On the Many-to-One Transport Capacity of a Dense Wireless Sensor Network and the Compressibility of Its Data

Daniel Marco, Enrique J. Duarte-Melo
Mingyan Liu, David L. Neuhoff

Electrical Engineering and Computer Science
University of Michigan, Ann Arbor

A Field-Gathering Scenario

- A two-dimensional field measured by uniformly distributed sensors in a fixed finite region
- Periodically, sensors take measurements, quantize, and encode with bits
- Collective measurements at any given time – snapshot of the field
- Data sent to a single receiver (data collector)
- Snapshot of the field reconstructed at the receiver
- Reconstruction distortion – per sample mean-squared error
- Data transmission via single hop or multi-hop
- Time is slotted, each sensor can send at most W bits in each slot

Questions of Interest

- (I) Can the network transport the amount of data necessary to reconstruct the field snapshot in a bounded number of slots per snapshot?
  1. How much data needs to be generated per snapshot to attain a given mean-squared error? – B_N
  2. How much data can be transported by the network per slot? – C_N

  Slots required per snapshot – B_N / C_N

- (II) The rate at which this quantity changes as the number of sensors increases

Questions of Interest

- (I) Can the network transport the amount of data necessary to reconstruct the field snapshot in a bounded number of slots per snapshot?
  1. How much data needs to be generated per snapshot to attain a given mean-squared error? – B_N
  2. How much data can be transported by the network per slot? – C_N

  Slots required per snapshot – B_N / C_N

- (II) The rate at which this quantity changes as the number of sensors increases

Compressibility – The random field

Assume

- Stationary 2-D random field X(u,v), real-valued random variable at point (u,v), u,v varying continuously
- Successive snapshots are independent

Do not assume

- Band-limited or non band-limited

Compressibility – Quantization and coding

- Sensors take samples of the random field at locations (u_i, v_i)

Constraint

- These samples are then individually scalar quantized, encoded and transported

Assume

- Identical quantizers that map a sample value to an integer indexing a quantization cell/bin
- Binary lossless encoding
- Probability that X(u,v) crosses some quantization threshold somewhere in the entire region G is greater than 0, otherwise the quantizer is too coarse to use

Compressibility – Reconstruction quality

- MSE of the reconstruction

$$\frac{1}{|G|}\int_G E(X(u,v)-\hat{X}(u,v))^2\,dudv$$

- Interpolation and quantization errors: MSE> 0
- In effect, lossy coding of the random field
- As N becomes large, MSE well approximated by

$$\frac{1}{N}\sum_{i=1}^{N} E(X(u_i,v_i)-\hat{X}(u_i,v_i))^2 = D$$

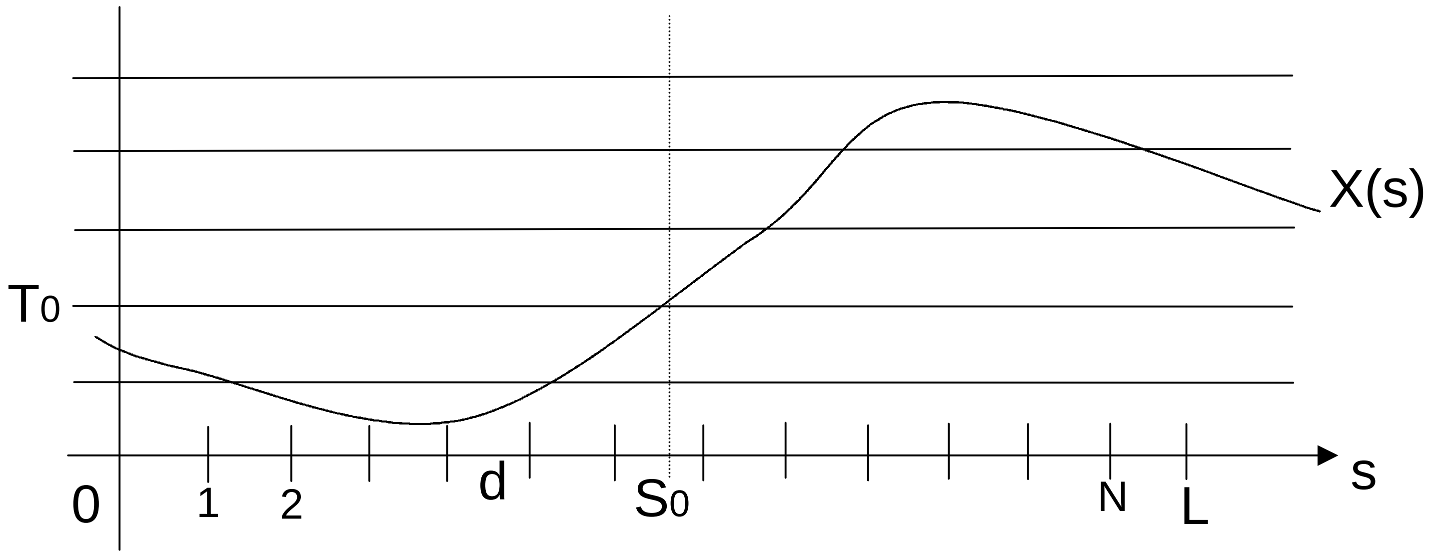
Compressibility – Data per snapshot

- $I_1, \cdots, I_N$ denote quantized integer values at sensor locations
- B_N denotes total number of bits required per snapshot

$$B_N \geq H(I_1, \cdots, I_N)$$

$$B_N \geq H(I_1, \cdots, I_N) \to \infty, N \to \infty$$

*Sketch of Argument Using 1-D**



$$H(I_1,\cdots,I_N) \geq H(\hat{S}_0) \rightarrow \infty$$

*Courtesy of Bruce Hajek

Compressibility – Data per sensor

- Using Slepian-Wolf
- b_N denotes number of bits required per sensor per snapshot

$$b_N = \frac{1}{N}\sum_{k=1}^{N} H(I_k \mid I_{k-1},\cdots,I_1)$$

$$\leq \frac{1}{N}\sum_{k=1}^{N} H(I_k \mid I_{k-1}) = \frac{H(I_1)}{N} + \frac{N-1}{N} H(I_2 \mid I_1)$$

$$\to H(I_2 \mid I_1)$$

$$b_N \to 0, N \to \infty$$

$$B_N \to \infty, N \to \infty$$

Questions of Interest

- (I) Can the network transport the amount of data necessary to reconstruct the field snapshot in a bounded number of slots per snapshot?
  1. How much data needs to be generated per snapshot to attain a given mean-squared error? – B_N
  2. How much data can be transported by the network per slot? – C_N

  Slots required per snapshot – B_N / C_N

- (II) The rate at which this quantity changes as the number of sensors increases

Many-to-One Transport Capacity – An Upper Bound

Assume

- Single data receiver/collector
- Cannot receive simultaneously from more than one sensor at a time
- C_N denotes transport throughput in bits per slot

$$C_N \leq W$$

Questions of Interest

- (I) Can the network transport the amount of data necessary to reconstruct the field snapshot in a bounded number of slots per snapshot?
  1. How much data needs to be generated per snapshot to attain a given mean-squared error? – B_N
  2. How much data can be transported by the network per slot? – C_N

  Slots required per snapshot – B_N / C_N

- (II) The rate at which this quantity changes as the number of sensors increases

Result I

$$B_N \rightarrow \infty, N \rightarrow \infty$$

$$C_N \leq W$$

$$B_N / C_N \rightarrow \infty, N \rightarrow \infty$$

- Any data compression scheme is insufficient for the required amount of data to be transported for the given quality within finite time

Questions of Interest

- (I) Can the network transport the amount of data necessary to reconstruct the field snapshot in a bounded number of slots per snapshot?
  1. How much data needs to be generated per snapshot to attain a given mean-squared error? – B_N
  2. How much data can be transported by the network per slot? – C_N

  Slots required per snapshot – B_N / C_N

- (II) The rate at which this quantity changes as the number of sensors increases

Rate of Growth

- Rate at which the number of slots per snapshot grows with the number of sensors can be characterized for certain combination of data dissemination and compression schemes
  - 1-D Gaussian field with two types of auto-correlation functions
  - Achievable many-to-one transport capacity

1-D Gaussian Field with Known Autocorrelation

- Using Slepian-Wolf coding (sub-optimally) each sensor value can be encoded with approximately $b_N = H(I_2 \mid I_1)$ bits
- Autocorrelation $R_x(s)$ with unit variance
- Infinite level uniform scalar quantizer with step size Δ

$$R_x(s) = e^{-|s|} \qquad B_N \approx \sqrt{2L} M_\Delta \sqrt{N} \log\left(\sqrt{\frac{N}{2L}}\right) \to \infty, N \to \infty$$

$$R_x(s) = e^{-s^2} \qquad B_N \approx \sqrt{2} L M_\Delta \left(\log \frac{N}{\sqrt{2} L}\right) \to \infty, N \to \infty$$

L is length of the line along which sensors are placed

Many-to-One Transport Capacity – A Lower Bound

Assume

- The field has circular shape with the collector at the center
- The collector cannot receive simultaneously from multiple sensors
- A sensor cannot receive and transmit simultaneously
- All sensors have transmission capacity W bits per slot
- Transmission from X_i to X_j is successful iff

$$X_{i,j} \le d; X_{k,j} > d + \delta, \delta \ge 0$$

[Gupta and Kumar] – "The capacity of wireless networks", IEEE Trans. IT, 46(2), 2000.

Achievable Transport Throughput

- c_N denotes the transport throughput per sensor per snapshot
- With high probability

$$c_N \geq \frac{W}{N} \cdot \frac{\pi r^2 - \sqrt{\varepsilon}}{4\pi r^2 + 4\pi r\delta + \pi\delta^2 + \sqrt{\varepsilon}}, N \to \infty$$

ε positive but arbitrarily small

$$c_N \geq \frac{W}{4N}, N \to \infty$$

$$C_N = \Theta(1)$$

Result II

- Number of slots required per snapshot

$$C_N = \Theta(1)$$

$$B_N = O(\sqrt{N}\log N), N \to \infty$$

$$B_N / C_N = O(\sqrt{N}\log N)$$

$$B_N = O(\log N), N \to \infty$$

$$B_N / C_N = O(\sqrt{N}\log N)$$

Summary

- Studied feasibility of using dense wireless sensor networks to transport field images
  - Compressibility of the sensor data
  - Many-to-one transport capacity
- Result 1: $B_N \to \infty, N \to \infty \qquad C_N = \Theta(1)$
- Result 2: characterized the rate of growth in specific cases

Discussion

- Independent scalar quantization of sensor value critical in deriving these results
- If jointly quantize a block, B_N remains bounded if block grows with N

Future Work

- There must be an optimal number of sensors that minimizes the number of slots per snapshot
- Limit on how densely sensors should be deployed or how densely a field should be measured
- Over-deployment and sensor suppression