Quiz 6, Math 115-11, Calculus I

March 14, 2007

Name:____

Extinguish all cell phones, pagers, beepers, etc. No headphones allowed. Calculators *are* allowed. Show appropriate work and provide units where appropriate.

1. For what values of a is $x + a\sin(x)$ increasing for all x?

2. An object on a spring oscillates about its equilibrium position at $y = 0$. Its distance from equilibrium is given as a function of time, t, by $y = e^{-t}\cos(t)$. Find the greatest distance the object goes above and below the equilibrium for $t \geq 0$.

Solutions:

1. Let $f(x) = x + a\sin(x)$. Then $f'(x) = 1 + a\cos(x)$. For f to be strictly increasing, we need $f'(x) > 0$ for all x, *i.e.*, $a\cos(x) > -1$ for all x. Since $\cos(x)$ takes values between -1 and $+1$, we can see that, if $|a| < 1$, then $|a\cos(x)| < 1$, so $a\cos(x) > -1$. On the other hand, since $\cos(0) = +1$, we know that $a\cos(0) = a$, so $a \leq -1$ will not make $a\cos(x) > -1$ for all x. Similarly, $\cos(\pi) = -1$, so $a\cos(\pi) = -a$, and so $a \geq +1$ will not work.

   We have shown that values of a in the range $-1 < a < +1$ all work and values in the ranges $a \leq -1$ and $a \geq +1$ will not work. So the complete specification of the a's that work is $\boxed{-1 < a < +1}$.

   There are different conventions as to whether "increasing" means "strictly increasing" or "increasing, but not necessarily strictly increasing." If you subscribe to an alternate convention, $\boxed{-1 \leq a \leq +1}$ would also be an acceptable answer. Don't mix conventions; i.e., don't write $-1 \leq a < 1$.

   A common mistake on this problem was to write $a > \frac{-1}{\cos(x)}$. This is wrong on two counts. First, note that multiplying through an equation by a negative number reverses the $<$ sign. For example, if $-x > -2$ then $x < 2$, by multiplying through by -1. In our case, we have $a\cos(x) > -1$ and we attempt to multiply through by $\frac{1}{\cos(x)}$, but $\frac{1}{\cos(x)}$ is neither always positive nor always negative, so we really can't multiply through and get something simple and useful. Second, we want our final answer to be a condition on a that does not depend on x.

2. The $e^{-t}\cos(t)$ problem is Example 4 in Section 4.3.