Homework 3, due Thurs, Feb 9, 2006

February 2, 2006

To be done in groups.

1 Unequal Divide and Conquer (revised from last time)

1.1 Guidelines

In these problems, you are not required to find c and n_0 and prove the result by induction. It is sufficient to give a proof that appeals to results in the chapter. For example, you can say, without further comment, "$5n^2 + 7n \leq O(n^2)$" or "$\sum_{j=1}^{n} j^3 = \Theta(\int_0^n x^3\,dx) = \Theta(n^4)$" or "a k-ary tree with height h has $\Theta(k^h)$ leaves and nodes" ($k > 1$). In fact, I'd prefer that you get some practice in this "bigger picture" way of thinking, and not always get bogged down with induction details.

1.2 The Problem

Exercises CLRS 4.2-4 and CLRS 4.2-5 illustrate that, if we use a divide-and-conquer approach to a problem, it is typically better to divide the problem into nearly equal-sized subproblems. The above and the last example of CLRS Section 4.2 also illustrate that it is not necessary that the subproblems be *exactly* the same size.

We now switch from *analyzing* to *designing* algorithms. Given a problem of size n, we will break it into two problems, of size $a(n)$ and $n - a(n)$, where $a(n)$ is a "common" function satisfying $1 \leq a(n) \leq n/2$. We assume the Divide and Recombine cost is linear, so we have recurrence

$$T(n) = T(a(n)) + T(n - a(n)) + cn.$$

All other things being equal, we'd want $a(n) = n/2$ to minimize $T(n)$. But sometimes there is a separate (direct or indirect) cost involved in making $a(n)$ exactly $n/2$ and it's easier to choose other $a()$'s. Below we investigate which $a()$'s are acceptable while still meeting certain overall cost requirements. Problem:

- How slowly-growing can $a(n)$ be and still make $T(n) \leq O(n \log^2(n))$?

Restrict attention to functions of the form $n^r(\log(n))^s$, where r and s are constant real numbers, and you only need to find s up to 1, additively. That is, your answer should be numbers r and s with

$$n^r(\log(n))^s \leq a(n) \leq n^r(\log(n))^{s+1}.$$

(If you've already found exactly the right s, please turn it in, but don't attempt this if you haven't started.)

2 New Problems

- CLRS 5.1-3, on biased coins. You may assume that different tosses of the coin are independent.
- CLRS 5.2-5, expected number of inversions