February 9, 2006

To be done in groups.

- CLRS 5-1, probabilistic counting.
- CLRS C.2-7, pairwise independence. (Solve the problem for \( n = 3 \) for partial credit.)
- (From Mitzenmacher and Upfal.) Suppose we have a sequence of items passing by one at a time. We want to maintain a sample of one item with the property that it is uniformly distributed over all the items that we have seen at each step. Moreover, we want to accomplish this without knowing the total number of items or storing all of the items that we see.

Consider the following algorithm, which stores just one item in memory at all times. When the first item appears, it is stored in the memory. When the \( k \)th item appears, it replaces the item in memory with probability \( 1/k \). Prove that this algorithm solves the problem.