R-Trees and GiST

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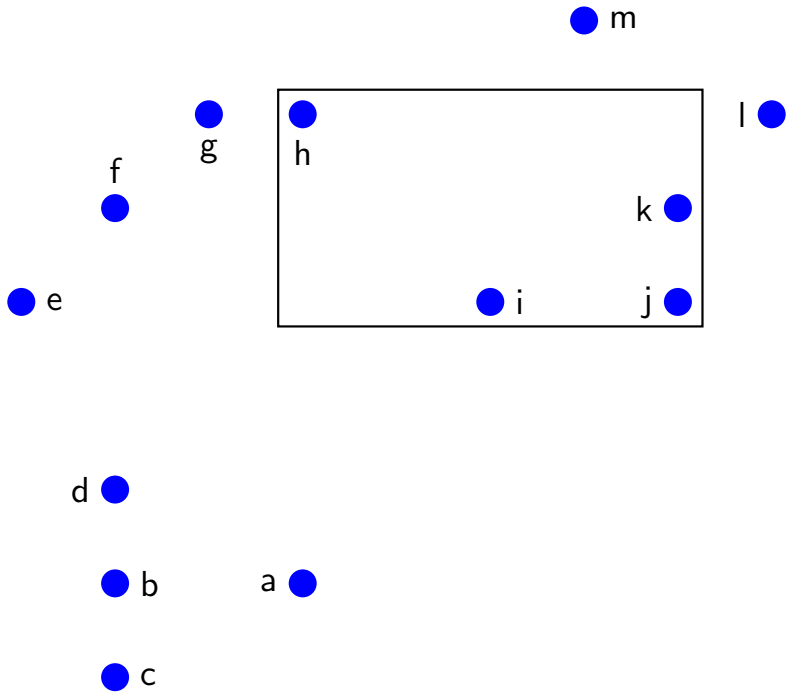
R-Tree Motivation



- **Range Query:** find objects in a given range (e.g. find all museums in New York City)
  - No index: need to scan through all objects. Inefficient!
  - B+-tree: clusters based only on one dimension. Inefficient!

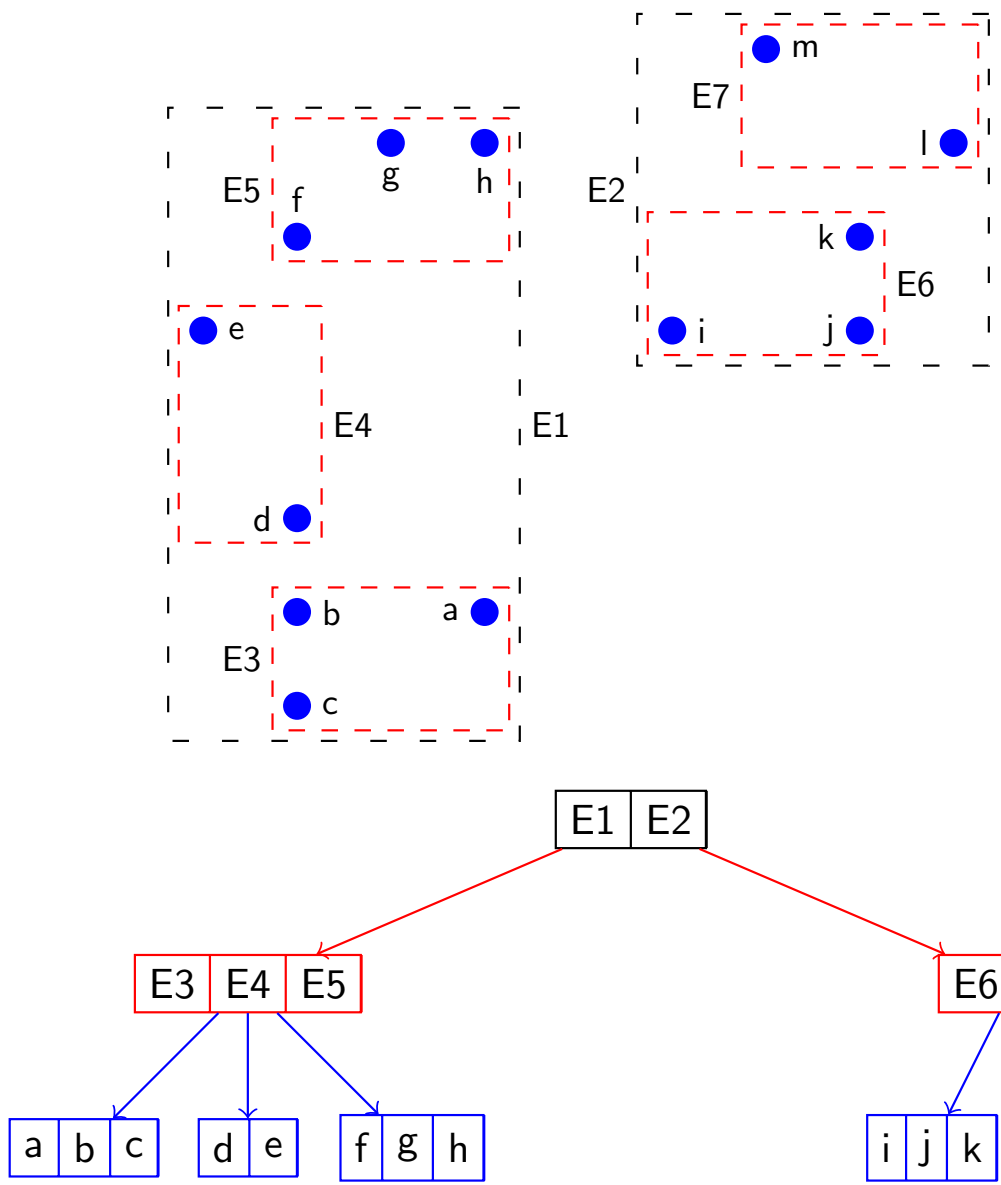
R-Tree Structure

- Non-leaf nodes contain entries in the form of *(I, child-pointer)*
  - *I* is an *n*-dimensional rectangle
- Leaf nodes contain entries in the form of *(I, tuple-identifier)*
- *M* is the maximum of entries which is usally given and *m* is the minimum of entries in one node

R-Tree Properties

1. Every leaf node contains between m and M index records unless it is the root; the root can have less entries than m
2. For each index record in a leaf node, I is the smallest rectangle that spatially contains the n-dimensional data object represented by the indicated tuple
3. Every non-leaf node has between m and M children unless it is the root
4. For each entry in a non-leaf node, i is the smallest rectangle that spatially contains the rectangles in the child node
5. The root node has at least two children unless it is a leaf
6. All leaves appear on the same level. That means the tree is balanced

R-Tree


m
E7
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E6 E7
a b c
d e
f g h
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l m

Search Algorithm

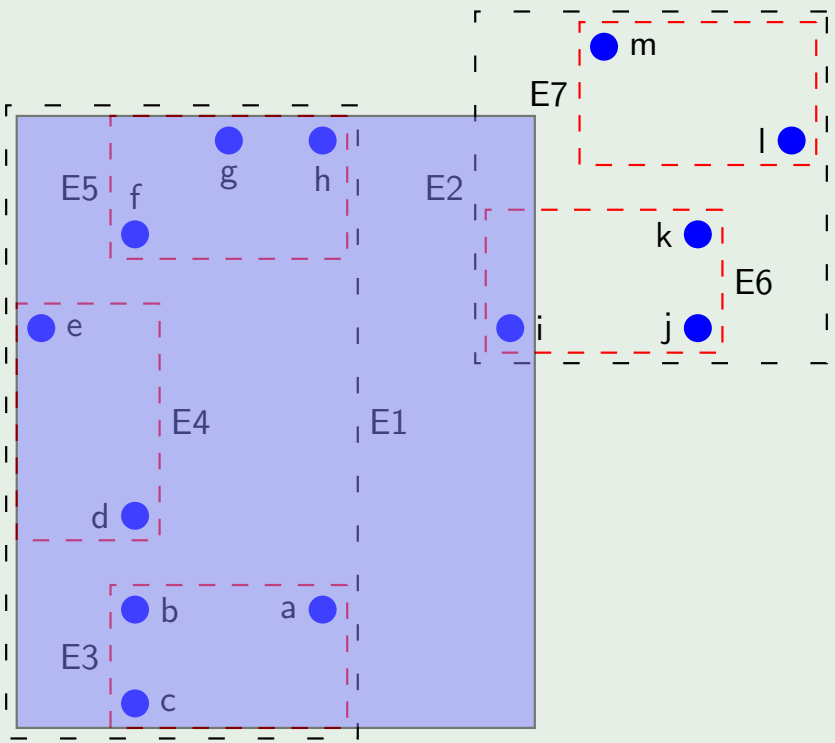
T is root node of an R-Tree, find all index records whose rectangles overlap a search rectangle S.

S1 [Search Subtree] If T no leaf then check each entry E, whether $E.I$ overlaps S. For all overlapping entries, start **Search** on the subtree whose root node is pointed to by $E.p$.

S2 [Search leaf node] If T is a leaf, then check each entry E whether $E.I$ overlaps S. If so, E is a suitable entry.

Search Algorithm

Example



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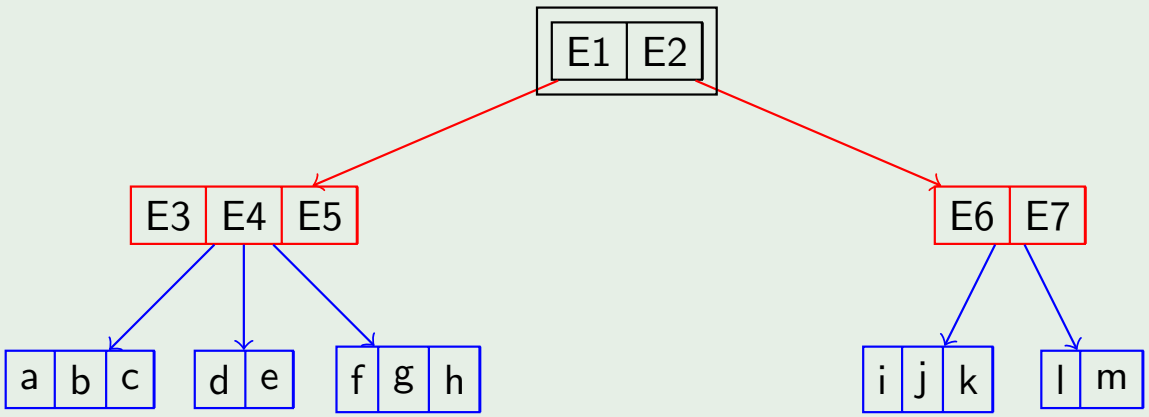

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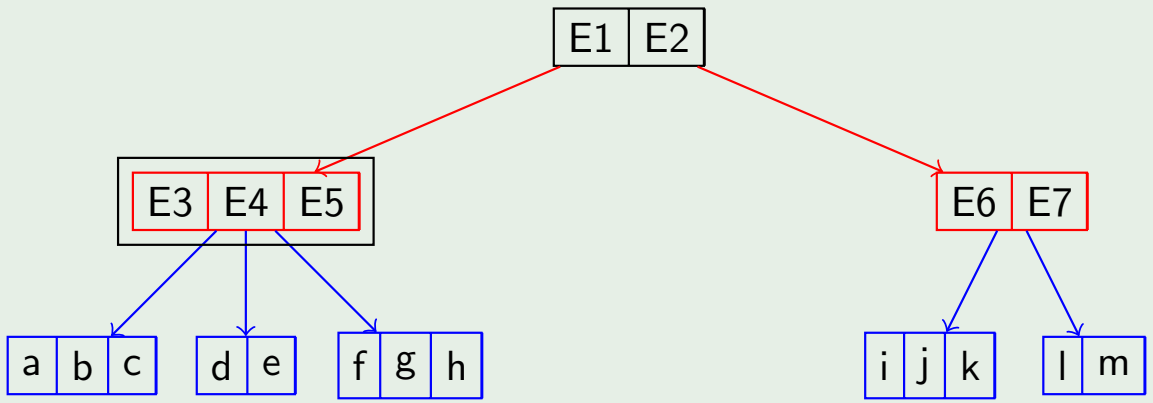

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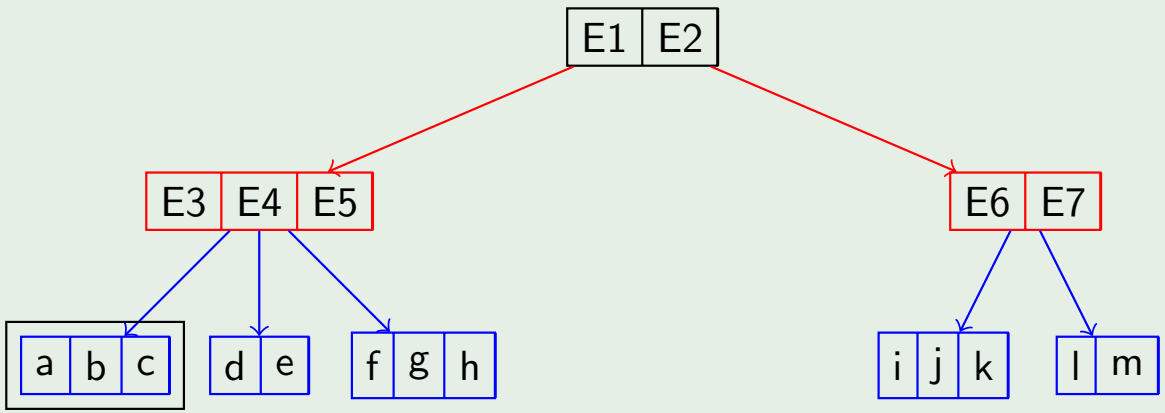

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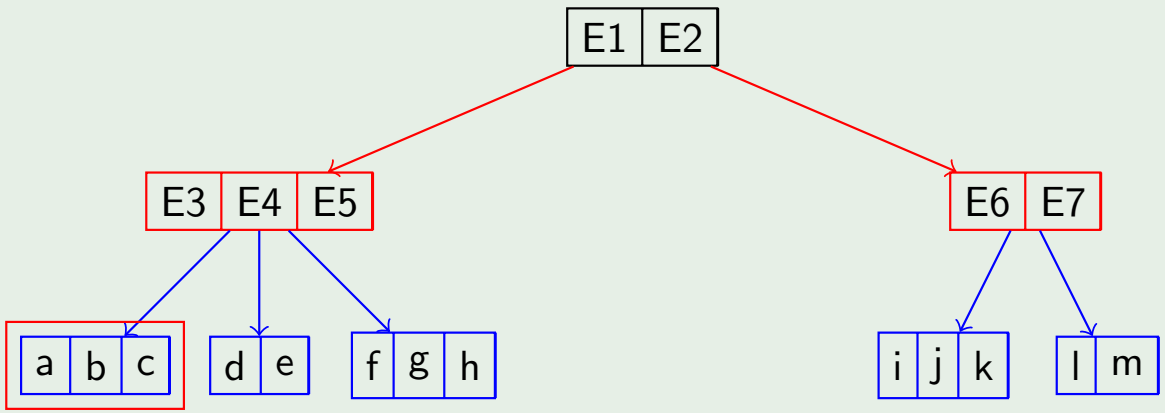

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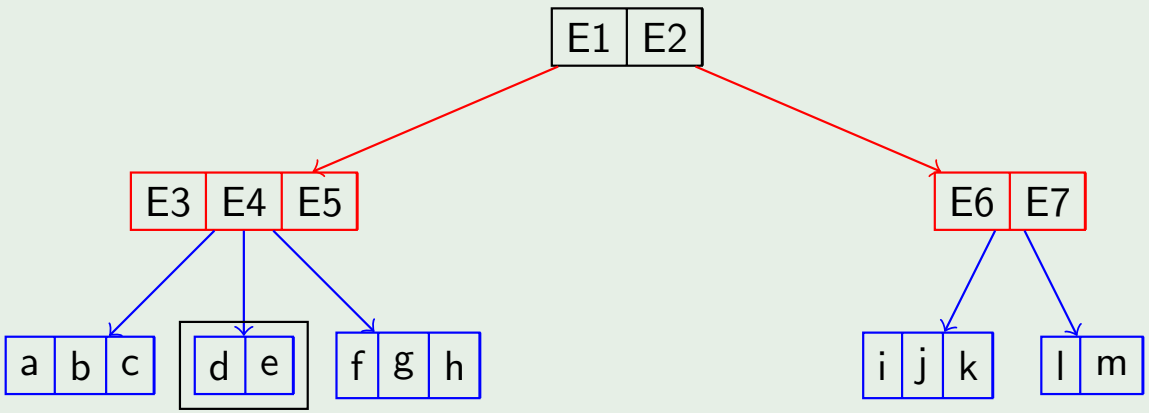

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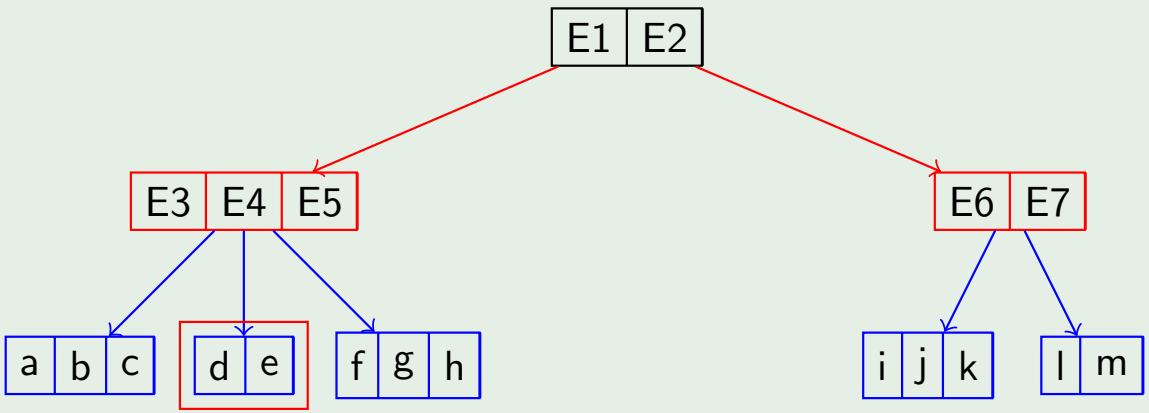

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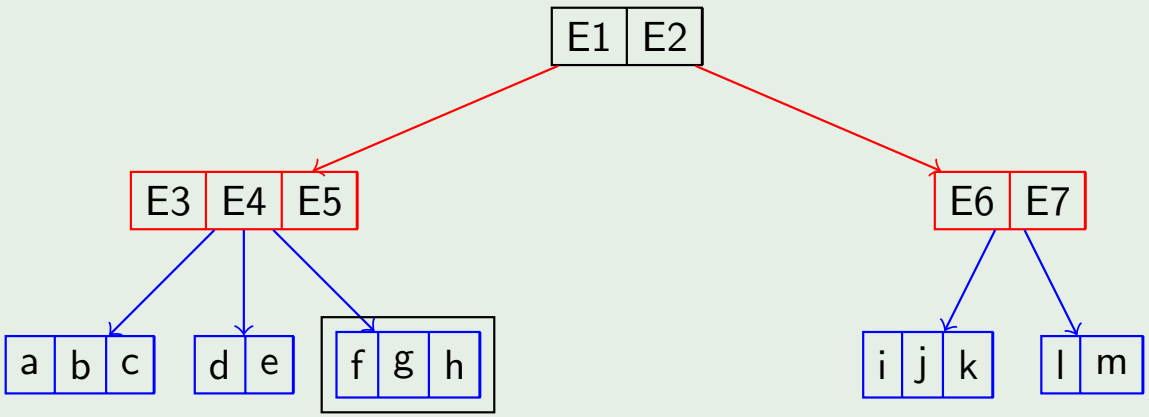

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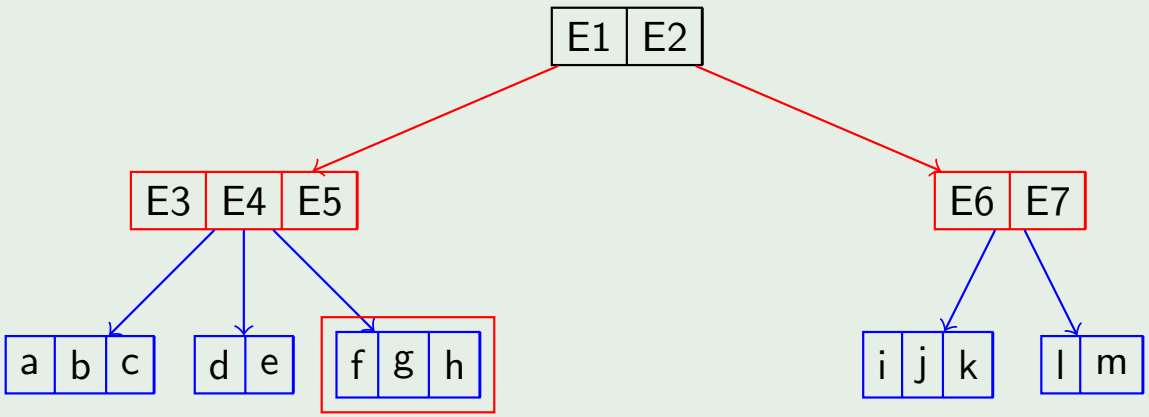

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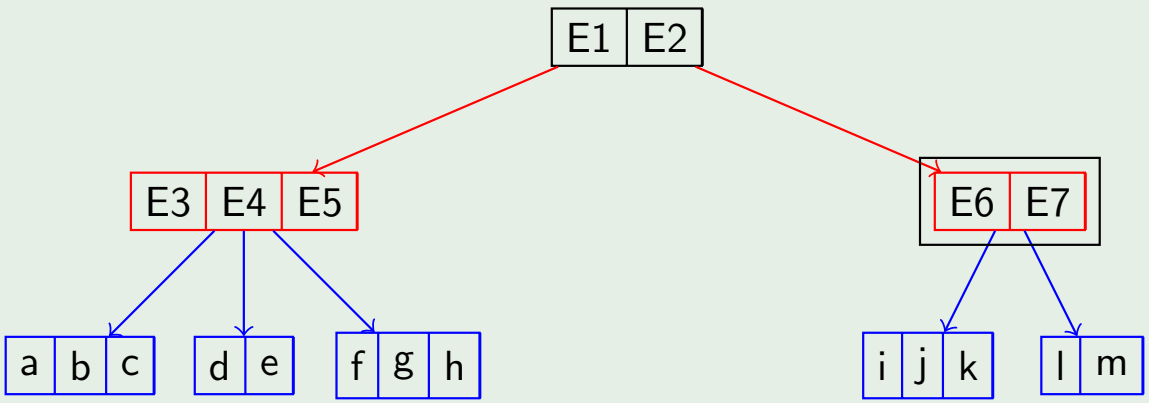

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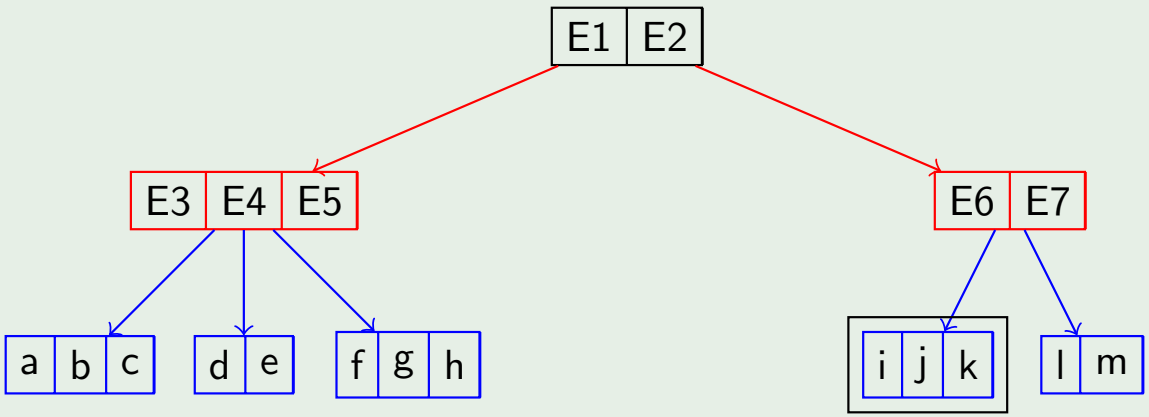

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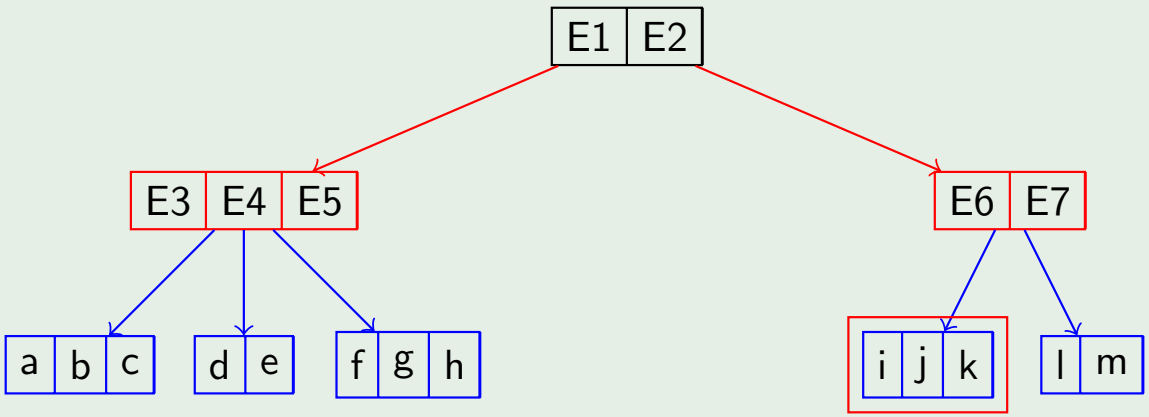

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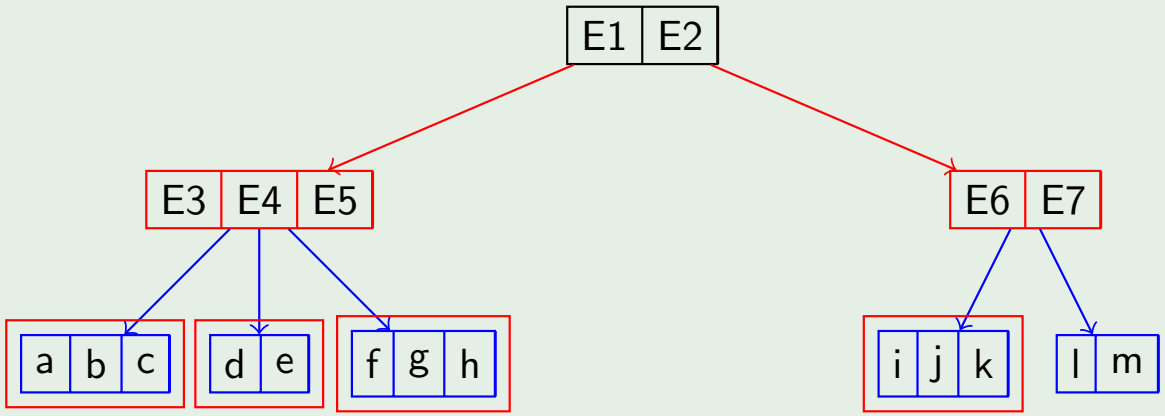


Insert Algorithm

New entry E will be inserted into a given R-Tree.

I1 [Find position for new record] Start **ChooseLeaf** to select a leaf node L in which to place E

I2 [Add record to leaf node] If node L has enough space for a new entry then add E. Else start **SplitNode** to obtain L and LL containing E and all the old entries of L.

I3 [Propagate changes upward] Start **AdjustTree** on node L and if a split was performed then also passing LL.

I4 [Grow tree taller] If node split propagation caused the root to split, create a new root whose children are the two resulting nodes.

ChooseLeaf Algorithm

ChooseLeaf selects a leaf node to place a new entry E.

CL1 [Initialize] Set N to be the root node.

CL2 [Leaf check] If N is a leaf then return N .

CL3 [Choose subtree] If N is not a leaf node then let F be the entry in N whose MBR $F.I$ needs least enlargement to include $E.I$. When there are more qualify entries in N , the entry with the rectangle of the smallest area is chosen.

CL4 [Descend until a leaf is reached] N is set to the child node F which is pointed to by $F.p$ and repeat from *CL2*

AdjustTree Algorithm

Leaf node L is upwarded to the root while adjusting covering rectangles. If necessary it comes to propagating node splits.

AT1 [Initalize] N is equal L.

AT2 [Check if done] stop if N is the root

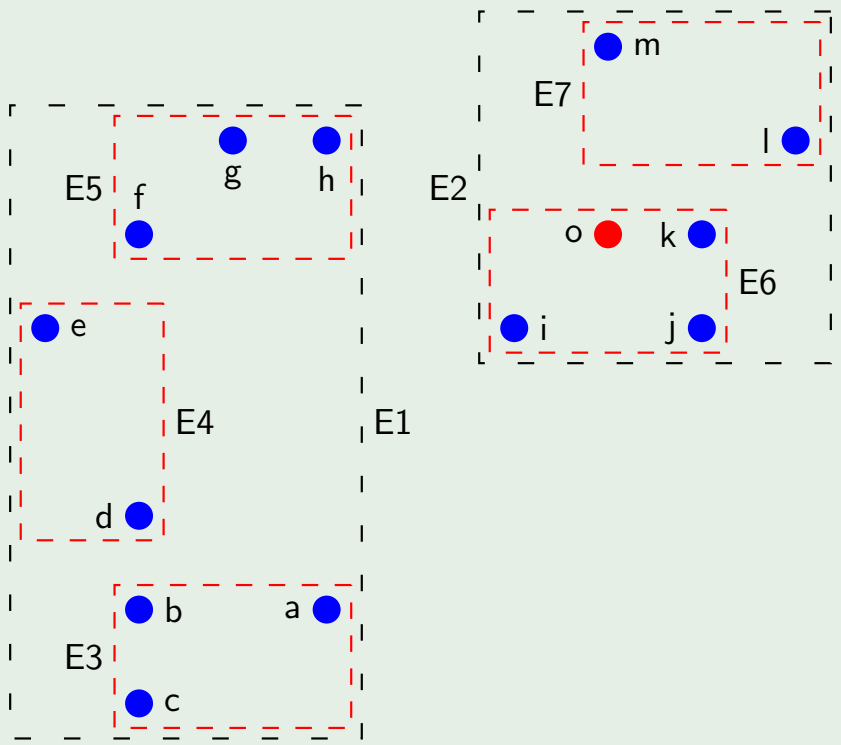
AT3 [Adjust covering MBR in parent entry] P is the parent node of N and E_N the entry of N in P. $E_N.I$ is adjusted so that all rectangles in N are tightly enclosed.

AT4 [Propagate node split upward] If N has a partner NN which was split previously then create a new entry with $E_{NN}.p$ pointing to NN and $E_{NN}.I$ enclosing all rectangles in NN. If there is room in P then add E_{NN}. Otherwise start **SplitNode** to get P and PP which include E_{NN} and all old entries of P.

AT5 [Move up to next level] N is equal L and if a split occurred then NN is equal PP. Repeat from *AT2*

Insert Algorithm

Example



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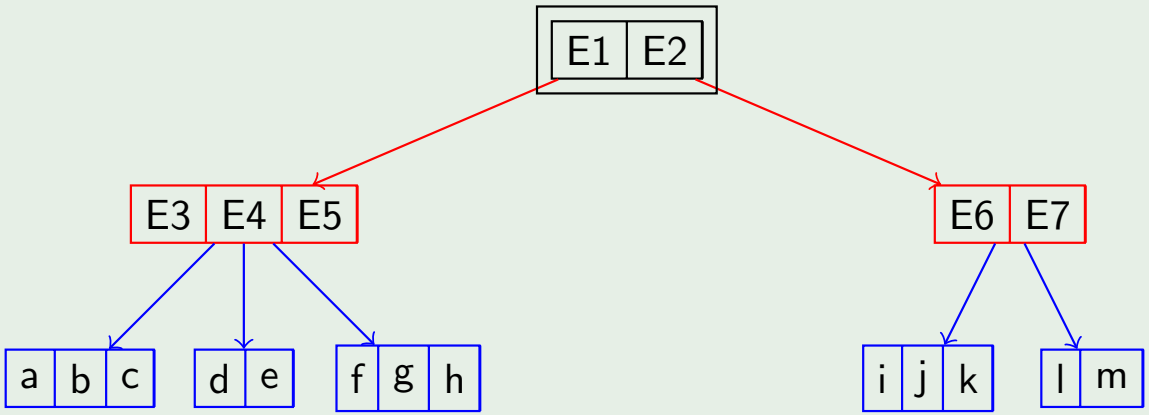

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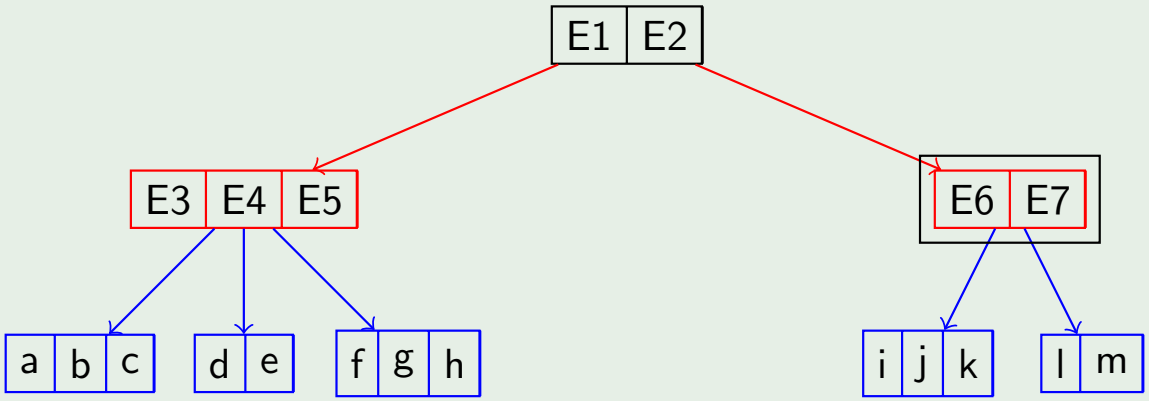

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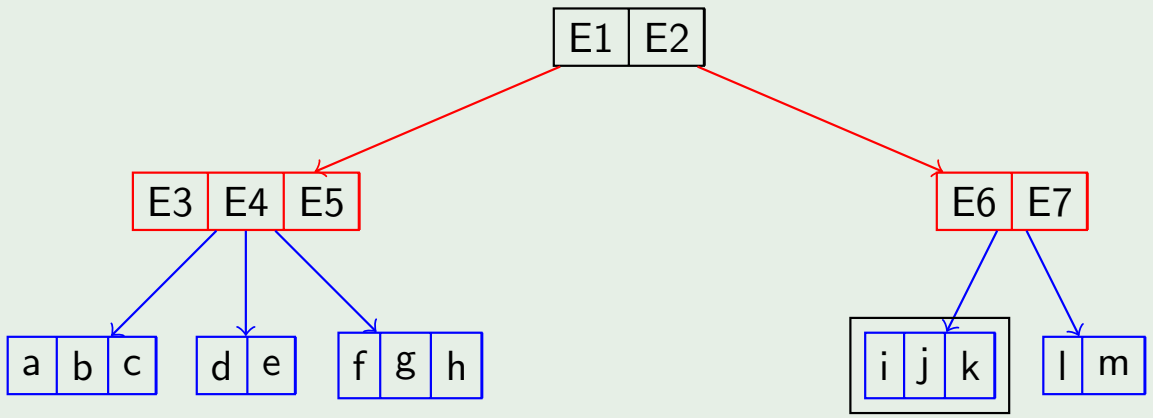

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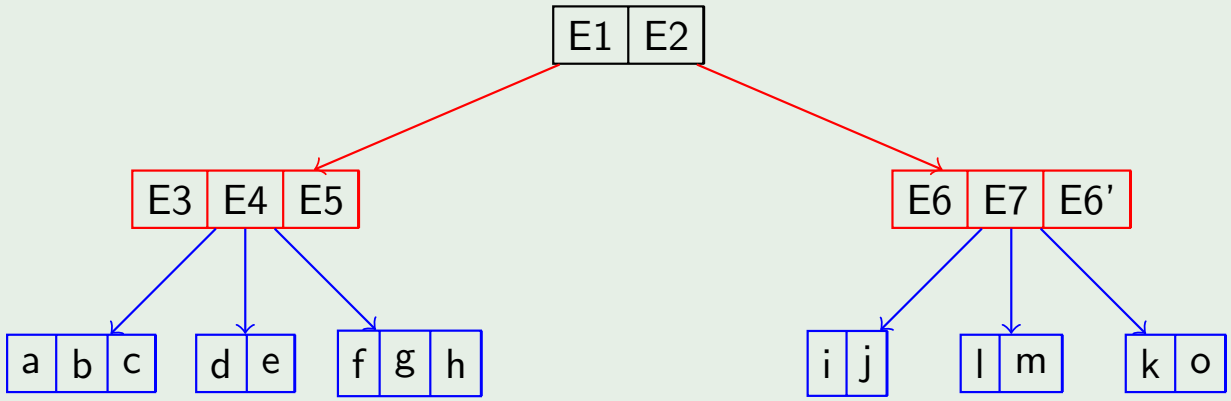


Delete Algorithm

Delete entry E from an R-Tree.

D1 [Find node containing record] Start **FindLeaf** to find the leaf node L con-taining E. If search unsuccessful then terminate.

D2 [Delete record] Remove E from L.

D3 [Propagate record] Start **CondenseTree** on L.

D4 [Shorten tree] If the root node has only one child after adjusting then make the child the new root.

FindLeaf Algorithm

Root node is T, the leaf node containing the index entry E is to find.

FL1 [Search subtree] If T is not a leaf, then check each entry F in T to determine when $F.I$ overlaps $E.I$. For all these entries **FindLeaf** starts on the subtree whose root is pointed to by $F.p$ until E is found or each entry has been checked.

FL2 [Search leaf node for record] If T is a leaf, then check each entry to see when it matches E. If E is found, then return T.

CondenseTree Algorithm I

Given is a leaf node L from which an entry has been deleted. If L has too few entries then eliminate it from the tree. After that, the remaining entries in L are reinserted in the tree. This procedure is repeated until the root. Also adjust all covering rectangles on the path to the root, making them smaller, if possible.

CT1 [Initalize] N is equal L. Initialize a list Q which consists of eliminate nodes as empty.

CT2 [Find parent entry] If N is the root, then go to *CT6*. Else P is the parent node of N , and E_N the entry of N in P.

CT3 [Eliminate underflow node] If N has fewer than m entries, then eliminate E_N from P and add N to list Q.

CT4 [Adjust covering rectangle] If N has not been deleted, then adjust $E_N.I$ to tightly contain all entries in N.

CT5 [Move up one level in tree] N is equal P and repeat from *CT2*.

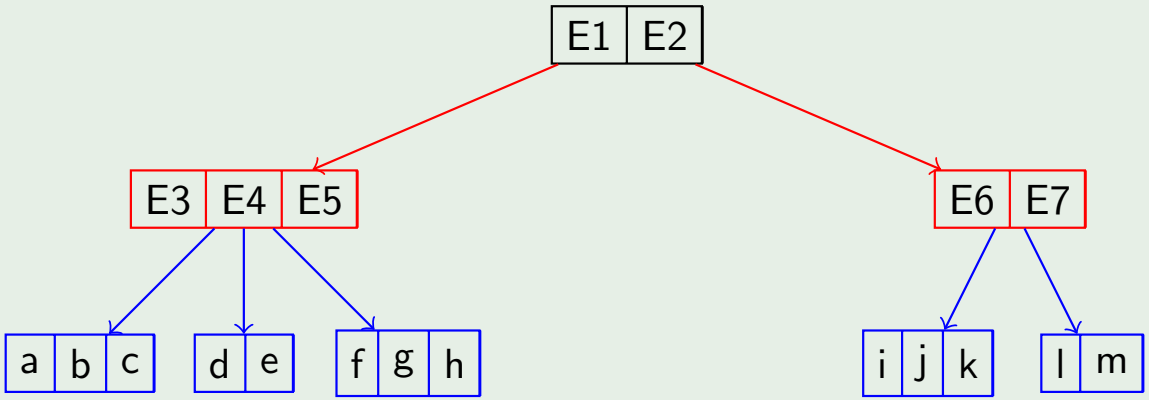
CondenseTree Algorithm II

CT6 [Re-insert orphaned entries] Every entry in Q is inserted. Leaf nodes are inserted like in *Insertion*. However, entries from higher-level nodes must be placed higher in the tree, so that leaves of their dependent subtrees will be on the same level as leaves of the main tree.

Delete Algorithm

Delete l where $m = 2$ and $M = 3$.

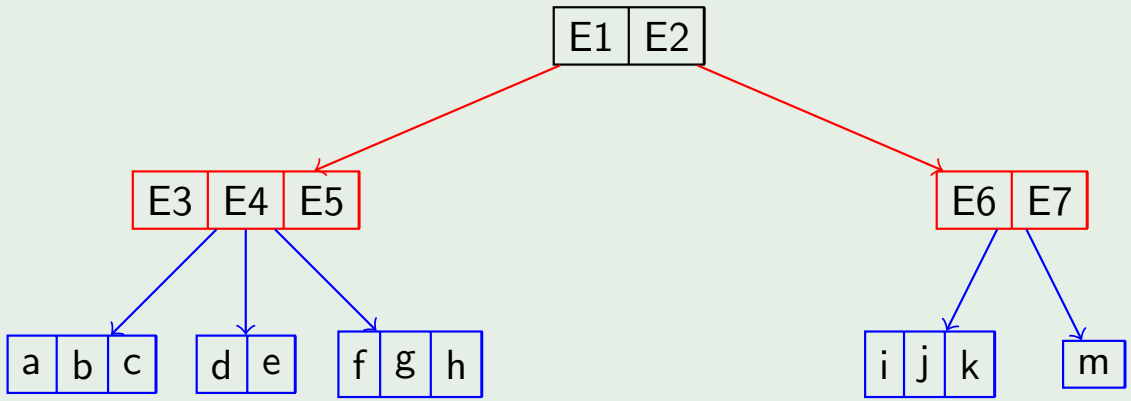
Example



Delete Algorithm

Deleting *l* left *E7* with an underflow.

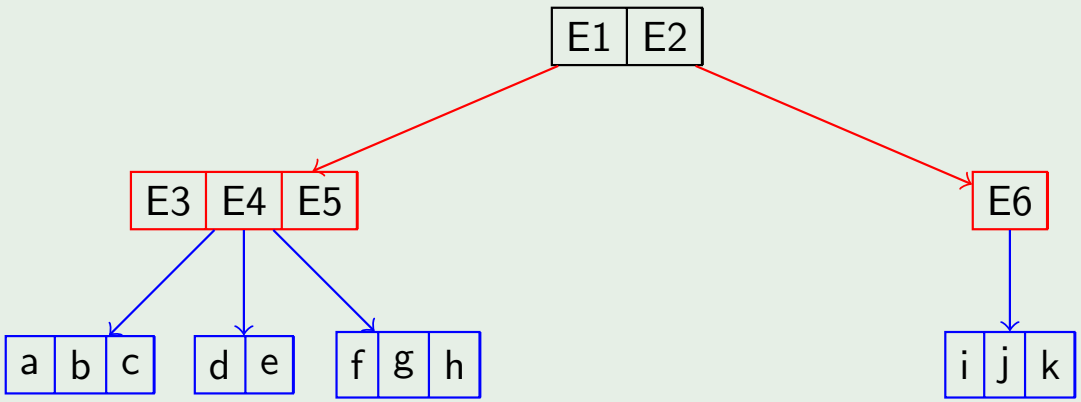
Example



Delete Algorithm

Save *m* and remove *E7*; *E2* now has an underflow.

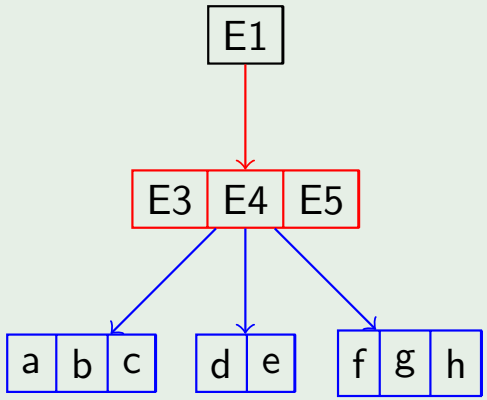
Example



Delete Algorithm

Save *E6* and remove *E2*.

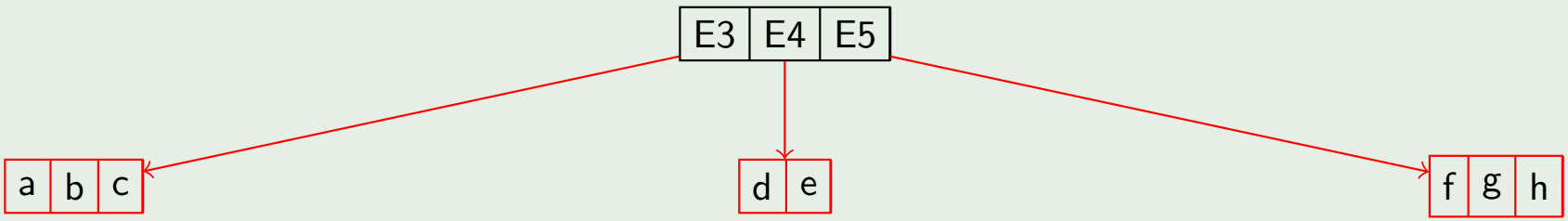
Example



Delete Algorithm

The root has only one child and thus the child becomes the new root.

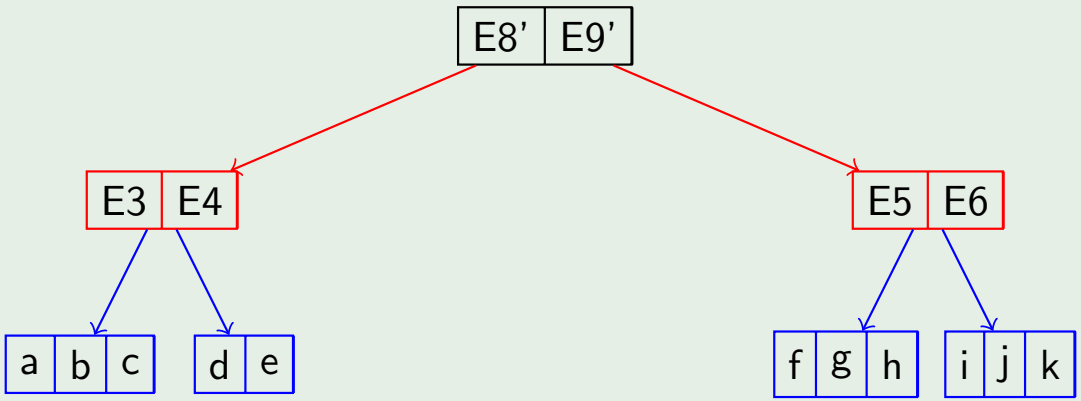
Example



Delete Algorithm

Insert *E6* which causes the root to split.

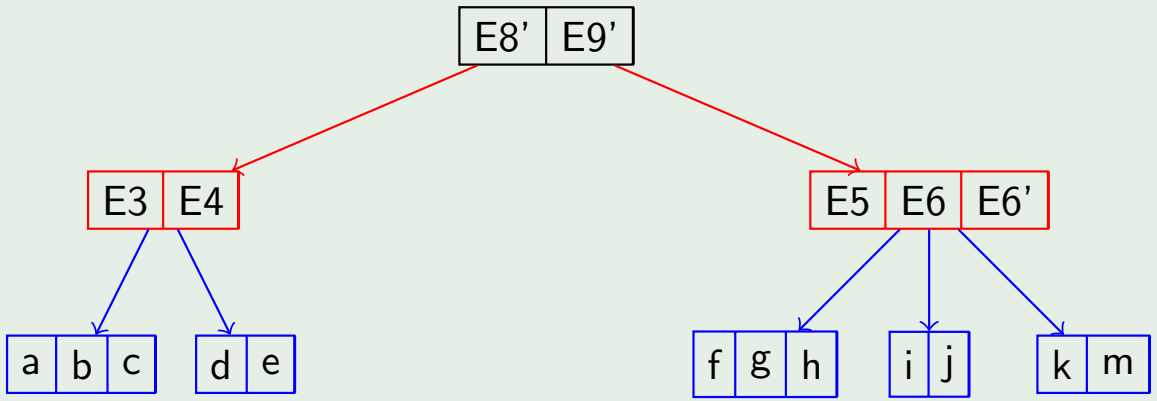
Example



Delete Algorithm

Insert *I*; *E6* is split.

Example



GiST Motivation

- New applications
  - Geographic information systems
  - Multimedia systems
  - CAD tools
  - Document libraries
  - Sequence databases
  - Fingerprint identification systems
  - Biochemical databases
- Rapid data type introduction; in need of equally adaptable search trees.

Prior Approaches

- Specialized Search Trees
  - e.g. Spatial Search Trees (R-Trees)
  - High cost of implementation and maintenance
- Search Trees for Extensible Data Types
  - e.g. B+-trees can be used to index any data with linear ordering
  - Extending data does not extend set of queries supported by tree

GiST: A New Hope

- A third direction for extending search tree technology
- Extensible in both the data types it can index and the queries it can support
- Allows new data types to be indexed in a manner that supports the queries natural to the types
- Unifies previously disparate structures used for currently common data types
  - e.g. B+-trees and R-trees can be implemented as extensions of the GiST. Single code base yet indexes multiple dissimilar applications.

Search Trees

- Balanced tree; high fanout
- **Search Key:** any arbitrary predicate that holds for each datum below the key
- **Search Tree:** hierarchy of categorizations, in which each categorization holds for all data stored under it in the hierarchy

Generalized Search Tree

- Balanced tree of variable fanout between kM and M
  - k is the minimum fill factor of the tree, $\frac{2}{M} \leq k \leq \frac{1}{2}$
  - The exception is the root which may have a fanout between 2 and M
- Non-leaf nodes *(p, ptr)*
  - *p*, predicate that is used as a search key
  - *ptr*, pointer to another tree node
- Leaf nodes *(p, ptr)*
  - *p*, predicate that is used as a search key
  - *ptr*, identifier of some tuple in the database

GiST Properties

1. Unless the node is root every node contains between kM and M index entries
2. For leaf nodes, p is true when instantiated with the values from the indicated tuple
3. For non-leaf nodes, p is true when instantiated with the values of *any* tuple reachable from *ptr*
4. The root has at least two children unless it is a leaf
5. All leaves appear on the same level

Key Methods

- Search
  - **Consistent**(E, q): Asks $E.p \wedge q$
- Characterization
  - **Union**(P): returns new predicate that holds for all tuples in P
- Categorization
  - **Penalty**(E_1, E_2): penalty for inserting E_2 into E_1
  - **PickSplit**(P): split P into two group of entries
- Compression
  - **Compress**(E): returns compressed representation of p
  - **Decompress**(E): returns an entry such (r, ptr) such that $p \rightarrow r$

Search Algorithm

Recursively descend all paths in tree whose keys are consistent with q.

S1 [Search subtrees] If R is not a leaf, check each entry E on R to determine whether Consistent(E, q). For all entries that are Consistent, invoke Search on the subtree whose root node is referenced by $E.ptr$.

S2 [Search leaf node] If R is a leaf, check each entry E on R to determine whether Consistent(E, q). If E is Consistent, it is a qualifying entry. At this point $E.ptr$ could be fetched to check q accurately, or this check could be left to the calling process.

Insert Algorithm

Find where E should go, and add it there, splitting if necessary to make room.

I1 [invoke ChooseSubtree to find where E should go] Let $L =$ ChooseSubtree(R, E, l)

I2 If there is room for E on L, install E on L (in order according to Compare, if IsOrdered.) Otherwise invoke Split(R, L, E).

I3 [propagate changes upward] AdjustKeys(R, L).

Delete Algorithm

Remove E from its leaf node. If this causes underflow, adjust tree accordingly. Update predicates in ancestors to keep them as specific as possible.

D1 [Find node containing entry] Invoke Search(R, E, p) and find leaf node L containing E. Stop if E not found.

D2 [Delete entry.] Remove E from L.

D3 [Propagate changes.] Invoke CondenseTree(R, L).

D4 [Shorten tree.] If the root node has only one child after the tree has been adjusted, make the child the new root

GiSTs over B+-Trees I

- **Consistent**(E, q) returns true if
  - q = Contains($[x_q, y_q)$, v): $(x_p < y_q) \wedge (y_p > x_q)$
  - q = Equal(x_q, v): $x_p \leq x_q < y_p$
- **Union**(P) returns $[MIN(x_1, \ldots, x_n), MAX(y_1, \ldots, y_n))$
- **Penalty**(E, F)
  - If E is the leftmost pointer on its node, returns $MAX(y_2 - y_1, 0)$
  - If E is the rightmost pointer on its node, returns $MAX(x_1 - x_2, 0)$
  - Otherwise, returns $MAX(y_2 - y_1, 0) + MAX(x_1 - x_2, 0)$
- **PickSplit**(P) Let the first $\lfloor \frac{|P|}{2} \rfloor$ entries in order go in the left group, and the rest in the right

GiSTs over B+-Trees II

- **Compress**(E) If E is the leftmost key on a non-leaf node return 0 bytes otherwise, returns $E.p.x$
- **Decompress**(E)
  - If E is the leftmost key on a non-leaf node let $x = -\infty$ otherwise let $x = E.p.x$
  - If E is the rightmost key on a non-leaf node let $y = \infty$. If E is other entry in a non-leaf node, let $=$ the value stored in the next key. Otherwise, let $y = x + 1$

GiSTs over R-Trees I

Key: $(x_{ul}, y_{ul}, x_{lr}, y_{lr})$

- Query predicates
  - Contains $((x_{ul1}, y_{ul1}, x_{lr1}, y_{lr1}), (x_{ul2}, y_{ul2}, x_{lr2}, y_{lr2}))$
    - Returns true if
      $(x_{ul1} \leq x_{ul2}) \wedge (y_{ul1} \geq y_{ul2}) \wedge (x_{lr1} \geq x_{lr2}) \wedge (y_{lr1} \leq y_{lr2})$
  - Overlaps $((x_{ul1}, y_{ul1}, x_{lr1}, y_{lr1}), (x_{ul2}, y_{ul2}, x_{lr2}, y_{lr2}))$
    - Returns true if
      $(x_{ul1} \leq x_{lr2}) \wedge (y_{ul1} \geq y_{lr2}) \wedge (x_{ul2} \leq x_{lr1}) \wedge (y_{lr1} \leq y_{ul2})$
  - Equal $((x_{ul1}, y_{ul1}, x_{lr1}, y_{lr1}), (x_{ul2}, y_{ul2}, x_{lr2}, y_{lr2}))$
    - Returns true if
      $(x_{ul1} = x_{ul2}) \wedge (y_{ul1} = y_{ul2}) \wedge (x_{lr1} = x_{lr2}) \wedge (y_{lr1} = y_{lr2})$

GiSTs over R-Trees II

- **Consistent**(E, q)
  - p contains $(x_{ul1}, y_{ul1}, x_{lr1}, y_{lr1})$, and q is either Contains, Overlaps, or Equals $(x_{ul2}, y_{ul2}, x_{lr2}, y_{lr2})$
  - Returns true if Overlaps $((x_{ul1}, y_{ul1}, x_{lr1}, y_{lr1}), (x_{ul2}, y_{ul2}, x_{lr2}, y_{lr2}))$
- **Union**(P) returns coordinates of the maximum bounding rectangles of all rectangles in P.
- **Penalty**(E, F) Compute q = Union(E,F) and return $area(q) - area(E.p)$
- **PickSplit**(P) Variety of algorithms are provided to best split the entries in a over-full node.
- **Compress**(E) Form the bounding rectangle of $E.p$
- **Decompress**(E) The identity function

Issues

- Performance
  - Data overlap
  - Lossy compression
- Implementation
  - Concurrency Control, Recovery and Consistency
  - Variable-Length Keys
  - Bulk Loading