total effort model in [34], and is similar to the effective investment model in [14] and the linear influence network game in [22]. The authors in [22] identify properties of the interdependence matrix affecting the existence and uniqueness of the Nash equilibrium in the linear influence model. Using a similar effective investment model, [14] determines a bound on the price of anarchy gap, i.e. the gap between the socially optimal and Nash equilibrium investments, dependent on the adjacency matrix. Our work on the above model fills a gap within this literature as well, by (1) introducing the study of exit equilibria, and (2) analyzing the general mechanism design problem, in both this model, as well as more general environments.

VI. Conclusion

We introduced the notion of exit equilibrium to study voluntary participation of users in mechanisms for provision of non-excludable public goods, such as security. This equilibrium concept accounts for both the spill-over of the public good produced by the participants on an outlier, as well as the continued influence of the outlier on the provision of the public good (here, the state of security). We have shown the fundamental result that, given these outside options, it is not possible to design a tax-based incentive mechanism to implement the socially optimal solution while guaranteeing voluntary participation and maintaining a weakly balanced budget, without additional information on the graph structure and users' preferences. We showed that despite the lack of a reliable mechanism for general problem instances, we can identify positive instances under restricted parameter conditions, for which it is possible to guarantee social optimality, weak budget balance, and voluntary participation using well-known incentive mechanisms. Alternatively, for instances in which the three properties are not attainable, we may be able to identify stable coalitions of participating users, by using an extended definition of exit equilibrium which accounts for possibly multiple outliers. Finally, we further extended our result to risk-averse users, highlighting the possible limitations of using cyber-insurance as a tool for improving the state of cyber-security to its socially desirable level.

An important implication of our result is that, when a designer lacks additional information about the specifics of the problem environment and users' preferences, she may choose to forgo the social optimality requirement, instead focusing on reliably attaining a sub-optimal solution while guaranteeing full voluntary participation and weak budget balance. Characterizing mechanisms that can lead to such sub-optimal solutions is a main direction of future work. Similarly, a designer may choose to guarantee the socially optimal outcome and voluntary participation, while incurring a budget deficit. The Pivotal mechanism is an instance of such mechanisms. Finding a bound on the deficit incurred by such mechanisms, or ideally, a mechanism leading to the smallest budget deficit, is another interesting direction of future work.

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Appendix A
The weighted effort model - effects of self-dependence

In this appendix, we consider the security game where users' utilities are given by (8) and interdependence matrix (9). Due to space considerations, we present the full characterization and analysis of the scenarios of Table I in the online appendix [23]. Here, for illustration, we include the formal analysis of the Externality and Pivotal mechanisms in case β.

For case β, the exit equilibrium $\hat{\mathbf{x}}^i$ is such that $\hat{x}^i_i = \frac{1}{(a-1)(a+N-1)}\log\left(\frac{a}{c}\right)^{a-1}(1+\frac{N-2}{a})^{-(N-1)}$ and $\hat{x}^i_j = \frac{1}{(a-1)(a+N-1)}\log\left(\frac{a}{c}\right)^{a-1}(1+\frac{N-2}{a})^{a}, \forall j \neq i$. Therefore, the costs of users at the SO and EE are given by:

$$\begin{aligned}
g_j(\mathbf{x}^*) &= \frac{c}{a+N-1}(1+\log\frac{a+N-1}{c}), \forall j\\
g_j(\hat{\mathbf{x}}^i) &= \frac{c}{a+N-2}\\
&\quad + \frac{c}{(a-1)(a+N-1)}\log\left(\frac{a}{c}\right)^{a-1}(1+\frac{N-2}{a})^{a}, \forall j\neq i\\
g_i(\hat{\mathbf{x}}^i) &= \frac{c}{a}\\
&\quad + \frac{c}{(a-1)(a+N-1)}\log\left(\frac{a}{c}\right)^{a-1}(1+\frac{N-2}{a})^{-(N-1)}
\end{aligned}$$

(i) Weak Budget Balance in the Pivotal mechanism: By (6), if $g_j(\mathbf{x}^*) \leq g_j(\hat{\mathbf{x}}^i), \forall j\neq i$, the Pivotal mechanism will carry a budget deficit. This happens in case β as:

$$\begin{aligned}
&\frac{c}{a+N-1}(1+\log\frac{a+N-1}{c}) \leq\\
&\quad \frac{c}{a+N-2} + \frac{c}{(a-1)(a+N-1)}\log\left(\frac{a}{c}\right)^{a-1}(1+\frac{N-2}{a})^{a}\\
\Leftrightarrow\ & \log\frac{a}{c}(1+\frac{N-1}{a}) \leq \frac{1}{a+N-2} + \log\frac{a}{c}(1+\frac{N-2}{a})^{\frac{a}{a-1}}\\
\Leftrightarrow\ & \log(1+\frac{N-1}{a}) \leq \frac{1}{a+N-2} + \frac{a}{a-1}\log(1+\frac{N-2}{a})\\
\Leftarrow\ & \log(1+\frac{N-1}{a}) \leq \frac{1}{a+N-2} + \log(1+\frac{N-2}{a})\\
\Leftrightarrow\ & \log(1+\frac{1}{a+N-2}) \leq \frac{1}{a+N-2}
\end{aligned}$$

The last line is true because $\log(1+x)\leq x$, for all $x>0$. Therefore, the Pivotal mechanism always carries a budget deficit.

(ii) Voluntary Participation in the Externality mechanism: The mechanism fails voluntary participation if and only if $g_i(\mathbf{x}^*) \geq g_i(\hat{\mathbf{x}}^i)$:

$$\begin{aligned}
&\frac{c}{a+N-1}(1+\log\frac{a+N-1}{c}) \geq\\
&\quad \frac{c}{a} + \frac{c}{(a-1)(a+N-1)}\log\left(\frac{a}{c}\right)^{a-1}(1+\frac{N-2}{a})^{-(N-1)}\\
\Leftrightarrow\ & 1+\log\frac{a}{c}(1+\frac{N-1}{a}) \geq\\
&\quad 1+\frac{N-1}{a} + \log\frac{a}{c}(1+\frac{N-2}{a})^{\frac{-(N-1)}{a-1}}\\
\Leftrightarrow\ & \log(1+\frac{N-1}{a}) + \frac{N-1}{a-1}\log(1+\frac{N-2}{a}) \geq \frac{N-1}{a}\\
\Leftarrow\ & \log(1+\frac{N-1}{a}) + \frac{N-1}{a}\log(1+\frac{N-1}{a}-\frac{1}{a}) \geq \frac{N-1}{a}
\end{aligned}$$

Let $z := \frac{N-1}{a}$, and define $f(z) := \log(1+z) + z\log(1+z-\frac{1}{a}) - z$ (i.e., we are assuming a fixed a). The derivative of this function wrt z is given by $\frac{1}{1+z} + \log(1+z-\frac{1}{a}) + \frac{z}{1+z-\frac{1}{a}} - 1 = \log(1+z-\frac{1}{a}) + \frac{\frac{1}{a}z}{(1+z)(1-\frac{1}{a}+z)}$. As this is positive for all $a>1$, we conclude that $f(z)$ is an increasing function in z. Furthermore, $\lim_{z\to 0} f(z) = 0$, which in turn means that $f(z)\geq 0, \forall z\geq 0$, and therefore, that the VP conditions in the Externality mechanism always fail to hold under these parameter settings.

APPENDIX B
THE WEIGHTED EFFORT MODEL - DOMINANT USER

In this appendix, we consider the weighted effort game with interdependence matrix (10), and solve for the socially optimal investment profile, and identify the possible exit equilibria, and parameter conditions under which each equilibrium is possible. It is straightforward to show that in a socially optimal investment profile $\mathbf{x}^*$, only user 1 will be exerting effort, so that:

$$x_1^* = \frac{1}{a}\ln\frac{aN}{c}, \quad x_j^* = 0, \forall j = 2, \ldots, N \ .$$

We next find the exit equilibria. First, if any non-dominant user $i \neq 1$ steps out of the mechanism, user 1 will continue exerting all effort, but at a lower level given by:

$$\hat{x}_1^i = \frac{1}{a}\ln\frac{a(N-1)}{c}, \quad \hat{x}_j^i = 0, \forall j = 2, \ldots, N \ .$$

Next, if user 1 steps out of the mechanism, there are two possible exit equilibria: if $a > N-1$, there will be enough externality for users $j \neq 1$ to continue free-riding, resulting in the following equilibrium investment levels:

$$\hat{x}_1^1 = \frac{1}{a}\ln\frac{a}{c}, \quad \hat{x}_j^1 = 0, \forall j = 2, \ldots, N \ .$$

However, when $a < N-1$, user 1 will free-ride on the externality of other users' investments, leading to the exit equilibrium:

$$\hat{x}_1^1 = 0, \quad \hat{x}_j^1 = \frac{1}{N-1}\ln\frac{N-1}{c}, \forall j = 2, \ldots, N \ .$$

A. Voluntary participation in the Externality mechanism

We now analyze the performance of the Pivotal and Externality mechanisms, under the different exit equilibria identified in the previous section, and summarized in Table II.

In the Externality mechanism, users' taxes are given by:

$$t_1^E(\mathbf{x}^*) = cx_1^*(\frac{1}{N} - 1)$$
$$t_j^E(\mathbf{x}^*) = \frac{c}{N}x_1^*, \forall j = 2, \ldots, N \ .$$

For non-dominant users $i \in \{2, \ldots, N\}$ to voluntarily participate in the mechanism, we require $u_i(\hat{\mathbf{x}}^i) \leq v_i(\mathbf{x}^*, t_i^E(\mathbf{x}^*))$:

$$\frac{c}{a(N-1)} \geq \frac{c}{aN} + \frac{c}{aN}\ln\frac{aN}{c} \Leftrightarrow \frac{1}{N-1} \geq \ln N + \ln\frac{a}{c} \ .$$

However, $\ln N \geq \frac{1}{N-1}, \forall N \geq 3$, and $a > c$. Therefore, the voluntary participation constraints will always fail to hold for free-riders in the Externality mechanism.

A perhaps more interesting aspect is that the voluntary participation of user 1, i.e., the main investor who is receiving a reward, may also fail to hold. Specifically, when $a < N-1$, user 1 will participate voluntarily if and only if $u_1(\hat{\mathbf{x}}^1) \leq v_1(\mathbf{x}^*, t_1^E(\mathbf{x}^*))$, which reduces to:

$$\frac{c}{N-1} \geq \frac{c}{aN} + \frac{c}{aN}\ln\frac{aN}{c} \ .$$

However, the above inequality does not necessarily hold. For example, with $N = 10$, $c = 0.45$, and $a < 5$, the above will fail to hold, indicating that the main investor will also prefer to opt out. It is also interesting to mention that when $a > N-1$, the voluntary participation of the main investor always holds.

B. Weak budget balance in the Pivotal mechanism

Finally, we analyze the total budget in the Pivotal mechanism. The taxes for the non-dominant users $i \neq 1$ will be given by:

$$t_i^P = \frac{c}{a}(\ln\frac{N}{N-1} - 1) \ .$$

The taxes for user 1 will depend on the realized exit equilibrium. If $a < N-1$, this tax is given by:

$$t_1^P = (N-1)\frac{c}{aN} - c(1 + \ln\frac{N-1}{c}) \ .$$

The sum of the Pivotal taxes under this parameter conditions will then be given by:

$$\sum_i t_i^P = c(\frac{N-1}{a}(\ln\frac{N}{N-1} - 1 + \frac{1}{N}) - (1 + \ln\frac{N-1}{c}))$$

Note that $\ln z - \frac{1}{z} < 0, \forall z < \frac{3}{2}$, and therefore, with $N \geq 3$, the above sum is always negative. We conclude that the Pivotal mechanism will always carry a deficit.

On the other hand, when $a > N-1$, the tax for the dominant user is given by:

$$t_1^P = (N-1)\frac{c}{aN} - (N-1)\frac{c}{a} = (N-1)\frac{c}{a}(\frac{1}{N} - 1) \ .$$

The sum of the Pivotal taxes will then be given by:

$$\sum_i t_i^P = \frac{c(N-1)}{a}\left(-1 + \ln\frac{N}{N-1} - 1 + \frac{1}{N}\right)$$

By the same argument as before, the above sum will always be negative, indicating a budget deficit in the Pivotal mechanism under this scenario as well.

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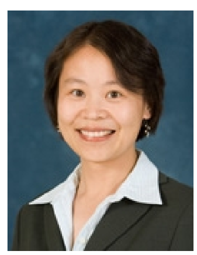

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