

It teaches solutions to complex math problems with step-by-step animation. The software has been developed by experienced software engineers with a Ph.D. in Computer Science, in conjunction with the faculty of the University of Michigan, and in consultation with elementary and high school mathematics teachers.

Math Guru

(Version 1)

Developed by

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Math Guru (v. 1) source code will be provided to freelance program developers who would like to enhance the features of Math Guru. The object code will be provided free to all school students upon request. For details, contact Prof. P. Mazumder at pinakimazum@gmail.com.

Flash Cards Elementary school arithmetic taught through animation.

Number Problems

Teach basic arithmetic techniques, sharpen verbal calculation skills.

- Addition and Subtraction
- Multiplication and Division
- Factors and LCM
- Fractions and Decimals
- Fractional Addition and Subtraction
- Fractional Multiplication and Division
- Brackets, Percentage, Average

Word Problems

Teach every-day application of math
The kid's practical math laboratory

- Dollars and Cents
- Direct Proportion
- Fractions and Percentage
- Average
- Simple Interest
- Ratio and Proportion

Features

- Grades K through 6 officially supported. Useful to kids through eighth grade.
- Comprehensive scoring and logging mechanism to keep track of your kid's progress
- Teaches solution methods for complex problems through animation
- Reduces kid's dependence on calculators by providing extensive practice for verbal arithmetic
- Problems ranging from single-digit to 8-digit precision

Math Guru

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Chapter 1

Preliminaries

1.1 A Note to Parents

This software is primarily intended to reduce the dependence of kids on calculators, and to teach them all kinds of verbal arithmetic problems encountered in daily life. Doing arithmetic is like riding a bicycle. You have to learn it, and practice it, before you can use it all your life. Similarly, Math Guru will only be of use if your kids use it regularly and practice it until they are experts.

All problems in Math Guru must be solved by verbal calculations. Using a calculator will reduce the benefits obtained from Math Guru. Mathguru is a computer tutoring program that requires very little parental monitoring. We have taken much of the burden of teaching the solution methods off of your shoulders. If we have done our work right, your kids should be able to learn how to solve the problems on their own, using Math Guru's animation features, with little or no help from you. Math Guru allows you to monitor your kid's progress by keeping a record of the

time spent, problems solved, and scores obtained during the last several sessions.

1.2 Installation

Math Guru will not run from the floppy disk. It must be installed on your computer. Start Windows. From the installation disk, run `install.exe`. Enter the source floppy drive name, such as **A:** or **B:**, and the destination directory, such as **C:MATHGURU**. Click **OK**. The software will be copied to the destination directory, and the icon will be installed.

Chapter 2

Getting Started

2.1 A Test drive of Math Guru

Step 1: Starting Math Guru. When you start Math Guru, it asks for your name. If you have entered the same name before, then it will set the problem sizes, and other settings the same as the last time you used it. If you enter a new name, then it will set the problem sizes to the maximum size.

After you enter your name, Math Guru will be ready to give you some arithmetic problems. When you are not solving any problems, the screen will contain some simple instructions on how to use Math Guru.

The menu bar on top of the screen contains the **Options** menu, for selecting various options, and the **Problems** menu for selecting problems. If you click on any menu, such as **Options**, with the mouse, it will open several menu items, as shown in Fig. 2.1. An item, such as **Score** under the **Options** menu is described in this manual as **Options** → **Score**. It is invoked by first clicking the mouse on the **Options** menu, and then on

the **Score** menu.

Step 2: Setting the Grade. Select **Problems** → **Grade**, i.e. click on the **Problems** menu, and select **Grade**. A box will open up, in which you can select the grade from K to 6. This will set all the problem sizes and problem types automatically. It will also select all of the problems that are suitable for that grade. These problems will be presented in mixed order.

The grade level may be close to your actual grade in school, but it may not be exact. Try different grades, and find one that is right for you. The right grade is the one that is not be too easy, or too hard. Start from there, and increase the grade as you learn more. In the next chapter, we will learn how to select each problem individually, so that you can learn and practice one problem at a time.

Step 3: Getting a New Problem, and solving it. Click the **New** button at the right edge of the screen. A new problem will be presented on the screen, as in Fig. 2.2.

Click the mouse in each box, and fill it with numbers. Click on a decimal point box to place the decimal point. After you have filled all the boxes, click the **Done** button on the right edge of the screen. The computer will mark all the errors in red. Click on the boxes marked in red, and use the backspace key to correct the numbers.

If all the numbers are correct, click the **Done** button to go to the next problem. Click the **Quit** button to stop solving problems.

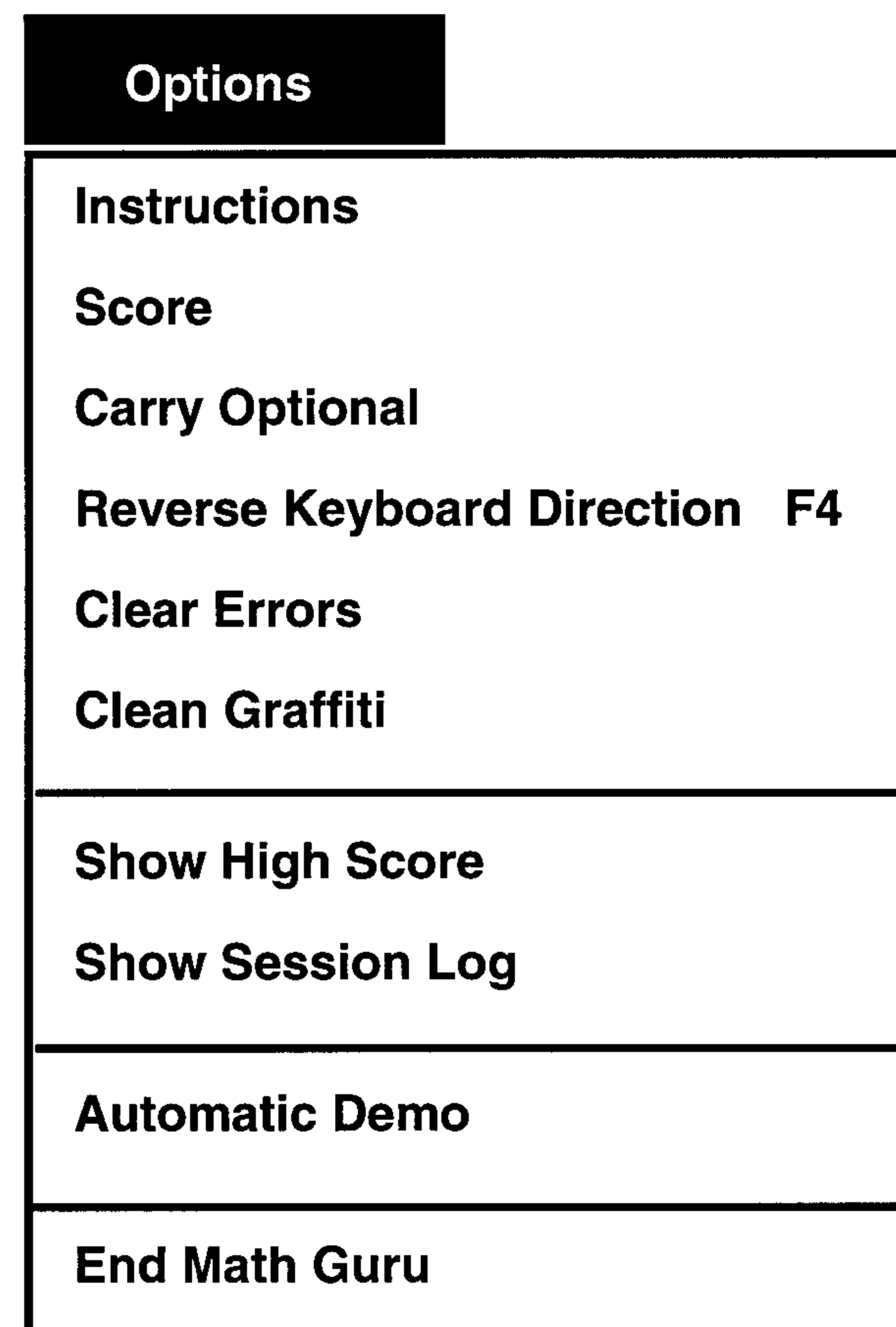


Figure 2.1: The Options Menu

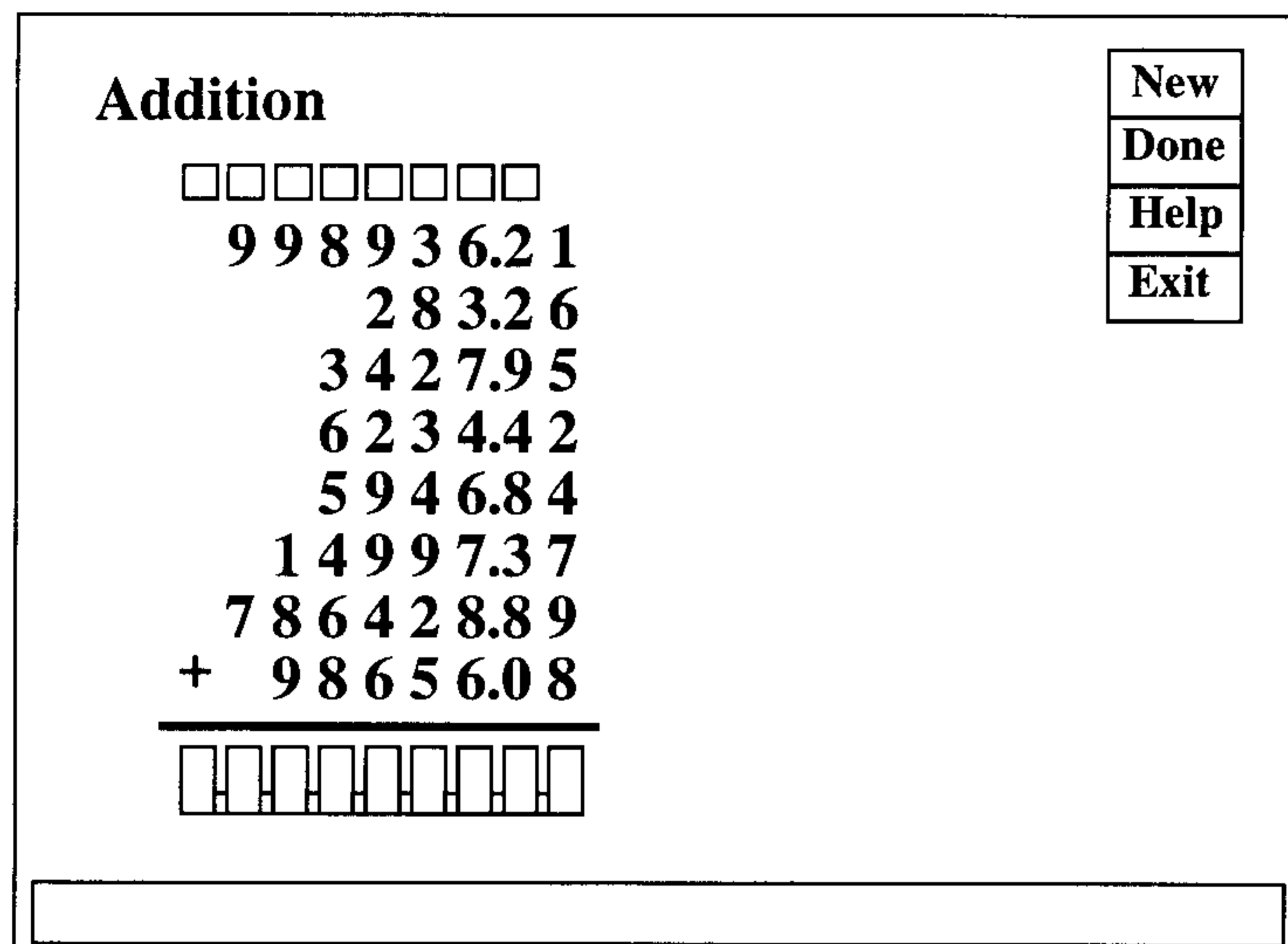


Figure 2.2: The Addition Problem

Step 4: Moving between the boxes with Arrow keys. Use the **left** and **right** arrow keys to move between the boxes. By pressing the **left** or **right** arrow keys, the computer automatically moves to the next box that you should fill, in the correct order. You can also go to any box by clicking the box with the mouse.

Step 5: Getting Help. If you want to know what a box is for, click the mouse in the box, and then click the **Help** button on the right edge of the screen. The computer will move some of the numbers to the bottom of the screen, and will ask you to solve a smaller part of the problem. After you have filled the **Help** boxes at the bottom of the screen, click on the **Done** button. If the boxes are filled correctly, the computer will move the result to where it is supposed to go. This will tell you what is

supposed to go in each box.

Step 6: Scrap Work. If you are solving large problems, you will need to do some scrap work. Click the mouse outside a box, and start typing. You can do any scrap work anywhere on the screen provided it is outside one of the number boxes.

Step 7: Reversing Keyboard Direction. If you are solving large problems with multiple digits in each box, you may need to enter numbers from right to left, as in addition and multiplication, or from left to right, as in division. Select **Options** → **Reverse Keyboard Direction** to reverse the keyboard direction. If the direction has been reversed, the keys will click. Reverse the direction once again to make it normal.

2.2 Setting Other Options

Carry Optional. If you do not want to fill out the carry squares, select **Options** → **Carry Optional**, as shown in Fig. 2.1. If you click on the **Options** menu again, you will see that the **Carry Optional** item has a check mark on it, indicating that the carry is optional. If you select **Options** → **Carry Optional** once again, the check mark will disappear, and the carry will become compulsory again. Carries only appear in addition, subtraction, and multiplication problems.

Carry boxes are important. For large problems, you should fill the carry boxes, so that if you made a mistake in one column, you would know what the carry was, so that you can easily correct your mistake. If

you do not fill out the carry boxes, then you may have to solve the entire problem again.

Scoring. If you want to turn on scoring, select the **Options** → **Score** option. When selected, it will be checked. Click on it once again to unselect it. When scoring is on, you will get a limited amount of time to solve each problem. If you cannot finish the problem on time you will lose 1/5 of your score. You will be given five chances with increasing time limits to finish the problem, after which you will get a zero score for the problem. When a problem is displayed, a score board will also be displayed, giving the time, number of problems solved with the scoring option, total score, and average score. The average score will be recorded in the high score file.

Scoring is like an exam. When scoring is on, you must do the problems very quickly, you cannot use the animation/help features, and you cannot go to the next problem without completing the present one. Use the scoring option for a problem after you have a lot of practice with that problem.

Scrap Work. If you want to do scrap work on the screen, click the mouse outside any box anywhere on the screen, and start typing. You can write anything anywhere for scrap work. Also, if you hold down the mouse button, and move the mouse, you can draw lines. This is useful for cancellation and some other problems. You will need scrap work only with the larger problems. To erase the scrap work, select the **Options** → **Clean Graffiti** option. The scrap work is automatically cleaned after each problem. Scrap work will be in green, and will be ignored by the

computer. If you click inside a box, and enter the numbers, they will be black, and will be checked by the computer. Although you can draw cartoons with this feature, but Math Guru is most useful for learning math.

Clear Errors. If you click the **Done** button when many of the boxes are blank, the computer will fill them with zeroes and mark them red. You can use **Options** → **Clear Errors** to empty all these red squares.

Show High Score and Session Log. The option **Options** → **Show High Score** displays the high score file. The high score of each person is maintained separately. Thus, kids of different ages and abilities can have their own personal high score. You can compare your high score with that of your friends. Your best **average** score on any day is saved as the high score. In order to enter your score in the high score file, you will have to solve at least 5 problems in one session. Then if your average for that session is higher than the previous high score, it will be saved.

The option **Options** → **Show Session Log** shows a record of your activities, such as the time for which Math Guru was running, the number of problems solved in that time, the score etc. The last 20 sessions (or less) are displayed, older sessions are erased. A session is the time for which you run the program, until you quit. In order to make a session log entry, you must quit the program either by using the **Options** → **End Math Guru** option, or the button in the top left corner of the window. If you turn off the computer while the program is running, a high score or session log entry will not be made.

Reverse Number Entry. In some large problems, such as addition, subtraction, or multiplication of large numbers, where you need to enter several digits in one box, you can enter the digits backwards, as you calculate one digit at a time. Press the **F4** key to start entering numbers backwards (right to left). There will be a click sound when you press any key. Press the **F4** key again to start entering numbers left to right again. If there is only one digit in a box, then there will be no significant difference either way. The keyboard direction can also be reversed from the **Options → Reverse Keyboard Direction** menu.

When entering numbers backwards, the cursor is to the left of the number, and the number is at the right edge of the number box. The direction of the backspace key is also reversed. Try it on a large number box with several digits and get familiar with it. See what happens when you press **F4** after entering half the number.

Reverse number entry also works for scrap work. Click the cursor anywhere outside a number box, and try it.

Cancel Help. If you clicked the **Help** button, but do not want to fill out all the Help squares at the bottom of the screen, then cancel the help by clicking on the **Quit** button at the right edge of the screen. You can get help again by clicking the **Help** button.

2.3 Selecting Problems and Problem Sizes

The **Problems** menu is shown in Fig. 2.3. There are two ways to select problems and problem sizes. The easy way is to set the grade from the **Problems → Grade** menu. This will select all the problems suitable

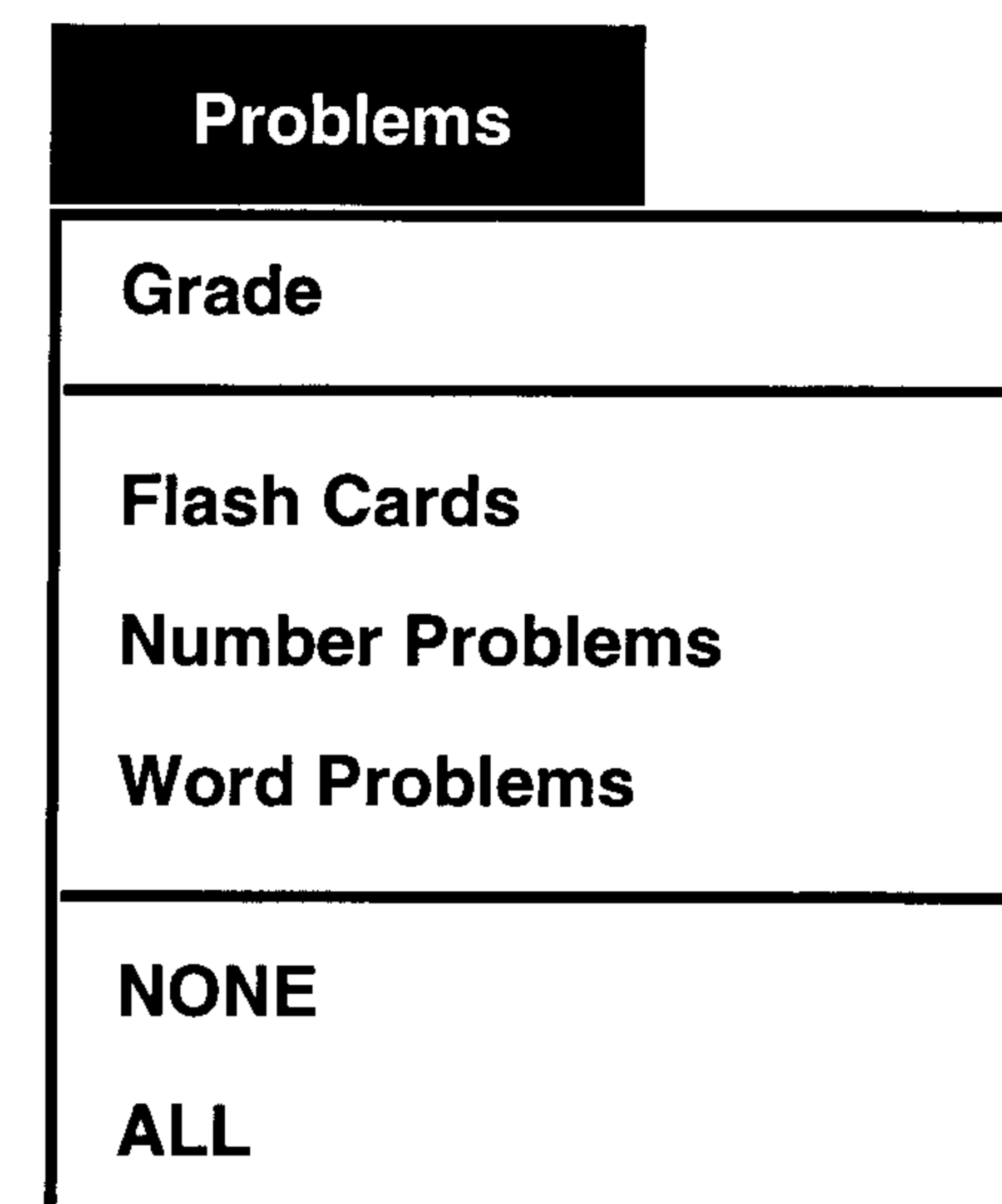


Figure 2.3: The Problems Menu

for that grade, and set appropriate sizes automatically. All the selected problems will then be presented in mixed order. If the problem sizes are not suitable for you, just select a lower or higher grade, or look at the method below.

In order to learn one problem at a time, you should select only that problem, and practice it. First select the **Problems → None** menu item. This will unselect all problems. Then select one of three options, **Flash Cards**, **Number Problems**, or **Word Problems** in the **Problems** menu. This manual contains a chapter on each type of problem. In each case, a box will open up, giving further options.

Flash Cards. Fig. 2.4 shows the flash card selection box. The box has four problems, **Addition**, **Subtraction**, **Multiplication**, and **Di-**

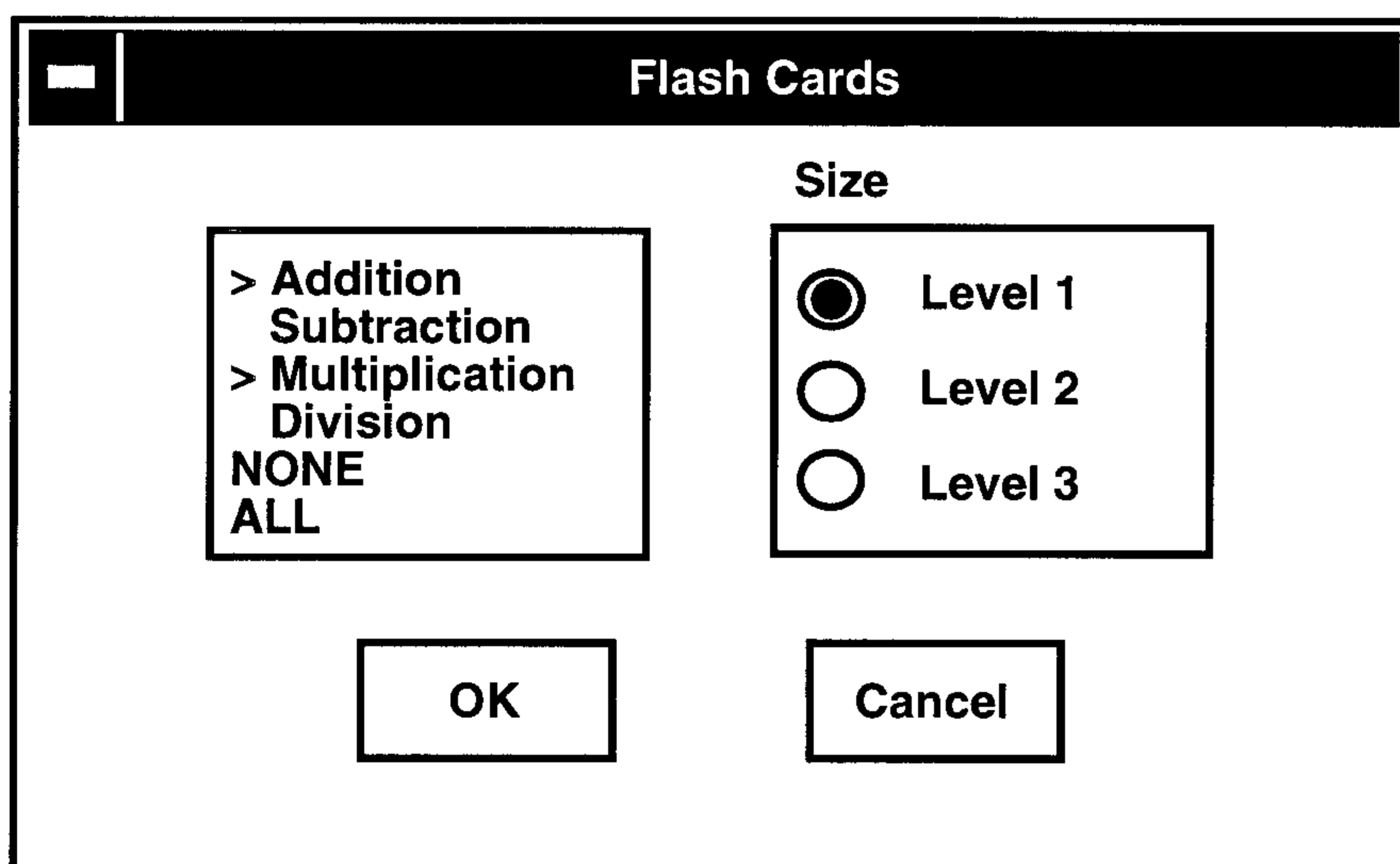


Figure 2.4: Flash Card Selection

vision. Each problem has three levels, **Small**, **Medium**, and **Large**. First click on a problem. A > sign will appear to the left of the problem, showing that the problem is selected. Then click on one of the levels.

In order to unselect the problem, click the problem again. The > sign next to the problem will disappear. To finalize your selection, click **OK**. Before you click OK, make sure that the problems you selected have a > sign to the left of them. If you click **Cancel**, the problems will remain the same as before you opened the box, and anything you clicked in the box will have no effect. The box also contains **All** and **None** options. If you click **All**, all four Flash Card problems will be selected, and a > sign will appear beside each of them. If you click **None**, all four flash card problems will become unselected.

Number Problems. If you select **Problems** → **Number Problems**, a box will open up containing all the number problems. Fig. 2.5 shows the number problem selection box. Click on the problems to select them. A > sign will appear to the left of the selected problems. For some problems, you can also select the number of numbers, and the level. For example, for addition problems at level 8, you get 8-digit numbers to add. Another option for some problems is integer or decimal.

Click on a problem, and then select its level, if available. Click on the problem again to unselect it. Make sure there is a > sign to the left of all of the problems that you selected, and then click **OK**.

Word problems. There are several word problem types. Each type has several individual problems. If you select **Problems** → **Word Problems**, a box will open up containing all the word problem types. Fig. 2.6 shows the word problem selection box. First click any problem type. A number of problem options will appear to the right of it. Click on one of the problem numbers. A > sign will appear beside the problem number to show that it is selected. Use the **ALL** or **NONE** options to select or unselect all word problems.

2.4 Tips on Getting Higher Scores

Each problem starts with a very small time limit. After the time expires, you lose 1/5 of your score, and you are given another chance with a larger time limit. In this way, you are given five chances with increasing time limits. For example, for a problem, the five time limits may be as follows.

Number Problems		
	Number of Numbers	Number of Digits
> Addition		
Subtraction		
> Multiplication		
Division		
Factors		
LCM		
Fractions		
Fractional Addition		
Fractional Subtraction		
Fractional Multiplication		
Brackets		
Percentage		
Average		
NONE		
ALL		
	2	2
	3	3
	4	4
	5	5
	6	6
	7	7
	8	8
	<input checked="" type="checkbox"/> Decimal	
	OK	
	Cancel	

Figure 2.5: Number Problem Selection

Word Problems	
Problem Type	Problem
Dollars and Cents, Level 1	> 1
Dollars and Cents, Level 2	2
Direct Proportion, Level 1	3
Direct Proportion, Level 2	> 4
Fractions and Percentage	> 5
Average	
Simple Interest	
Ratio and Proportion	
Other	
Exam, Random	
Exam, Selected	
NONE	
ALL	
	OK
	Cancel

Figure 2.6: Word Problem Selection

Time limit (Sec.)	100	+ 150	+ 225	+ 340	+ 510	+ unlimited
Score	100	80	60	40	20	zero

This means that for a problem that starts out with a time limit of 100 seconds, you will get a total of 1325 seconds (22 min.) before your score is reduced to zero. The time limits are adjusted so that only very smart kids, or kids who practice very hard can solve the problem within the first chance. With a little practice, any good student should be able to solve problems within the second chance.

In order to solve the problems quickly, and get higher scores, try the following method.

While solving the problem, devote your full concentration to the problem. Do not panic. Do not look at the clock often. Give more importance to accuracy than to speed. Try to solve a little slowly, but get it right in the first try. If you solve it incorrectly, you will have to re-do a lot of work and you will waste a lot of time. Do not try to make your scrap work beautiful. Fig. 2.7 shows a scrap work multiply and divide. I can do it in just a few seconds. Can you?

Practice the problems daily. Use Math Guru for at least one hour every day. Use the **Flash Card Multiplication** problem to practice your tables daily. Unless you are an expert at tables, you cannot do well in mathematics. There is no scoring option with the flash card problems. Allow yourself **one** second and **one** try for each flash card problem. If you hit the **Done** button and the answer is incorrect, that means that you need more practice.

Larger problems have a higher score. Even if you can do them in the third or fourth chance, you may be able to improve your high score. So

$$\begin{array}{r}
 4068 \\
 \times 23 \\
 \hline
 12204 \\
 8136 \\
 \hline
 93564
 \end{array}
 \qquad
 \begin{array}{r}
 141 \\
 26 \overline{) 3681} \\
 \underline{26} \\
 108 \\
 \underline{104} \\
 41 \\
 \underline{26} \\
 15
 \end{array}$$

Figure 2.7: Scrap work Multiply (4068 x 23) and divide (3681 / 26)

learn how to solve larger problems.

Your high score is the average score for the day. If you get a good score in the first problem, do not relax. A bad score in the next problem can lower your average. Remember, you must solve at least 5 problems in order to make an entry into the high score file. So, for a high average, you must do well on all 5 problems.

Use the keyboard instead of the mouse. See the following section on using the keyboard.

Finally, if you do get a poor score, don't worry. Try again the next day for a better score.

2.5 Using the keyboard

If you use the keyboard instead of the mouse, your work can be faster, and you can get a higher score.

Use the **left** and **right** arrow keys to move between the boxes. By

pressing the **left** or **right** arrow keys, the computer automatically moves to the next box that you should fill, in the correct order.

For the addition, subtraction, multiplication, and division problems, use the **left** arrow key to go forward, and the **right** arrow key to go back. This is because these problems are usually solved from right to left. Use the **down** arrow key to fill the decimal point box. Use the **up** arrow key to go to the carry box. This is useful if you have selected **Options** → **Carry Optional**, and the computer does not automatically go to the carry boxes.

For all other problems, use either the **right** or **down** arrow keys to go forward, and the **left** or **up** arrow keys to go back.

Use the **Enter** or **Return** key to activate the **Done** button. Use the **Esc** key to activate the **Quit** button. Use the **h** or **?** key to activate the **Help** button. Use the **n** key to activate the **New** button, in order to get a new problem. Use the **F4** key to reverse the keyboard direction.

Chapter 3

Flash Cards

The **Flash Cards** contain simple arithmetic for small kids. They teach the physical concept of addition, subtraction, multiplication, and division in terms of counting apples. They also teach the place-value system, or the meaning of the units, tens, and hundreds digits, and their relationship to addition, subtraction, and multiplication. Flash Card Multiplication problems are very important for learning multiplication tables. Kids of all ages should practice and master their tables first.

3.1 Flash Card Addition

The screen layouts for flash card addition are given in Fig. 3.1. Two small numbers are presented for verbal addition. If the help button is clicked, each number is animated into the corresponding number of apples, and you are told that adding the two numbers is equivalent to counting all the apples. When you click the **Continue** button, the apples are separated into the units, and tens boxes, and you are asked to count the number

of rows of apples in each box, and enter the result below. When you click the **Done** button, these numbers are moved to the main flash card answer box.

3.2 Flash Card Subtraction

Two small numbers are presented for verbal subtraction. If the help button is clicked, the larger number is animated into the corresponding number of apples. When you click the **Continue** button, the apples for the smaller number are taken away, and put in a shaded box. The remaining apples are moved to the units and tens boxes. You are asked to count the number of rows of apples in each box, and enter the result below. When you click the **Done** button, these numbers are moved to the main flash card answer box. An example is shown in Fig. 3.2

3.3 Flash Card Multiplication

Multiplication is presented as a problem of counting apples in rows and columns. The number of rows and the number of columns are the two numbers being multiplied. The apples are separated into the units, tens, and hundreds boxes. An example is shown in Fig. 3.3

3.4 Flash Card Division

The dividend is animated into apples. These apples are then separated into a number of groups equal to the divisor. The number of apples in

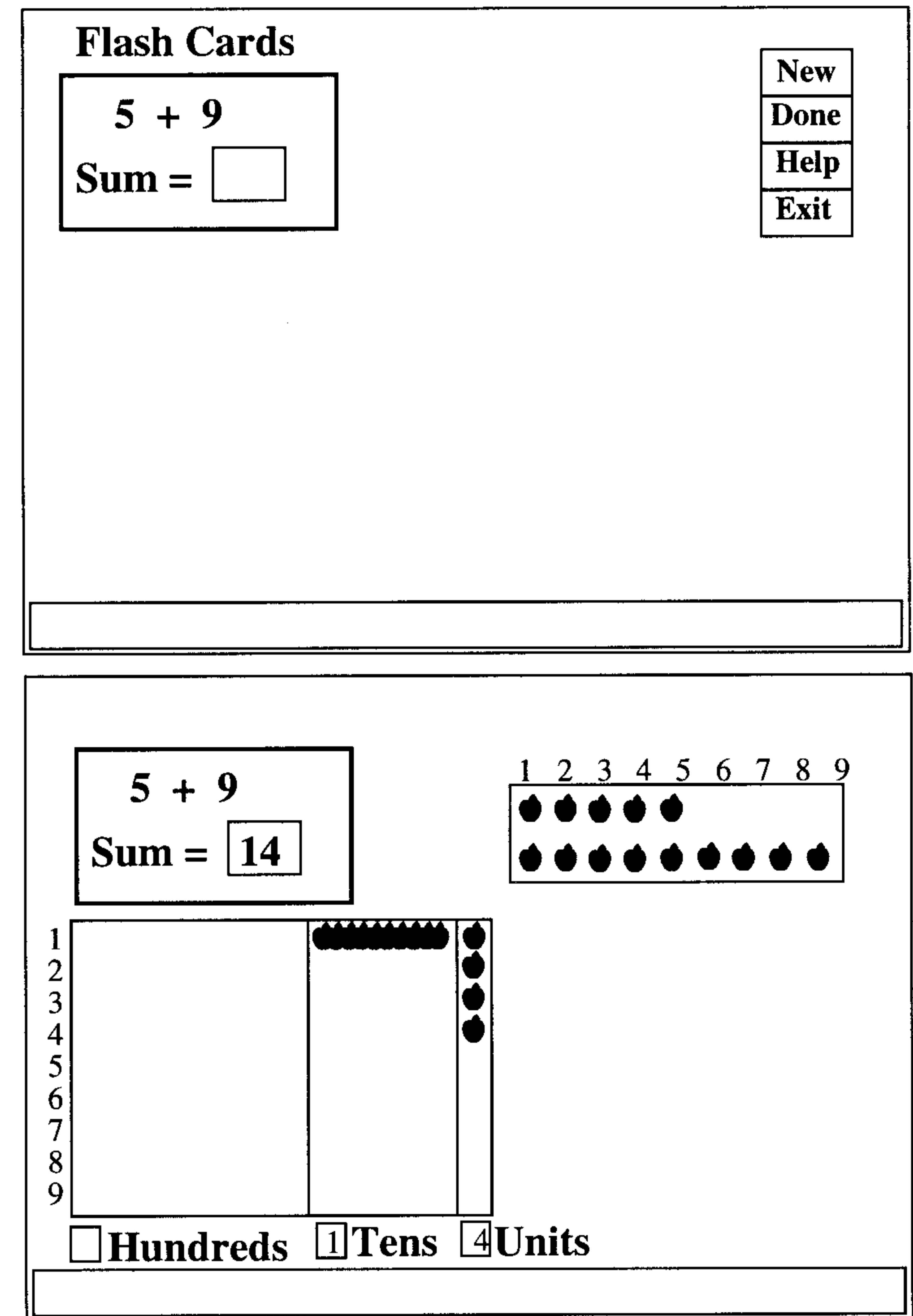


Figure 3.1: Flash Card Addition

Flash Cards

$9 - 5$
 Sum =

New
 Done
 Help
 Exit

$9 - 5$
 Sum =

1	2	3	4	5	6	7	8	9
●	●	●	●	●	●	●	●	●
●	●	●	●	●				

1	2	3	4	5	6	7	8	9
							●	●
							●	●
							●	●

Hundreds Tens Units

Figure 3.2: Flash Card Subtraction

Flash Cards

3×4
 Sum =

New
 Done
 Help
 Exit

3×4
 Sum =

1	2	3	4	5	6	7	8	9
●	●	●	●					
●	●	●	●					
●	●	●	●					

1	2	3	4	5	6	7	8	9
							●	●
							●	●
							●	●

Hundreds Tens Units

Figure 3.3: Flash Card Multiplication

each group is the quotient, and the number of apples remaining, which could not be divided equally between the groups, are the remainder. An example is shown in Fig. 3.4

Flash Cards

10 / 4
Quotient =
Remainder =

New

Done

Help

Exit

Animation, step 1

10 / 4
Quotient =
Remainder =

Divide 10 apples into 4 equal parts

1 2 3 4 5 6 7 8 9 10

● ● ● ● ● ● ● ● ● ●

1 2

1 ● ●

2 ● ●

3 ● ●

4 ● ●

Figure 3.4: Flash Card Division

Chapter 4

Number Problems

4.1 Addition

From the **Problems** menu, select **Number Problems**. A box will open up, showing all the number problems. Click on **Addition**. A > sign will appear to the left of the problem selected. To unselect the problem, click on the problem once again. Choose the number of numbers, and the level. The number of digits in each number depends on the level. You can also choose whether the numbers contain a decimal point or not. Make sure that there is a > sign to the left of the problems you have selected. Then click **OK**. Click the **New** button.

Use the **Left** arrow key to move forward to the next box. Use the **Right** arrow key to go back to the previous box. Add the numbers in each column. Write the answer in the box below the column, and the carry in the box above the next column.

Example:

1	2	3	4	4	3	3	4						
	2	8	2	4	8	4	3	3					
			2	5	1	9	6	9					
			2	9	9	5	0	3	1				
				2	9	4	3	8					
					8	7	8	0	5				
				6	9	5	9	7	4	7	9		
					+	2	6	8	0	0	0	5	9
1	2	8	0	1	0	2	1	4					

New
Done
Help
Quit

Help:

$$3 + 9 = 12 + 1 = 13 + 8 = 21 + 5 = 26 + 9 = 35 + 9 = 44$$
$$4 + 3 = 7 + 6 = 13 + 3 = 16 + 3 = 19 + 7 = 26 + 5 = 31$$
$$3 + 4 = 7 + 9 = 16 + 4 = 20 + 8 = 28 + 4 = 32$$
$$3 + 8 = 11 + 1 = 12 + 5 = 17 + 9 = 26 + 7 = 33 + 7 = 40$$
$$4 + 4 = 8 + 5 = 13 + 9 = 22 + 2 = 24 + 8 = 32 + 9 = 41$$
$$4 + 2 = 6 + 2 = 8 + 9 = 17 + 5 = 22 + 8 = 30$$
$$3 + 8 = 11 + 2 = 13 + 9 = 22 + 6 = 28$$
$$2 + 2 = 4 + 6 = 10 + 2 = 12$$

4.2 Subtraction

From the **Problems** menu, select **Number Problems**. A box will open up, showing all the number problems. Click on **Subtraction**. A > sign will appear to the left of the problem selected. Choose the level. The number of digits in each number depends on the level. You can also choose whether the numbers contain a decimal point or not. Make sure that there is a > sign to the left of the problems you have selected. Then click **OK**. Click the **New** button.

Use the **Left** arrow key to move forward to the next box. Use the **Right** arrow key to go back to the previous box. Subtract the numbers in each column. Write the answer in the box below the column, and the borrow in the box above the next column.

There are two ways to calculate borrows in subtraction. The method used here is very simple, and you do not have to look several digits ahead. Instead the borrow is determined just by the current digit. It is always 0 (no borrow) or 1 (borrow). The borrow that you determine for the current digit will then be used for the next digit. Use the Help feature to determine what is supposed to go in every box. We strongly recommend this system.

If you are more familiar with another system of borrow, then from the **Options** menu, select **Carry Optional**. Then use the borrow squares to write whatever numbers you want, and if the difference (the large squares) is correct, the computer will not complain about the borrows.

Example:

$ \begin{array}{r} \boxed{1}\boxed{1}\boxed{0}\boxed{0}\boxed{1}\boxed{0}\boxed{0} \\ 87139456 \\ - 9833821 \\ \hline \boxed{7}\boxed{7}\boxed{3}\boxed{0}\boxed{5}\boxed{6}\boxed{3}\boxed{5} \end{array} $	<table border="1" style="margin-left: auto; margin-right: auto;"> <tr><td style="padding: 2px;">New</td></tr> <tr><td style="padding: 2px;">Done</td></tr> <tr><td style="padding: 2px;">Help</td></tr> <tr><td style="padding: 2px;">Quit</td></tr> </table>	New	Done	Help	Quit
New					
Done					
Help					
Quit					

Help:

$$\boxed{0}6 - 1 = \boxed{5}$$

$$\boxed{0}5 - 2 = \boxed{3}$$

$$\boxed{1}4 - 8 = \boxed{6}$$

$$\boxed{0}9 - 3 = \boxed{6} - 1 = \boxed{5}$$

$$\boxed{0}3 - 3 = \boxed{0}$$

$$\boxed{1}1 - 8 = \boxed{3}$$

$$\boxed{1}7 - 9 = \boxed{8} - 1 = \boxed{7}$$

$$\boxed{0}8 - 0 = \boxed{8} - 1 = \boxed{7}$$

4.3 Multiplication

From the **Problems** menu, select **Number Problems**. A box will open up, showing all the number problems. Click on **Multiplication**. A >

sign will appear to the left of the problem selected. Choose the level. The number of digits in each number depends on the level. You can also choose whether the numbers contain a decimal point or not. Make sure that there is a > sign to the left of the problems you have selected. Then click **OK**. Click the **New** button.

Use the **Left** arrow key to move forward to the next box. Use the **Right** arrow key to go back to the previous box.

Example:

$ \begin{array}{r} 1022 \\ 0011 \\ 4068 \\ \times \quad 23 \\ \hline 00000 \\ 12204 \\ 08136 \\ \hline 093564 \end{array} $	<table border="1" style="margin: auto;"> <tr><td>New</td></tr> <tr><td>Done</td></tr> <tr><td>Help</td></tr> <tr><td>Quit</td></tr> </table>	New	Done	Help	Quit
New					
Done					
Help					
Quit					

Help:

$3 \times 8 = \boxed{24}$

$3 \times 6 = \boxed{18} + 2 = \boxed{20}$

$3 \times 0 = \boxed{0} + 2 = \boxed{2}$

$3 \times 4 = \boxed{12}$

$2 \times 8 = \boxed{16}$

$2 \times 6 = \boxed{12} + 1 = \boxed{13}$

$2 \times 0 = \boxed{0} + 1 = \boxed{1}$

$2 \times 4 = \boxed{8}$

$2 + 3 = \boxed{5}$

$2 + 1 = \boxed{3}$

$1 + 8 = \boxed{9}$

4.4 Division

From the **Problems** menu, select **Number Problems**. A box will open up, showing all the number problems. Click on **Division**. A > sign will appear to the left of the problem selected. Choose the level, and whether the numbers contain a decimal point or not. Click the **New** button.

Use the **Left** arrow key to move forward to the next box. Use the **Right** arrow key to go back to the previous box. Subtract the numbers in each column. Write the answer in the box below the column, and the borrow in the box above the next column.

Example:

		1	4	1	
26		3	6	8	1
		2	6		
		1	0	8	
		1	0	4	
		0	4	1	
		0	2	6	
		1	5		

New

Done

Help

Quit

Help:

$$36 = 26 \times \boxed{1} + ?$$

$$26 \times 1 = \boxed{26}$$

$$36 - 26 = \boxed{10}$$

$$108 = 26 \times \boxed{4} + ?$$

$$26 \times 4 = \boxed{104}$$

$$108 - 104 = \boxed{04}$$

$$41 = 26 \times \boxed{1} + ?$$

$$26 \times 1 = \boxed{26}$$

$$41 - 26 = \boxed{15}$$

4.5 Factors

In this problem, you find the prime numbers which can divide a given number without any remainder. These prime numbers are called factors of the given number.

From the **Problems** menu, select **Number Problems**. A box will open up, showing all the number problems. Click on **Factors**. Choose the level, either 1, or 2. Make sure that there is a > sign to the left of the problems you have selected. Then click **OK**. Click the **New** button.

Factor problems are presented as follows. At the top of the screen is a list of prime numbers. You should memorize these, and some more. Below the Prime numbers is the number to be factorized.

In the first box (to the left of the number), you try the primes one at a time starting with the smallest. When you find a prime which can divide the number, then you divide them, enter the quotient in box below the number. Then you repeat the same step with the quotient. Find the smallest prime factor which can divide the quotient, and write the prime factor to the left of the quotient, and the new quotient below. In the last box at the bottom, you should get 1.

4.5.1 Using Help

When you click on the help button, the computer tries to divide the number by one prime at a time. For some primes, simple tests are presented to determine whether the number is divisible without actually dividing it. You are asked some questions in order to perform these tests. You should memorize these tests, and perform them verbally. If the prime is divisible, then you need to fill the quotient box. If you need help in

dividing, click the quotient help box at the bottom of the screen, and click the help button again.

Example:

Primes: 2 3 5 7 11 13 17 19

2	1870
5	935
11	187
17	17
	1

New
Done
Help
Quit

Help:

If the last digit is even(0, 2, 4, 6, 8), then the number is divisible by 2.

Last digit of dividend =
 1870 / 2 Quotient = Remainder =

If the sum of digits is divisible by 3, then the number is divisible by 3.

Sum of digits of dividend =
 935 / 3 Quotient = Remainder =

If the last digit is 0 or 5, then the number is divisible by 5.

Last digit of dividend =
 935 / 5 Quotient = Remainder =

187 / 7 Quotient = Remainder =
 187 / 11 Quotient = Remainder =

4.6 LCM

The **Lowest Common Multiple** or the **Greatest Common Divisor** is the smallest number which is divisible by all of the given numbers without any remainder. For example, if the two given numbers are 6 and 8, then the smallest number which is divisible by both 6 and 8 is 24. To find this number, you multiply all of the factors of 8, i.e. 2 x 2 x 2, and then you multiply all the factors of 6 that have not been included already, i.e. 3.

From the **Problems** menu, select **Number Problems**. A box will open up, showing all the number problems. Click on **LCM**. Choose the level, either 1, or 2. Make sure that there is a > sign to the left of the problems you have selected. Then click **OK**. Click the **New** button.

The factorization part is similar to the **Factors** problem. After finding the factors of all the numbers, you select the factors that should go into the LCM, and then multiply them. Use the **Help** button to find out which ones they are.

Example:

Primes: 2 3 5 7 11 13 17 19

2	1870	2	30	3	105
5	935	3	15	5	35
11	187	5	5	7	7
17	17		1		1
	1				

1870 x 3 = 5610 x 7 = 39270

New

Done

Help

Quit

4.7 Fractions and Decimals

These problems provide an introduction to fractions, and their relation to decimals. Simplification of fractions, conversion between proper and improper fractions, and conversion to and from decimals is included.

A **proper fraction** is one in which the numerator is less than the denominator. It may have a whole number to the left of it. An **improper fraction** is one in which the numerator is greater than the denominator.

A fraction can be simplified by dividing both the numerator and denominator by the same factor. For example given a fraction $\frac{18}{30}$, the numerator and denominator can first be divided by 2 to give $\frac{9}{15}$, and then by 3 to give $\frac{3}{5}$. This is the most simplified form of the fraction, since there is no other prime number which divides both the numerator and

denominator without a remainder. $\frac{18}{30}$ and $\frac{3}{5}$ are equal fractions.

From the **Problems** menu, select **Number Problems**. A box will open up, showing all the number problems. Click on **Fractions And Decimals**. Then click **OK**. Click the **New** button.

Fraction problems are presented as follows. In line 1, a proper fraction is presented. First the fraction is simplified by cancellation, if possible. You may do cancellation by pressing the mouse button, and moving the mouse to draw a line through the numbers. However, the computer will check only the numbers you write in the boxes.

Next, the simplified proper fraction is converted to an improper fraction as follows: Improper Numerator = Whole Number x Proper Denominator + Proper Numerator.

The Denominator remains the same.

Next, the proper fraction is converted to a decimal number by dividing the numerator by the denominator to the required number of decimal places.

In line 2, an improper fraction is presented. First, this is simplified. Then it is converted to a proper fraction by dividing the numerator with the denominator. The whole number is the quotient, and the proper numerator is the remainder. The denominator remains the same.

Next, the proper fraction is converted to a decimal number by dividing the numerator by the denominator to the required number of decimal places.

In line 3, a decimal number is presented. First, it is converted to a proper fraction. The whole part of the decimal number is the whole number of the fraction. The decimal part becomes the numerator, and the denominator consists of 1 and a number of zeroes equal to the number

of decimal places. The proper fraction is then simplified, and converted to an improper fraction.

Example:

$1\frac{8}{20} = 1\frac{\boxed{2}}{\boxed{5}} = \frac{\boxed{7}}{\boxed{5}} = \boxed{1.4}$ $\frac{\boxed{76}}{\boxed{14}} = \frac{\boxed{38}}{\boxed{7}} = \boxed{5}\frac{\boxed{3}}{\boxed{7}} = \boxed{5.428}$ $1.425 = \boxed{1}\frac{\boxed{425}}{\boxed{1000}} = \boxed{1}\frac{\boxed{17}}{\boxed{40}} = \frac{\boxed{57}}{\boxed{40}}$	<table border="1" style="margin: auto; border-collapse: collapse;"> <tr><td style="padding: 2px 10px;">New</td></tr> <tr><td style="padding: 2px 10px;">Done</td></tr> <tr><td style="padding: 2px 10px;">Help</td></tr> <tr><td style="padding: 2px 10px;">Quit</td></tr> </table>	New	Done	Help	Quit
New					
Done					
Help					
Quit					

Help:

$$\left. \begin{array}{l} 8 \\ 20 \end{array} \right\} / \boxed{4} = \left\{ \begin{array}{l} \boxed{2} \\ \boxed{5} \end{array} \right.$$

$$1 \times 5 = \boxed{5} + 2 = \boxed{7}$$

4.8 Fractional Addition and Subtraction

In these problems, you add or subtract fractional numbers.

From the **Problems** menu, select **Number Problems**. A box will open up, showing all the number problems. Click on **Fractional Addition**, or **Fractional Subtraction**. Choose the level, either 1, 2, or 3. In Level 1 you are asked to add single-digit fractions. In level 3, very large

numbers are presented, and this level is for the expert. With fractional addition, you can also choose the number of fractions that you want to add, either 2, 3, or 4. After selecting the problems and levels, make sure that there is a > sign to the left of the problems you have selected. Then click **OK**. Click the **New** button.

Fractional addition and subtraction problems are presented as follows. Line 1 gives the fractions to be added or subtracted.

On line 2, you simplify the fractions by cancellation, if possible, otherwise, write them as they are. You may do cancellation by pressing the mouse button, and moving the mouse to draw a line through the numbers. However, the computer will check only the numbers you write in the boxes. Until this stage, the fractions are proper fractions, with the numerator less than the denominator, and possibly a whole number to the left of each fraction.

On line 3, you convert proper fractions to improper fractions as follows:

Improper Numerator = Whole Number x Proper Denominator + Proper Numerator.

The Denominator remains the same at this stage.

On line 4, you compute the LCM or Lowest Common Multiple (also known as GCD, or Greatest Common Denominator) of all the denominators in the previous step. See the separate Number Problem on LCM for a detailed explanation. Then you calculate all the numerators and write them above this common denominator. The numerators are computed as follows:

New Numerator = LCM / Improper Denominator x Improper Numerator.

This ensures that new numerator / LCM is the same as the original fraction. At this stage, you can check your work, since the division (LCM / Improper Denominator) in this calculation should produce a whole number with no remainder.

On line 5, you add or subtract the numerators from the previous step. The denominator is the same as the LCM. This is the answer in the Improper Fraction form.

On line 6, you first simplify it by cancellation, if possible, then convert it to a proper fraction with a whole number, and the numerator less than the denominator. This is the final answer. To convert to proper fraction, divide the numerator by the denominator. The quotient is the whole number, and the remainder is the proper numerator. The denominator is the same as in the simplified improper fraction.

4.8.1 Using Help

Simplifying Fractions. On line 2 and Line 6, you need to simplify fractions by cancellation. To get help in simplification, click in any box on line 2, and then click the **Help** button. If there is no common factor, the numerator and denominator will be moved to the next line unchanged. If there is a common factor, then the numerator and denominator will move to the bottom of the screen. Next to the numerator and denominator, after the / sign, there is a help box for entering the common factor. For example if the numerator and denominator are 35 and 12, they are both divisible by 2 as well as by 3, so their common factor is 6. After the equal sign, there are two more help boxes to enter the simplified numerator and denominator. These are calculated by dividing the numerator and

denominator with the common factor respectively. If you need additional help with this division, click in the two right-most help boxes at the bottom of the screen, and then click the **Help** button again. After you have filled all three help boxes, click the **Done** button. If the answer is right, the simplified numerator and denominator will move back to their boxes.

LCM. Click in the common denominator box on line 4, and then click the **Help** button. First fill in all the prime factors of each denominator in ascending order, with the smallest on the left, and the largest on the right. Then, at the bottom of the screen, choose the factors which go into the LCM, and multiply them. If the LCM is calculated correctly, the computer will not check any of the other help boxes, but will move the LCM to its final position.

Example:

$$\begin{aligned}
 & 2\frac{3}{5} + \frac{2}{8} + 4\frac{3}{4} + 5\frac{1}{5} \\
 &= 2\frac{\boxed{3}}{\boxed{5}} + \frac{\boxed{1}}{\boxed{4}} + 4\frac{\boxed{3}}{\boxed{4}} + 5\frac{\boxed{1}}{\boxed{5}} \\
 &= \frac{\boxed{13}}{\boxed{5}} + \frac{\boxed{1}}{\boxed{4}} + \frac{\boxed{19}}{\boxed{4}} + \frac{\boxed{26}}{\boxed{5}} \\
 &= \frac{\boxed{52}}{\boxed{20}} + \frac{\boxed{5}}{\boxed{20}} + \frac{\boxed{95}}{\boxed{20}} + \frac{\boxed{104}}{\boxed{20}} \\
 &= \frac{\boxed{256}}{\boxed{20}} \\
 &= \frac{\boxed{64}}{\boxed{5}} = \boxed{12}\frac{\boxed{4}}{\boxed{5}}
 \end{aligned}$$

New

Done

Help

Quit

Example:

$$\begin{aligned}
 & 2\frac{10}{12} - 1\frac{55}{3150} \\
 &= 2\frac{\boxed{5}}{\boxed{6}} - 1\frac{\boxed{11}}{\boxed{630}} \\
 &= \frac{\boxed{17}}{\boxed{6}} - \frac{\boxed{641}}{\boxed{630}} \\
 &= \frac{\boxed{1785}}{\boxed{630}} - \frac{\boxed{641}}{\boxed{630}} \\
 &= \frac{\boxed{1144}}{\boxed{630}} \\
 &= \frac{\boxed{572}}{\boxed{315}} = \boxed{1}\frac{\boxed{257}}{\boxed{315}}
 \end{aligned}$$

New

Done

Help

Quit

Help:

$$\begin{aligned}
 \left. \begin{array}{l} 2 \\ 8 \end{array} \right\} / \boxed{2} &= \left\{ \begin{array}{l} \boxed{1} \\ \boxed{4} \end{array} \right. \\
 2 \times 5 &= \boxed{10} + 3 = \boxed{13} \\
 \text{LCM} &= \boxed{2} \times \boxed{2} = \boxed{4} \times \boxed{5} = \boxed{20} \\
 13 \times 20 / 5 &= \boxed{52} \\
 1 \times 20 / 4 &= \boxed{5} \\
 52 + 5 + 95 + 104 &= \boxed{256} \\
 \left. \begin{array}{l} 256 \\ 20 \end{array} \right\} / \boxed{4} &= \left\{ \begin{array}{l} \boxed{64} \\ \boxed{5} \end{array} \right. \\
 64 / 5 &\text{Quotient} = \boxed{12} \text{ Remainder} = \boxed{5}
 \end{aligned}$$

4.9 Fractional Multiplication and Division

In these problems, you multiply or divide fractional numbers.

From the **Problems** menu, select **Number Problems**. A box will open up, showing all the number problems. Click on **Fractional Multiplication And Division**. Then choose the number of fractions that you want to multiply and divide, either 2, 3, or 4. Make sure that there is a > sign to the left of the problems you have selected. Then click **OK**. Click the **New** button.

Fractional multiplication and division problems are presented as fol-

lows. Line 1 gives the fractions to be multiplied and divided.

On line 2, you simplify the fractions by cancellation, if possible, otherwise, write them as they are. You may do cancellation by pressing the mouse button, and moving the mouse to draw a line through the numbers. However, the computer will check only the numbers you write in the boxes. Until this stage, the fractions are proper fractions, with the numerator less than the denominator, and possibly a whole number to the left of each fraction.

On the left side of line 3, you convert proper fractions to improper fractions as follows:

Improper Numerator = Whole Number x Proper Denominator + Proper Numerator.

The Denominator remains the same at this stage.

On the right side of line 3, you invert the fractions which are divisors (ie. to the right of a / sign), and do not invert fractions that are to the right of an X sign. At this time all / signs are changed to X and then you need to multiply all the resulting fractions. At this step, you simplify any of the numerators and denominators and write the results in the boxes above and below the fractions.

On line 4, you copy the simplified numerators and denominators from the previous step.

On line 5, you multiply all the numerators from the previous step, to get the product numerator, and multiply all the denominators to get the product denominator. This is the improper fraction answer, and is already simplified. Then you calculate the proper fraction answer, including the whole number, and the proper numerator and denominator.

Example:

$1\frac{3}{6} / 3\frac{1}{2} \times 5\frac{6}{8} \times 4\frac{1}{3}$	<div style="border: 1px solid black; padding: 2px; width: 40px; text-align: center;">New</div> <div style="border: 1px solid black; padding: 2px; width: 40px; text-align: center;">Done</div> <div style="border: 1px solid black; padding: 2px; width: 40px; text-align: center;">Help</div> <div style="border: 1px solid black; padding: 2px; width: 40px; text-align: center;">Quit</div>
$= 1\frac{\boxed{1}}{\boxed{2}} / 3\frac{\boxed{1}}{\boxed{2}} \times 5\frac{\boxed{3}}{\boxed{4}} \times 4\frac{\boxed{1}}{\boxed{3}}$	$\boxed{1} \quad \boxed{1}$
$= \frac{\boxed{3}}{\boxed{2}} / \frac{\boxed{7}}{\boxed{2}} \times \frac{\boxed{23}}{\boxed{4}} \times \frac{\boxed{13}}{\boxed{3}}$	$= \frac{\boxed{3}}{\boxed{2}} \times \frac{\boxed{2}}{\boxed{7}} \times \frac{\boxed{23}}{\boxed{4}} \times \frac{\boxed{13}}{\boxed{3}}$
$= \frac{\boxed{1}}{\boxed{1}} \times \frac{\boxed{1}}{\boxed{7}} \times \frac{\boxed{23}}{\boxed{4}} \times \frac{\boxed{13}}{\boxed{1}}$	$\boxed{1} \quad \boxed{1}$
$= \frac{\boxed{299}}{\boxed{28}} = \boxed{10}\frac{\boxed{19}}{\boxed{28}}$	

Help:

$$\begin{array}{l}
 \left. \begin{array}{l} 3 \\ 6 \end{array} \right\} / \boxed{3} = \left\{ \begin{array}{l} \boxed{1} \\ \boxed{2} \end{array} \right. \\
 1 \times 2 = \boxed{2} + 1 = \boxed{3} \\
 23 \times 13 = \boxed{299} \\
 7 \times 4 = \boxed{28} \\
 299 / 28 \quad \text{Quotient} = \boxed{10} \quad \text{Remainder} = \boxed{19}
 \end{array}$$

4.10 Brackets

From the **Problems** menu, select **Number Problems**. A box will open up, showing all the number problems. Click on **Brackets**. Then choose the number of numbers. Make sure that there is a > sign to the left of the problems you have selected. Then click **OK**. Click the **New** button.

First solve the innermost brackets, and write the result in the box on the next line. Then proceed with the outer brackets.

Example:

$\begin{aligned} & (((0.66 + 0.50) / ((3.3 + 0.30) / 76)) - 0.82) \\ = & ((\boxed{1.2} / (\boxed{3.6} / 76)) - 0.82) \\ = & ((\boxed{1.2} / \boxed{0.047}) - 0.82) \\ = & (\boxed{25.5} - 0.82) \\ = & \boxed{24.7} \end{aligned}$	<table border="1"><tr><td>New</td></tr><tr><td>Done</td></tr><tr><td>Help</td></tr><tr><td>Quit</td></tr></table>	New	Done	Help	Quit
New					
Done					
Help					
Quit					

Help:

$$0.66 + 0.5 = \boxed{1.2}$$

$$3.3 + 0.30 = \boxed{3.6}$$

$$3.6 / 76 = \boxed{0.047}$$

$$1.2 / 0.047 = \boxed{25.5}$$

$$25.5 - 0.82 = \boxed{24.7}$$

4.11 Average

Two to six numbers are given and their average is calculated. First, all the numbers are added. The sum is then divided by the number of numbers to calculate the average.

From the **Problems** menu, select **Number Problems**. A box will open up, showing all the number problems. Click on **Average**. Then click **OK**. Click the **New** button.

Example:

Numbers:	52.13	
	318.10	
	225.15	
Sum	=	$\boxed{595.38}$
Number of numbers	=	$\boxed{3}$
Average	=	$\frac{\boxed{595.38}}{\boxed{3}}$
	=	$\boxed{198.46}$

New
Done
Help
Quit

4.12 Percentage

In this problem one number is represented as a percentage of another.

From the **Problems** menu, select **Number Problems**. A box will open up, showing all the number problems. Click on **Percentage**. Then

click **OK**. Click the **New** button.

Percentage problems are presented as follows. In line 1, two numbers are given, and one is to be converted to a percentage of the other. For this purpose, the first number is divided by the second, and multiplied by 100 to find the percentage. Line 2 says that the first number is a percentage of the second. The percentage is the same as that calculated in line 1.

In Line 3, a percentage and a number are given, and we are asked to calculate what is the value of the given percentage of the given number. This is done by dividing the percentage by 100 and multiplying by 100.

Example:

$\frac{5}{35} \times 100 = \boxed{14.285}\%$	<table border="1"><tr><td>New</td></tr><tr><td>Done</td></tr><tr><td>Help</td></tr><tr><td>Quit</td></tr></table>	New	Done	Help	Quit
New					
Done					
Help					
Quit					
$5 = \boxed{14.285}\% \text{ of } 35$					
$75\% \text{ of } 35 = \frac{75}{100} \times 35 = \boxed{26.25}$					

Chapter 5

Word Problems

5.1 Dollars and Cents Level 1

This menu item contains some money problems.

5.1.1 Shopping Problem Level 1

You go shopping, buy some items, and pay some money. You are asked to calculate the change.

Example: Ann bought 3 apples at 17c. each, 4 bags of rice at \$ 1.33 each, 1 head of lettuce at 61 c. each, and 3 oranges at 8 c. each. She paid \$ 10. What is the change?

For each item, you multiply the quantity with the unit price given to get the total price paid for the item. Then you add all the total prices to get the total money spent on buying the items. Then you subtract this total from the money you handed over to the store clerk to get the change.

Solution:

ITEM	QTY.	UNIT PR.	TOTAL PR.	
Apples	3	0.17	0.51	New Done Help Quit
Bags of rice	4	1.33	5.32	
Head of lettuce	1	0.61	0.61	
Oranges	3	0.08	0.24	
Paid:	\$ 10	Total:	6.68	
		Change:	3.32	

Help:

Apples:	$3 \times 0.17 =$	0.51
Bags of rice:	$4 \times 1.33 =$	5.32
Head of lettuce:	$1 \times 0.61 =$	0.61
Oranges:	$3 \times 0.08 =$	0.24
Total:	$0.51 + 5.32 + 0.61 + 0.24 =$	6.68
Change:	$10 - 6.68 =$	3.32

5.1.2 Percentage Profit Problem

Example: A book salesman sells each book for \$ 13, and his profit is 17%. How many books must he sell to make a profit of at least \$ 32 a day?

The profit is always stated as the percentage of the buying price. How-

ever, the problem gives the selling price. If the buying price is considered as 100%, and the profit is 17% of the buying price, then the selling price is $100 + 17 = 117\%$ of the buying price. Therefore, the buying price is equal to $\frac{\text{selling price} \times 100}{117}$. The profit on each book is equal to the selling price - the buying price. The total profit should be at least \$ 32. Divide the total profit by the profit on each book. In this case, the result of the division is 16.93, i.e. 16.93 books need to be sold to make exactly \$ 32 profit. Since only whole books must be sold, so take the next higher number or 17, for a profit slightly higher than \$ 32.

Solution:

Selling price of each book	=	\$ 13	New Done Help Quit
Profit on each book	=	17%	
Selling price	=	117 % of buying price	
Buying price	=	\$ 11.11	
Profit on each book	=	\$ 1.89	
Total profit at least	=	\$ 32	
Books sold	=	17	

Help:

Selling price: $100 + 17 = \boxed{117}$ % of buying price
 Buying price $13 / 117 \times 100 = \$ \boxed{11.11}$
 Profit on each book $13 - 11.11 = \$ \boxed{1.89}$
 Books sold $32 / 1.89 = \boxed{16.9312}$
 If not exactly divisible, then take the next higher integer = $\boxed{17}$

5.2 Dollars and Cents Level 2

Example 1: Johnny gets \$ 14 for mowing a lawn, \$ 8 for raking a yard, and \$ 26 for cleaning a house. If he cleans 2 houses, and rakes 3 yards, how many lawns does he have to mow in order to get at least \$ 170 for a pair of skates?

Multiply the number of jobs by the price for each job, and add them together. This subtotal will give the earnings from the houses and the yards. At this point we need to check whether Johnny already has enough money to buy the skates. If the subtotal is greater than or equal to the price of skates, then Johnny needs to mow zero lawns. If the subtotal is less than the price of the skates, then subtract the subtotal from the price of the skates. This gives the additional money needed from the lawns. Divide this amount by the price of mowing each lawn. In this case they are not exactly divisible, and the result is 6.7143 lawns. If only 6 lawns are mowed, Johnny gets only \$ 84, and he does not have enough money to buy the skates. Therefore, the answer should be the next higher number 7.

Solution:

2 houses at \$ 26 each	= \$	<input type="text" value="52"/>	<input type="button" value="New"/> <input type="button" value="Done"/> <input type="button" value="Help"/> <input type="button" value="Quit"/>
3 yards at \$ 8 each	= \$	<input type="text" value="24"/>	
Total collected so far	= \$	<input type="text" value="76"/>	
Total required for skates	= \$	170	
Is extra money required?			
Extra money required	= \$	<input type="text" value="94"/>	
Money for each lawn	= \$	14	
Lawns needed to be mowed	=	<input type="text" value="7"/>	

Help:

Houses: $2 \times 26 = \$ \boxed{52}$
 Yards: $3 \times 24 = \$ \boxed{24}$
 Total collected so far: $52 + 24 = \$ \boxed{76}$
 Extra money required: $170 - 76 = \$ \boxed{94}$
 Lawns needed to be mowed $94 / 14 = \boxed{6.7143}$

If not exactly divisible, then take the next higher integer = $\boxed{7}$

Example 2: Susan has 6 lb. candy. The average candy is 3 oz. She sold 2 candies to John, and 3 candies to Joe for 25 c. each. What is the value of the remaining candy?

Find the total amount of candy in ounces by multiplying the number of pounds by 16. Then divide by the weight of each candy to find the total number of candies. Round off to the nearest whole number, since

there are only whole candies in the package. Out of these, find out the number of candies sold by adding the number sold to John and Joe. Subtract to find the number of candies left. The price of each candy is given. Multiply by the number of candies left to find the total price of the remaining candies.

Solution:

Total candy	=	6 lb. =	<input type="text" value="96"/>	oz.	<input type="button" value="New"/>
Each candy	=	3 oz.			<input type="button" value="Done"/>
Number of candies	=		<input type="text" value="32"/>		<input type="button" value="Help"/>
Candies sold	=	2 + 3 =	<input type="text" value="5"/>		<input type="button" value="Quit"/>
Candies left	=		<input type="text" value="27"/>		
Price per candy	=	\$	0.25		
Value of the candy left	=	\$	<input type="text" value="6.75"/>		

Help:

Total candy	=	6 x 16	=	<input type="text" value="96"/>	oz.
Number of candies	=	96 / 3	=	<input type="text" value="32"/>	
Candies left	=	32 - 5	=	<input type="text" value="27"/>	
Value of the candy left	=	27 x 0.25	=	\$ <input type="text" value="6.75"/>	

Example 3: Donna buys 3 chairs and 8 tables, and pays a total of \$ 162. If a chair costs \$ 8 less than a table, then what is the cost of a table?

Solution:

Total Paid =	\$ 162 for 3 chairs and 8 tables	<input type="button" value="New"/>
A table costs	\$ 8 more than a chair	<input type="button" value="Done"/>
If she had bought 3 more tables instead of chairs,		<input type="button" value="Help"/>
then she would have paid \$	<input type="text" value="24"/> extra,	<input type="button" value="Quit"/>
then the total would be	\$ <input type="text" value="186"/> for <input type="text" value="11"/> tables	
Each table costs	\$ <input type="text" value="16.91"/>	

Help:

extra paid	3 x 8	=	\$ <input type="text" value="24"/>
then the total would be	162 + 24	=	\$ <input type="text" value="186"/>
for	8 + 3	=	<input type="text" value="11"/> tables
Each table costs	186 / 11	=	\$ <input type="text" value="16.91"/>

5.2.1 The Grocery Shopping Problem, Level 2

Example: Ron bought 2.2 lb. sugar at \$ 1.32 /lb., 4.8 lb. almonds at 49 c./oz., 4.3 oz. cinnamon at 45 c./oz., 4.5 oz. flour at \$ 1.22 /lb., and 3.5 oz. candy at \$ 1.02 /lb. He paid \$ 50. What is the change?

In these problems, the weight of each item is provided in either pounds, or ounces, and the price is also either in dollars per pound, or dollars

per ounce. They must be converted to the same units before they are multiplied. For example, if the price is given in \$ /lb., and the quantity purchased is in oz., divide by 16 to find out the quantity in lb. If the price is given in \$ /oz., and the quantity purchased is in lb., multiply by 16 to find out the quantity in oz.

Solution:

ITEM	QTY.	UNIT PR.	QTY.	TOTAL PR.	
sugar	2.2 lb.	1.32 /lb.	<input type="text" value="2.2"/> lb.	<input type="text" value="2.90"/>	<input type="button" value="New"/>
almonds	4.8 lb.	0.49 /oz.	<input type="text" value="76.8"/> oz.	<input type="text" value="37.63"/>	<input type="button" value="Done"/>
cinnamon	4.3 oz.	0.45 /oz.	<input type="text" value="4.3"/> oz.	<input type="text" value="1.94"/>	<input type="button" value="Help"/>
flour	4.5 oz.	1.22 /lb.	<input type="text" value="0.28"/> lb.	<input type="text" value="0.34"/>	<input type="button" value="Quit"/>
candy	3.5 oz.	1.02 /lb.	<input type="text" value="0.22"/> lb.	<input type="text" value="0.22"/>	
Paid: \$ 50		Total:		<input type="text" value="43.03"/>	
		Change:		<input type="text" value="6.97"/>	

Help:

sugar total price: $1.32 \times 2.2 = \boxed{2.90}$
 almonds quantity: $4.80 \times 16 = \boxed{76.8}$
 almonds total price: $0.49 \times 76.8 = \boxed{37.63}$
 cinnamon total price: $0.45 \times 4.3 = \boxed{1.94}$
 flour quantity: $4.50 / 16 = \boxed{0.28}$
 flour total price: $1.22 \times 0.28 = \boxed{0.34}$
 candy quantity: $3.50 / 16 = \boxed{0.22}$
 candy total price: $1.02 \times 0.22 = \boxed{0.22}$
 Total: $2.90 + 37.63 + 1.94 + 0.34 + 0.22 = \boxed{43.03}$
 Change: $50 - 43.03 = \boxed{6.97}$

5.3 Direct Proportion, Level 1

This is a very important type of problem. Hundreds of events in our daily life are controlled by direct proportion. This menu item contains many such examples, which look quite different from each other, but their solution method is similar.

Example: A construction company builds 9 houses in 3 months. How many houses will it build in 15 months?

Solution:

Direct Proportion, more time, more houses.

3 months are needed for building 9 houses.
15 months are needed for building $\frac{9}{3} \times 15$
= 45 houses.

New
Done
Help
Quit

5.4 Direct Proportion, Level 2

This menu item contains problems that can be solved using two steps of Direct Proportion.

Example: If the average spray can covers 252 sq. ft., and the average door is 28 sq. ft., how many spray cans are required to paint 26 doors?

Solution:

1 door equals 28 sq. ft.
26 doors equal $\frac{28}{1} \times 26$
= 728 sq. ft.

252 sq. ft. can be covered by 1 can
728 sq. ft. can be covered by $\frac{1}{252} \times 728$
= 2.89 cans

New
Done
Help
Quit

5.5 Fractions and Percentage

5.5.1 Percentage Problem

Example 1: Mother baked a batch of cookies. Cindy ate 8 cookies. If there were 46 cookies in the batch, what percentage of the batch was eaten?

Solution:

Eaten:	8	
Total:	46	
Percentage eaten	=	$\frac{8}{46} \times 100$
	=	17.39%

New
Done
Help
Quit

Example 2: Mother baked a batch of cookies. Cathy ate 11 cookies. If there were 61 cookies left, what percentage of the batch was eaten?

First we need to find out how many cookies there were in the batch. This is the sum of the cookies eaten and the cookies remaining. Then the percentage eaten equals the number of cookies eaten, divided by the total number of cookies, times hundred.

Solution:

Eaten:	11	
Remaining:	61	
Total:		72
Percentage eaten	=	$\frac{11}{72} \times 100$
	=	15.28%

New
Done
Help
Quit

5.5.2 Fraction Problem

Example: Chuck spends 1/7 of his money on books, and 1/4 of the remaining money on clothes. He saves the remaining money in a bank. If the money saved is \$ 35, how much money did he have in the beginning?

Solution:

		New
		Done
		Help
		Quit
1/4 of the money left after buying books is spent on clothes		
<input type="text" value="3"/> /4 of the money left after buying books is saved	= \$ 35	
Money left after buying books	= \$	<input type="text" value="46.67"/>
1/7 of the total money is spent on buying books		
<input type="text" value="6"/> /7 of the total money is left after buying books	= \$	<input type="text" value="46.67"/>
Total money	= \$	<input type="text" value="54.45"/>

Help:

Money left after buying books = $35 \times 4 = \text{input } 140 / 3 = \text{input } 46.67$

Total money = $46.67 \times 7 = \text{input } 326.69 / 6 = \text{input } 54.45$

5.6 Average

Example: Group A contains 20 students, with average age 12 years, 10 months, group B contains 26 students, with average age 17 years, 3 months, and group C contains 35 students, with average age 18 years, 10 months. What is the average age of all the students?

This problem teaches you how to calculate the average of averages. Simple averages are covered in the number problems. The ages are converted from years and months to months, the average of each group is

multiplied by the total number of students in the group, to get their total ages. The subtotals of all the groups are then added up, and divided by the total number of students in all groups to get the overall average.

Solution:

				New
				Done
				Help
				Quit
Group	Students	Average Age		
A	20	12 years 10 months =	<input type="text" value="154"/> months	<input type="text" value="3080"/>
B	26	17 years 3 months =	<input type="text" value="207"/> months	<input type="text" value="5382"/>
C	35	18 years 10 months =	<input type="text" value="226"/> months	<input type="text" value="7910"/>
Total			<input type="text" value="81"/>	<input type="text" value="16372"/>
Average =		<input type="text" value="202.12"/> months = <input type="text" value="16"/> years, <input type="text" value="10.12"/> months		

Help:

A - months: $12 \times 12 = \boxed{144} + 10 = \boxed{154}$

A - product: $20 \times 154 = \boxed{3080}$

B - months: $17 \times 12 = \boxed{204} + 3 = \boxed{207}$

B - product: $26 \times 207 = \boxed{5382}$

C - months: $18 \times 12 = \boxed{216} + 10 = \boxed{226}$

C - product: $35 \times 226 = \boxed{7910}$

Total students: $20 + 26 + 35 = \boxed{81}$

Total age: $3080 + 5382 + 7910 = \boxed{16372}$

Average: $16372 / 81 = \boxed{202.12}$ months

Average - years: $202.12 / 12 = \boxed{16}$

If not an integer, then take the next lower integer

Average - months: $202.12 - 16 \times 12 = \boxed{10.12}$

5.7 Simple Interest

Example 1: John puts \$ 3671 in a bank at 3.9% interest. What will the total amount be after 10.2 years?

Solution:

Principal P	= \$ 3671		
Rate R	= 3.9 %	= $\boxed{0.039}$	
Time T	= 10.2 years		
Interest I	= P x R x T	= $\boxed{1460.32}$	
Final amount, A	= P + I	= $\boxed{5131.32}$	

New

Done

Help

Quit

Help:

Rate R = $3.9 / 100 = \boxed{0.039}$

Interest I = $3671 \times 0.039 = \boxed{143.169} \times 10.2 = \boxed{1460.32}$

Final amount, A = $3671 + 1460.32 = \boxed{5131.32}$

Example 2: Sandy puts \$ 22563 in a bank at 4.7% interest. In how much time will the money amount to a total of \$ 45126?

Solution:

Principal P	= \$ 22563	
Rate R	= 4.7 %	= <input type="text" value="0.047"/>
Total Amount, A	= \$ 45126	
Interest I	= A - P	= <input type="text" value="22563"/>
Time T	= I / (P R)	= <input type="text" value="21.28"/>

New

Done

Help

Quit

Help:

Rate R = 4.7 / 100 =

Interest I = 45126 - 22563 =

Time T = 22563 / (22563 x 0.047) =

5.8 Ratio and Proportion

Example 1: \$ 8764 is divided among three people, so that Ann gets 4 times as much as Joe, and Ron gets 2 times as much as Joe. How much money does Ann get

Solution:

Joe gets		1 share
Ron gets 2 times the share of Joe	=	<input type="text" value="2"/> shares
Ann gets 4 times the share of Joe	=	<input type="text" value="4"/> shares
Total shares	=	<input type="text" value="7"/>
Total money	= \$	8764
Each share	= \$	<input type="text" value="1252"/>
Ann got	= \$	<input type="text" value="5008"/>

New

Done

Help

Quit

Help:

Total shares 1 + 2 + 4 =

Each share 8764 / 7 = \$

Ann got 4 x 1252 = \$

Example 2: \$ 3410 is divided among three people, so that Ann gets 8 times as much as Phil, and Phil gets 3 times as much as Bob. How much money does Ann get?

Note the difference between this example and the previous one. The major difference is in how to calculate Ann's shares.

Solution:

Bob gets		1 share	
Phil gets 3 times the share of Bob	=	<input type="text" value="3"/> shares	
Ann gets 8 times the share of Phil	=	<input type="text" value="24"/> shares	
Total shares	=	<input type="text" value="28"/>	
Total money	= \$	3410	
Each share	= \$	<input type="text" value="121.786"/>	
Ann got	= \$	<input type="text" value="2922.86"/>	

New

Done

Help

Quit

Help:

$$\begin{aligned}
 \text{Ann gets} & 8 \times 3 = \text{ shares} \\
 \text{Total shares} & 1 + 3 + 24 = \text{} \\
 \text{Each share} & 3410 / 28 = \$ \text{} \\
 \text{Ann got} & 24 \times 121.786 = \$ \text{}
 \end{aligned}$$

5.9 Other Problems

This menu item contains miscellaneous problems that cannot be categorized under the above groups.

Example 1: A and B together have \$ 283.30, B and C have \$ 232.03, and C and A have \$ 406.05. How much money does each person have?

Solution:

A and B have		\$ 283.30	
B and C have		\$ 232.03	
C and A have		\$ 406.05	
Adding all, twice the money of A, B, and C =	\$	<input type="text" value="921.38"/>	
A, B and C together have	\$	<input type="text" value="460.69"/>	
A has	\$	<input type="text" value="228.66"/>	
B has	\$	<input type="text" value="54.64"/>	
C has	\$	<input type="text" value="177.39"/>	

New

Done

Help

Quit

Help:

$$\begin{aligned}
 \text{Twice A, B, C} & 283.30 + 232.03 + 406.05 = \text{} \\
 \text{A, B, and C have} & 921.38 / 2 = \text{} \\
 \text{A has} & 460.69 - 232.03 = \text{} \\
 \text{B has} & 460.69 - 406.05 = \text{} \\
 \text{C has} & 460.69 - 283.30 = \text{}
 \end{aligned}$$

Example 2: The quotient is 4 times the divisor, and the divisor is 4 times the remainder. If the sum of the quotient, divisor, and remainder is 42, what is the dividend?

Solution:

Divisor	= 4 x Remainder	<table border="1"> <tr><td>New</td></tr> <tr><td>Done</td></tr> <tr><td>Help</td></tr> <tr><td>Quit</td></tr> </table>	New	Done	Help	Quit
New						
Done						
Help						
Quit						
Quotient	= 4 x Divisor = <input type="text" value="16"/> x Remainder					
Total	= Divisor + Quotient + Remainder					
	= 4 x Remainder + <input type="text" value="16"/> x Remainder					
	+ Remainder					
	= <input type="text" value="21"/> x Remainder = 42					
Remainder	= <input type="text" value="2"/>					
Divisor	= <input type="text" value="8"/>					
Quotient	= <input type="text" value="32"/>					
Dividend	= <input type="text" value="258"/>					

Help:

Quotient = 4 x 4 x Remainder = x Remainder
 Total = 4 + 16 + 1 =
 Remainder = 42 / 21 =
 Divisor = 4 x 2 =
 Quotient = 16 x 2 =
 Dividend = 8 x 32 = + 2 =

Example 3: What number nearest to 2547 is exactly divisible by 77?

Solution:

2547 / 77		<table border="1"> <tr><td>New</td></tr> <tr><td>Done</td></tr> <tr><td>Help</td></tr> <tr><td>Quit</td></tr> </table>	New	Done	Help	Quit
New						
Done						
Help						
Quit						
Quotient	= <input type="text" value="33"/>					
Remainder	= <input type="text" value="6"/>					
Numbers close to 2547 divisible by 77	= <input type="text" value="2541"/> and <input type="text" value="2618"/>					
Closest to 2547	= <input type="text" value="2541"/>					

Help:

Numbers close to 2547 divisible by 77 = 2547 - 6 =
 Numbers close to 2547 divisible by 77 = 2541 + 77 =

Example 4: A cop ran after a thief from 479 ft. behind. The thief ran 661 ft./min., and the cop ran 702 ft./min. How far did the thief run before he was caught?

Solution:

Speed of cop	= 702	<table border="1"><tr><td>New</td></tr><tr><td>Done</td></tr><tr><td>Help</td></tr><tr><td>Quit</td></tr></table>	New	Done	Help	Quit
New						
Done						
Help						
Quit						
Speed of thief	= 661					
Extra speed of cop	= <input type="text" value="41"/> ft./min.					
Extra distance run by cop	= 479 ft.					

The extra distance was covered by the extra speed during the entire time that the cop and the thief ran.

Total time of run	= <input type="text" value="11.68"/> min.
Distance run by thief	= <input type="text" value="7720.48"/> ft.

Help:

$$\begin{aligned} \text{Extra speed of cop} &= 702 - 661 = \text{41 ft./min.} \\ \text{Total time of run} &= 479 / 41 = \text{11.68 min.} \\ \text{Distance run by thief} &= 661 \times 11.68 = \text{7720.48 ft.} \end{aligned}$$

Example 5: A package of paper contains 300 sheets. If a xerox machine jams on 1 out of 23 sheets, how many copies of an 87 page report can be made from each package of paper?

Solution:

Sheets per pack	= 300	<table border="1"><tr><td>New</td></tr><tr><td>Done</td></tr><tr><td>Help</td></tr><tr><td>Quit</td></tr></table>	New	Done	Help	Quit
New						
Done						
Help						
Quit						
Sheets wasted: 1 out of	23					
Sheets wasted from each pack	= <input type="text" value="13"/>					
Sheets used from each pack	= <input type="text" value="287"/>					
Report size	= 87					

Number of copies	= <input type="text" value="3.3"/>
------------------	------------------------------------

Help:

$$\begin{aligned} \text{Sheets wasted from each pack} &= 300 / 23 = \text{13} \\ \text{Sheets used from each pack} &= 300 - 13 = \text{287} \\ \text{Number of copies} &= 287 / 87 = \text{3.3} \end{aligned}$$