Verifiable Hierarchical Protocols with Network Invariants on Parametric Systems

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Abstract—We present Neo, a framework for designing pre-verified protocol components that can be instantiated and connected in an arbitrarily large hierarchy (tree), with a guarantee that the whole system satisfies a given safety property. We employ the idea of network invariants to handle correctness for arbitrary depths in the hierarchy. Orthogonally, we leverage a parameterized model checker (Cubicle) to allow for a parametric number of children at each internal node of the tree. We believe this is the first time these two distinct dimensions of configuration have been together tackled in a verification approach, and also the first time a proof of an observational preorder (as required by network invariants) has been formulated inside a parametric model checker. Aside from the natural up/down communication between a child and a parent, we allow for peer-to-peer communication, since many real protocol optimizations rely on this paradigm. The paper details the Neo theory, which is built upon the Input-Output Automata formalism, and demonstrates the approach on an example hierarchical cache coherence protocol.

I. INTRODUCTION

Formal verification of large-scale, modern systems protocols is currently challenging. Although theorem proving is theoretically able to verify arbitrary protocols, the manual effort required to guide a theorem prover through the verification of a modern protocol is prohibitive. Model checkers are more widely used, but they cannot handle complex, large-scale protocols. As a result of the state explosion problem, model checking proofs are successful for only a handful of protocol components—generally not sufficient to exercise all the behaviors exhibited in industrial-scale systems. Hence, there is strong motivation for architects to design protocols specifically to be verifiable with state-of-the-art model checking tools. Our solution is to construct a set of protocol components, instances of which are composed into an arbitrary hierarchy, where each component instance is independently scaled. The components are pre-verified in such a way as to guarantee that the resulting large and complex system is always correct.

Our approach involves the combination of two distinct ideas from the model checking literature: network invariants and parameterized model checking. Consider the hierarchical protocol depicted in Fig. 1. We would like to design the leaf $L$, internal $I$, and root $R$ nodes, so that any arbitrary nesting in the vertical direction, and any arbitrary (and independent) branching degree (number of children) at each internal and root node, yields a system that is correct. Arbitrary nesting is handled by network invariants; in particular we require (and verify) that $L$ is a network invariant. This means that the observational behaviors of $L$ subsume that of any larger composition of components. For instance, the behavior along communication channel $c_2$ over-approximates that of $c_1$, $c_3$, etc. We formulate network invariants in a novel way that not only captures the observational behaviors (messages) across an interface, but also captures what we call the summary state of a sub-hierarchy. These summary states are integral in defining the safety property, which, like the system itself, is hierarchically defined.

Beyond the hierarchical nesting afforded by network invariants, we employ parameterized model checking to allow arbitrary branching degrees. This entails that we prove the observational pre-order containment required of network invariants parametrically in a model checker; we believe this is novel. Hence, $L$ serves as a network invariant for not just a particular $I$, but for all members of an infinite family $I(1), I(2), I(3), \ldots$, where $I(n)$ is an internal node configured to connect to $n$ children.

We emphasize that network invariants and parameterized model checking are both necessary ingredients in this story; neither is capable of solving what the other does. Network invariants deal with the connection of instances of components into arbitrarily complex hierarchies, with relatively simple interfaces between constituents; parameterized model checkers typically do not support such a notion of “parameterization” when the structure is nontrivial (e.g. a tree). On the other hand, network invariants are not appropriate to deal with an internal node that is parameterized on the number of children. An example is a directory in a cache coherence protocol—the directory is an array, with one entry per child. There is no clear way to formulate this type of tightly coupled parameterization as the composition of components along with a network invariant. Fortunately, parameterized model checkers are usually targeted at precisely this style of parameterization.

Previous research on network invariants [12], [15], [1], [16], [31] tends to focus on “flat” compositions of processes with rather trivial structure; processes are arranged in a linear or circular array with only neighbor-to-neighbor communication, or the other extreme wherein each process talks to all others. The work of Clarke, Grumberg, and Jha [7], is the most closely

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1We use the terms component and node interchangeably.
Network Invariants

Parameterized Model Checking

Fig. 1. A Neo hierarchy. Nodes labels $R$, $I$, and $L$ respectively indicate root, internal and leaf nodes. Solid lines indicate parent/child communication channels, while dotted lines indicate peer/peer communication. Arbitrary nesting of tree structures in the vertical direction is handled by network invariants. Arbitrary branching widths in the horizontal direction is handled by parameterized model checking. The leaf $L$ acts as a network invariant, which means for example that the behavior along communication channel $c_2$ over-approximates that of $c_1$, $c_3$, etc.

related to us, since they allow hierarchical structures. Like us, they require that a small process serves as a network invariant for all (larger) composite processes.\footnote{In cases where a single terminal (what we call a leaf) process fails to be a networks invariant, they are able to instead employ a non-terminal, but suitably small, composition of processes.} However, we extend their work in several ways:

- As mentioned above, we use parameterized model checking to facilitate arbitrary branching degree
- We use an asynchronous/interleaved execution semantics (I/O automata), while Clarke et al. use a synchronous.
- We make a (modest, but important) extension to allow processes to be given “identifiers.”
- Our example cache protocol is significantly more complex than their example (a protocol that computes a parity function over the leaves of a tree).
- We express our state invariant property using summary functions, which makes the property’s structure naturally echo that of the Neo system’s hierarchy.

Other related work looks at the problem of verifying hierarchical protocols with two levels, using abstraction and assume/guarantee reasoning \cite{5}. Similar to us, smaller systems are verified to conclude coherence of a system for which model checking is intractable, but the approach involves manual effort and it’s unclear if it scales to more elaborate hierarchies.

Parameterized model checking approaches have been widely explored in the literature \cite{13, 2, 9}. The research includes disparate techniques such as: assume/guarantee-style abstraction \cite{20, 21, 6, 14, 32}, predicate abstraction \cite{17}, invisible invariants \cite{25}, flows \cite{22}, regular sets \cite{4}, Satisfiability Modulo Theories (SMT) \cite{11}. We elected to use Cubicle \cite{8} as it has a clean language, has published encouraging results, and is being actively maintained. Though our work is rather agnostic to the underlying model checking technique, we believe our leveraging of a parametric model checker to parametrically prove an observational pre-order is novel.

Some prior work has proposed designing systems from pre-verified components to enable scalable verification. Zhang et al. propose designing cache coherence protocols such that caches are organized in a tree hierarchy, with any scale of the system being observationally equivalent to a pre-verified small-scale system \cite{33}. Unfortunately, \cite{33} is not rigorously formalized. Furthermore, the definition of observational equivalence used focuses only on matching states and ignores actions, which could permit safety violations in a larger scale system. \cite{30} and \cite{29} present performance optimizations to \cite{33} and \cite{19} adapts \cite{33} to designing verifiable power management protocols. Hence, these works inherit \cite{33}’s flaws.

Beu et al. propose a template that allows one to link pre-verified cache coherence protocols into a hierarchy by allowing directories of lower tiers to seek permissions from higher tiers \cite{8}. However, the work is also not rigorously formalized. Also, the pre-verified protocols are not verified in an environment where they interact with higher tiers, which could permit incoherence when they are actually linked into a hierarchy.

To illustrate our verification methodology, we design a hierarchical cache coherence protocol called NeoGerman by composing a parameterized German protocol \cite{6} into a Neo hierarchy. We prove that our protocol is a Neo system, which implies that it behaves correctly for any arbitrary configuration of the hierarchy. While the flat German protocol is trivial, we are not aware of any work that verifies an arbitrary-dimension hierarchical version. We believe our framework is applicable to more sophisticated protocols, and we pick the hierarchical German protocol only to illustrate our approach.

II. FORMALIZING THE NEO FRAMEWORK

Our framework can be thought of as a class of transition systems for which certain properties hold, as a result of which any member of this class is amenable to a much simpler verification methodology. We hope many systems protocols can be shown (or designed) to fit this class and thus inherit the simplified verification. In this section, we will define this class of transition systems and prove that given some automatedly verifiable antecedents, all members of this class are safe.

For any $n \geq 0$, we define $\mathbb{N}_n = \{0, 1, \ldots, n-1\}$; note that $\mathbb{N}_0 = \emptyset$. Also, if $x = (x_0, \ldots, x_k)$ is a tuple or list, we denote $x_i$ by $x[i]$.

A. I/O Automata Theory

We start by giving a short description of the well-known I/O automata theory upon which our framework is formalized. We will only go into enough detail as is sufficient for our work; for a more complete description of I/O automata, see \cite{28}.

An action signature $S$ is a partition of a set $\text{acts}(S)$ of actions into three disjoint sets: $\text{in}(S)$, $\text{out}(S)$, and $\text{int}(S)$, respectively called the input, output, and internal actions. The...
set \( \text{int}(S) \cup \text{out}(S) \) is denoted by \( \text{local}(S) \). An I/O automaton (IOA) \( A \) consists of the following:

- an action signature \( S \), denoted \( \text{sig}(A) \)
- a set \( \text{states}(A) \) called the states
- a nonempty set \( \text{start}(A) \subseteq \text{states}(A) \) called the start states
- a transition relation \( \text{steps}(A) \subseteq \text{states}(A) \times \text{acts}(S) \times \text{states}(A) \)

\( \text{acts}(S) \) is also referred to as \( \text{acts}(A) \), \( \text{in}(S) \) is also referred to as \( \text{in}(A) \), etc. The set \( \text{in}(A) \cup \text{out}(A) \) of external actions is referred to as \( \text{ext}(A) \).

An execution fragment \( e \) of \( A \) is a sequence \( e = s_0, a_1, s_1, \ldots, a_k, s_k \) such that, for each \( i \), \((s_i, a_{i+1}, s_{i+1}) \in \text{steps}(A) \). If \( s_0 \in \text{start}(A) \), then \( e \) is an execution of \( A \). The set of executions of \( A \) is denoted by \( \text{execs}(A) \). If a state \( s \) is the final state of an execution, then \( s \) is said to be reachable.

A set \( \{S_0, \ldots, S_n\} \) of action signatures is said to be compatible if for all \( i \neq j \), \( \text{out}(S_i) \cap \text{out}(S_j) = \emptyset \) and \( \text{int}(S_i) \cap \text{acts}(S) = \emptyset \). A set of IOA are said to be compatible if their action signatures are compatible.

The \( n \)-way composition \( S = \prod_{i=0}^{n-1} S_i \) of compatible action signatures \( \{S_0, \ldots, S_{n-1}\} \) is an action signature with \( \text{in}(S) = \bigcup_{i=0}^{n-1} \text{in}(S_i) \setminus \bigcup_{i=0}^{n-1} \text{out}(S_i), \text{out}(S) = \bigcup_{i=0}^{n-1} \text{out}(S_i) \setminus \bigcup_{i=0}^{n-1} \text{in}(S_i) \), and \( \text{int}(S) = \bigcup_{i=0}^{n-1} \text{int}(S_i) \cup \bigcup_{i=0}^{n-1} \text{out}(S_i) \cap \bigcup_{i=0}^{n-1} \text{in}(S_i) \).

The \( n \)-way composition \( C = \prod_{i=0}^{n-1} C_i \) of compatible IOA \( \{C_0, \ldots, C_{n-1}\} \) is an IOA with the following:

- \( \text{sig}(C) = \prod_{i=0}^{n-1} \text{sig}(C_i) \)
- \( \text{states}(C) = \text{states}(C_0) \times \cdots \times \text{states}(C_{n-1}) \)
- \( \text{start}(C) = \text{start}(C_0) \times \cdots \times \text{start}(C_{n-1}) \)
- \( \text{steps}(C) \) is a set of tuples of the form \((s, a, s') \in \text{states}(C) \times \text{acts}(C) \times \text{states}(C) \) that satisfy the following for all \( i \):
  - \( a \in \text{acts}(C_i) \) implies \((s[i], a, s'[i]) \in \text{steps}(C_i) \)
  - \( a \notin \text{acts}(C_i) \) implies \((s[i], s'[i]) \in \text{steps}(C_i) \)

For IOA \( C = \prod_{i=0}^{n-1} C_i \), for \( s \in \text{states}(C) \) and for all \( i \), define \( s|C_i = s[i] \). Let \( e = s_0, a_1, s_1, \ldots, a_k, s_k \) be an execution of \( C \). Then, for all \( i \), define \( e|C_i \) as the sequence derived by modifying \( e \) as follows. Delete each \( a_j, s_j \) if \( a_j \notin \text{acts}(C_i) \). Then, replace all remaining \( s_j \) with \( s_j|C_i \).

**Lemma 1.** Let IOA \( C = \prod_{i=0}^{n-1} C_i \) and \( e \in \text{execs}(C) \). Then, for all \( i \), \( e|C_i \in \text{execs}(C_i) \).

**Proof.** See Tuttle et al. \[28\].
Lemma 2. For Neo system $\Omega$, if $\Omega$ is an open Neo system, then $\text{in}(\Omega) = D \cup P(m-1)$ and $\text{out}(\Omega) = U \cup P(m-1)$. If $\Omega$ is a closed Neo system, then $\text{ext}(\Omega) = \emptyset$.

Proof. We are tasked with proving the following statements:

$$\text{in}(\Omega) = D \cup P(m-1) \quad \text{and} \quad \text{out}(\Omega) = U \cup P(m-1) \tag{2}$$

$$\text{ext}(\Omega) = \emptyset \tag{3}$$

First, we will prove (2) by structural induction on the definition of Open Neo systems. (1) holds for $m$-peer leaf processes, by virtue of a leaf being a $(0, m)$ internal node.

For the inductive case, assume (2) holds for each $\Omega_i$. We will show that it holds for $\Omega$.

Let $P'$ be the set $\{(p, i, i) : p \in P \text{ and } i < n\}$. Observe that $\text{in}(\Omega) \subseteq \text{in}(A) \cup \bigcup_{i=0}^{n-1} \text{in}(\Omega_i)$. That implies $\text{in}(\Omega) \subseteq D \cup P(m-1) \cup D(n) \cup (P(n, n) \setminus P')$ (by inductive hypothesis and definition of $\phi_i$). $D(n) \subseteq \text{out}(A)$ and $(P(n, n) \setminus P') \subseteq \bigcup_{i=0}^{n-1} \text{out}(\Omega_i)$ cannot appear in the input signature of $\Omega$, by definition of action signature composition. However, no elements of $D$ and $P(m-1)$ are elements of $\text{out}(A) \cup \bigcup_{i=0}^{n-1} \text{out}(\Omega_i)$. Furthermore, $D$ and $P(m-1)$ are subsets of $\text{in}(\Omega)$. Hence, by definition of action signature composition, $\text{in}(\Omega) = D \cup P(m-1)$.

By similar reasoning as above, $\text{out}(\Omega) = U \cup P(m-1)$.

Next, we will prove (3), assuming $\Omega$ is a closed Neo system. $\text{in}(A) = U(n)$, by definition of root node. Using results from above, $\text{ext}(\Omega) \subseteq U(n) \cup D(n) \cup (P(n, n) \setminus P')$. But $U(n)$, $D(n)$, and $P(n, n) \setminus P'$ cannot appear in $\text{ext}(\Omega)$ because all their elements involve communication between components of $\Omega$, as shown above. Hence, $\text{ext}(\Omega) = \emptyset$. □

C. Neo System Safety

1) Summary of States: Let $\text{Sum}$ be a finite set of summary states that contain a distinguished state $\text{bad}$. We associate summary functions with each $L(m)$ and the elements of $I$s and $R$s as follows:

- $\text{sum}_{L(m)}$ has type $\text{states}(L(m)) \rightarrow \text{Sum}$
- For each $n$-child $A \in I$s $\cup R$s, $\text{sum}_A : \text{states}(A) \times \text{Sum}^n \rightarrow \text{Sum}$ is a “bad preserving” function, i.e., $\text{bad} \in \{s_0, \ldots, s_{n-1}\}$ implies $\text{sum}_A(s_0, \ldots, s_{n-1}) = \text{bad}$.

We extend the above elemental $\text{sum}_A$ functions to summarize the state of an arbitrary non-leaf Neo system $\Omega$ as follows. $\Omega$ is $[\Omega]$, where $A \in I$s $\cup R$s and $\Omega_0, \ldots, \Omega_{n-1}$ are open Neo systems. Then $\text{sum}_{\Omega} : \text{states}(\Omega) \rightarrow \text{Sum}$ is defined by

$$\text{sum}_{\Omega}(s_0, \ldots, s_{n-1}) = \text{sum}_A(s_0, \text{sum}_{\Omega_0}(s_0), \ldots, \text{sum}_{\Omega_{n-1}}(s_{n-1}))$$

2) Summary Sequence of Executions: Given an execution $e = s_0, \alpha_1, \ldots, \alpha_k, s_k$ of a Neo system $\Omega$, we define the summary sequence $\text{sum}(e)$ as follows. Let $\alpha'_i = \alpha_i$ if $\alpha_i \in \text{ext}(\Omega)$, otherwise $\alpha'_i = \lambda$. We start with the sequence

$$\text{sum}_{\Omega}(s_0, \alpha_1', \ldots, \alpha_k', \text{sum}_{\Omega}(s_k))$$

and delete all elements $\alpha'_i, \text{sum}_{\Omega}(s_i)$ such that $\alpha'_i = \lambda$ and $\text{sum}_{\Omega}(s_i) = \text{sum}_{\Omega}(s_{i-1})$.

3) Safety Definition: For Neo system $\Omega$ and state $s \in \text{states}(\Omega)$, we say that $s$ is safe if $\text{sum}_{\Omega}(s) \neq \text{bad}$. We say that $\Omega$ itself is safe if all its reachable states are safe. The primary goal of this paper is to establish that all Neo systems are safe, by only proving a handful of lemmas about the “ingredient” IOAs ($\text{Ls, Is, Rs}$) and their summary functions.

D. Neo Pre-order $\leq$

We define a pre-order $\leq$ on open Neo systems. Given two $m$-peer open Neo systems $\Omega_1$ and $\Omega_2$ that are either both tagged or untagged, the relation $\Omega_1 \leq \Omega_2$ holds if, for all executions $e_1$ of $\Omega_1$, there exists an execution $e_2$ of $\Omega_2$ such that $\text{sum}(e_1) = \text{sum}(e_2)$.

Lemma 3. $\leq$ is transitive.

Proof. Assume $\Omega_1 \leq \Omega_2$ and $\Omega_2 \leq \Omega_3$. Let $e_1$ be an arbitrary execution of $\Omega_1$. Let $e_2$ be some execution of $\Omega_2$ such that $\text{sum}(e_1) = \text{sum}(e_2)$; $e_2$ exists, by $\Omega_1 \leq \Omega_2$. Let $e_3$ be some execution of $\Omega_3$ such that $\text{sum}(e_2) = \text{sum}(e_3)$; $e_3$ exists, by $\Omega_2 \leq \Omega_3$. Then, $e_3$ is an execution of $\Omega_3$ such that $\text{sum}(e_1) = \text{sum}(e_2) = \text{sum}(e_3)$. □

Lemma 4. $\Theta \leq \Omega$ if and only if $\phi_i(\Theta) \leq \phi_i(\Omega)$.

Proof. $(\Rightarrow)$ Define the function

$$\text{shiftInv}(i, j) = \begin{cases} j & \text{if } j < i \\ j - 1 & , \text{otherwise} \end{cases}$$

Observe that $\text{shiftInv}(i, \text{shift}(i, j)) = j$ and $\text{shift}(i, \text{shiftInv}(i, j)) = j$. Assume $\Theta \leq \Omega$. Let $x$ be an execution of $\phi_i(\Theta)$. We generate an execution $X$ of $\phi_i(\Omega)$ such that $\text{sum}(x) = \text{sum}(X)$ as follows.

Generate a new sequence $x'$ by modifying $x$ to reverse the tagging of actions in $\Theta$. Every internal or external action of the form $(a, i)$ is replaced with $a$, every input action of the form $(p, j, i)$ is replaced with $(p, \text{shiftInv}(i, j))$, and every output action of the form $(p, i, j)$ is replaced with $(p, \text{shift}(i, j))$.

Because $\text{shift}(i, \text{shiftInv}(i, j)) = j$, if we tagged the actions of both sides of the equation $\text{sum}(x') = \text{sum}(X')$ using $\text{shift}$, we would get $\text{sum}(x) = \text{sum}(X)$.

$(\Leftarrow)$ Uses a similar argument as the other direction. □

Lemma 5. Let Neo systems $\Omega = A \cdot \prod_{i=0}^{n-1} \phi_i(\Omega_i)$ and $\Theta = A \cdot \prod_{i=0}^{n-1} \phi_i(\Theta_i)$, where $A \in I$s $\cup R$s. Suppose for some $k$, $\Omega_k \leq \Theta_k$ and for all $i \neq k$, $\Omega_i = \Theta_i$. Then, for all executions $e$ of $\Omega$, there exists an execution $e'$ of $\Theta$ such that $\text{sum}(e) = \text{sum}(e')$. □
Proof. WLOG, fix $k = n - 1$. Let $C = A \cdot \prod_{i=0}^{n-2} \phi_i(\Omega_i) = A \cdot \prod_{i=0}^{n-2} \phi_i(\Theta_i)$. Let $B = \phi_k(\Omega_k)$ and $G = \phi_k(\Theta_k)$. Then, $\Omega = C \cdot B$ and $\Theta = C \cdot G$.

Let $e = e_0, a_1, \ldots, a_k, s_k$ be an arbitrary execution of $\Omega$. Let $d \in \text{execs}(G)$ be such that $\text{sum}_B(e|B) = \text{sum}_C(d)$. We know $d$ exists by Lemma 1 and because $\Omega_k \leq \Theta_k$ implies $\phi_k(\Omega_k) \leq \phi_k(\Theta_k)$ (Lemma 4).

For some state $y$, define

$$y * e = (y, s_0, \alpha_1, \ldots, \alpha_k, (y, s_k))$$

i.e. $y$ is simply pre-pended as a component to every state. Similarly $e * y$ post-pends $y$ to every state of $e$. Let us call $e * y$ a transition $(p, \alpha, (p', a'))$ of $C.B$ quiet if we have

- $\alpha \in \text{int}(B) \cup \text{int}(C)$
- $p = p'$, and
- $\text{sum}(a) = \text{sum}(a')$.

A transition is called loud if it is not quiet. Given an execution $e \in \text{execs}(C.B)$, the loud-length of $e$ is the number of loud transitions in $e$. An execution fragment is called quiet if all its transitions are quiet (i.e. the loud-length is 0).

We proceed by induction on loud-length of $e$. We will prove the stronger statement that the $e'$ asserted to exist by the lemma statement satisfies:

(I) $\text{sum}(e') = \text{sum}(e)$,

(II) $e'|G = d$, and

(III) $e'$ and $e$ agree on the final state of $C$.

**Base Case:** The loud-length of $e$ is 0, which means

$$e = (p, a_0), \alpha_1, (p, a_1), \alpha_2, \ldots, \alpha_k, (p, a_k)$$

where all for all $1 \leq i \leq k$ we have $\alpha_i \in \text{int}(B) \cup \text{int}(C)$ and $\text{sum}(a_i) = \text{sum}(a_0)$. Note that $\text{sum}(e)$ is just the single-state execution $\text{sum}(p, \text{sum}(a_0))$, and $\text{sum}(e|B) = \text{sum}(a_0)$.

It follows that $d = b_0, \beta_1, \ldots, \beta_m, b_m$, where $\beta_i \in \text{int}(G)$ and $\text{sum}(b_i) = \text{sum}(b_0) = \text{sum}(a_0)$ for all $1 \leq i \leq m$. Now set

$$e' = p * d$$

Clearly $e' \in \text{execs}(C \cdot G)$ and (I), (II), and (III) all hold.

**Inductive Step:** Suppose the lemma holds for all executions of loud-length $n$, and consider an execution $e \in \text{execs}(C \cdot B)$ of loud-length $n + 1$. Let us factor $e$ as follows:

$$e = e_1, (p, a), \alpha, (p', a'), e_2$$

where $e_1$ and $e_2$ are execution fragments of $C \cdot B$ and $(p, a), (p', a') \in \text{states}(C \cdot B)$ are such that

- $e_1, (p, a)$ has loud-length $n$.
- $(p, a), \alpha, (p', a')$ is loud, and
- $(p', a'), e_2$ is quiet.

Now let $d$ be any execution of $G$ such that $\text{sum}(e|B) = \text{sum}(d)$. We now case split into three cases.

**Case 1:** $\alpha \notin \text{sig}(B)$. Then we must have $a = a'$ and $\alpha \in \text{sig}(C)$, and clearly $\text{sum}(d) = \text{sum}((e_1, (p, a))|B)$. By the inductive hypothesis, there exists $e'_1 \in \text{execs}(C \cdot G)$ such that $\text{sum}(e'_1) = \text{sum}(e_1, (p, a)), e'_1|G = d$, and $e'_1$ has final state $(p, b)$ for some $b \in \text{states}(G)$. Let us set

$$e' = e'_1, \alpha, (p', b)$$

Clearly (II) and (III) hold of $e'$; (I) can be seen to hold by noting that $\text{sum}(e') = \text{sum}(e'_1), f = \text{sum}(e_1, (p, a)), f$ where $f = \alpha, \text{sum}(p', \text{sum}(b))$ if $\alpha \in \text{ext}(C \cdot G) = \text{ext}(C \cdot B)$ or $\text{sum}(p, \text{sum}(b)) \neq \text{sum}(p', \text{sum}(b))$, otherwise $f = \alpha$. In either case we can see (I) holds by inspection of $d$ and noting $a = a'$.

**Case 2:** $\alpha \in \text{ext}(B) = \text{ext}(G)$ (Lemma 2). We can then factor $d$ as:

$$d = d_1, b, \alpha, b', d_2$$

where $d_1$ and $d_2$ are execution fragments of $G$ and $b, b' \in \text{states}(G)$ are such that $b', d_2$ is quiet. Clearly $\text{sum}((e_1, (p, a))|B) = \text{sum}(d_1, b)$, and thus, from the inductive hypothesis, there exists an execution $e'_1$ of $C \cdot G$ such that $e'_1|G = d_1, b \cdot \text{sum}(e'_1) = \text{sum}(e_1, (p, a))$, and $e'_1$ has final $C$-state $p$. Letting $b''$ be the final state of $d_2$, let us set

$$e' = e'_1, \alpha, (p', b'), (p' * d_2)$$

One can confirm that (I), (II), and (III) hold of $e'$.

**Case 3:** $\alpha \in \text{int}(B)$. Since $(p, a), (p', a')$ is loud, it follows that and $\text{sum}(a) \neq \text{sum}(a')$. We can then factor $d$ as:

$$d = d_1, b, \beta, b', d_2$$

where $\beta \in \text{int}(G), \text{sum}(b) = \text{sum}(a) \neq \text{sum}(b') = \text{sum}(a')$, and $b', d_2$ is quiet. If we replace $\alpha$ with $\beta$ in the $e'$ of Case 2 (5), (I), (II), and (III) hold.

**Lemma 6.** Let $\Theta = A \cdot \prod_{i=0}^{n-1} \phi_i(\Theta_i)$ and $\Omega = A \cdot \prod_{i=0}^{n-1} \phi_i(\Omega_i)$ be open Neo systems such that $\Theta_i \leq \Omega_i$, for all $i$. Then, $\Theta \leq \Omega$.

**Proof.** Consider the following sequence $\Pi$ of processes constructed by incrementally replacing an $\Theta_i$ with a corresponding $\Pi_i = A \cdot \prod_{i=0}^{n-1} \phi_i(\Theta_i)$, $\Pi_0 = \Theta$, and $\Pi_i = \Pi_{i-1} \cdot \phi_i(\Omega_i)$ for all $i$. Then $\Theta \leq \Omega$.

**Lemma 7.** (Leaf as a Network Invariant) Suppose that the $m$-peer open Neo system $\Omega_L = A \cdot \prod_{i=0}^{n-1} \phi_i(L(n))$ satisfies $\Omega_L \leq L(m)$. Then, for any $m$-peer open Neo system $\Omega$, $\Omega \leq L(m)$.

**Proof.** If $\Omega$ is an $m$-peer leaf, then $\Omega = L(m) \leq L(m)$. Otherwise, let $\Omega = A \cdot \prod_{i=0}^{n-1} \phi_i(\Omega_i)$ be an $m$-peer open Neo system. Assuming that each $\Omega_i \leq L(n)$ (inductive hypothesis), we will prove, by structural induction on the construction of $\Omega$, that $\Omega \leq L(m)$. By Lemma 6 and inductive hypothesis, $\Omega \leq L(m)$. By transitivity of $\leq$ (Lemma 5) and inductive hypothesis, $\Theta \leq L(m)$.

**Theorem 1.** (Every Neo system is safe.) Suppose that for each $n$-child node $A \in R_s \cup I_s$, $\Omega_L = A \cdot \prod_{i=0}^{n-1} \phi_i(L(n))$ is safe.
Furthermore, suppose that if $A$ is an $n$-peer internal node, then $\Omega_L \leq L(m)$. Then all Neo systems are safe.

Proof. Let $\Omega$ be an (open or closed) Neo system $[\Omega]$. From the assumptions of this lemma and Lemma 7, $\Omega_i \leq L(n)$ for all $i$. Let $e$ be an arbitrary execution of $\Omega$. By an $n$-fold application of Lemma 5, there exists an execution $e'$ of $\Omega_L$ such that $\text{sum}(e') = \text{sum}(e)$. By definition of $\text{sum}$, if no state $\in e'$ summarizes to $\text{bad}$, then no state $e$ summarizes to $\text{bad}$. Therefore $\Omega$ is safe.

The significance of Theorem 1 is that if we establish $\Omega_L$’s safety, for all $A \in R_s \cup I_s$, and $\Omega_L \leq L(m)$, for all $A \in I_s$, then any configuration of the Neo nodes (which would typically be closed) is safe. Parameteric model checking comes into play, since when the elements of $R_s \cup I_s$ are parameterized (by number of children $n$ and number of peers $m$), these safety and preorder checks are parameterized verification problems.

III. MAPPING PROTOCOLS’ SAFETY TO NEO SAFETY

We have defined safety of a Neo system (Sect. IV-D1) to mean that no reachable state summarizes to $\text{bad}$, which is somewhat removed from the actual invariant one might be interested in. Here, we illustrate how an invariant of interest can be expressed in the form of the Neo safety definition. The key is that the summary functions must be forced to return $\text{bad}$ whenever the specific safety property of interest is violated.

A. Cache Coherence

In a typical MOESI cache coherence protocol [26], $\text{Sum} = \{I, S, O, E, M, bad\}$ and cache coherence means that if any leaf summarizes to $M$ or $E$, then all other leaves must summarize to $I$. To ensure that Neo system safety (no reachable state summarizes to $\text{bad}$) implies all reachable states are cache coherent, we require some simple constraints on $\text{sum}_A$ for each $A \in I_s \cup R_s$. Let us define an ordering $<$ on $\text{Sum}$ by $I < S, O < E, M < \text{bad}$. Recalling that $\text{sum}_A$ has type $\text{states}(A) \times \text{Sum}^n \rightarrow \text{Sum}$, where $n$ is the arity of $A$, the cache coherence constraint on $\text{sum}_A$ is as follows:

- Whenever there exists distinct $i, j \in \mathbb{N}_n$ such that $s_i \in \{M, E\}$ and $s_j \neq I$, we require $\text{sum}_A(s_a, s_0, \ldots, s_{n-1}) = \text{bad}$, and
- For all $i \in \mathbb{N}_n$, $s_i \leq \text{sum}_A(s_a, s_0, \ldots, s_{n-1})$ (i.e. $\text{sum}_A$ is monotonically increasing with $<$)

Lemma 8. If $\text{sum}_A$ satisfies the cache coherence constraint for all $A \in I_s \cup R_s$, then Neo safety implies cache coherence.

Proof. Let $s$ be a state of a Neo system $\Omega$. We argue, by structural induction on $\Omega$, that whenever $s$ contains a cache coherency violation, $\text{sum}_{\Omega_L}(s) = \text{bad}$. The base case $\Omega \in I_s$ holds vacuously, since a leaf in isolation cannot violate cache coherency. Now choose $A \in I_s \cup R_s$ with arity $n$. Then $s = (s_a, s_0, \ldots, s_{n-1})$, where $s_a \in \text{states}(A)$ and $s_i \in \text{states}(\Omega_i)$, $0 \leq i < n$. If $s$ contains a cache coherency violation then there exists a leaf $L$ in $s$ that summarizes to $M$ or $E$, and a distinct leaf $L'$ that summarizes to something other than $I$. If $L$ and $L'$ are both components of $\Omega_i$ for some $i$, then from our inductive hypothesis, $\text{sum}_{\Omega_i}(s_i) = \text{bad}$, and from Sect. II-C1 we have that $\text{sum}_{\Omega_i}(s) = \text{bad}$. On the other hand, suppose $L$ and $L'$ are respectively components in $\Omega_i$ and $\Omega_j$ with $i \neq j$. Since the cache coherency constraint requires $\text{sum}$ to be monotonic, it follows that $E \leq \text{sum}_{\Omega_i}(s_i)$ and $S \leq \text{sum}_{\Omega_i}(s_j)$, and again the cache coherency constraint requires $\text{sum}_{\Omega_i}(s) = \text{bad}$.

We envision that other types of Neo systems will need similar side arguments to relate Neo safety to a more concrete property of interest, and such arguments will be as straightforward as what was required to prove Lemma 8 above.

B. Distributed Lock Management (DLM)

Distributed Lock Management (DLM) protocols are used to ensure safe access to shared resources such as disks and files. Several DLM protocols are based on the DEC VMS’s DLM implementation [27], including the Oracle Cluster File System (OCFS2) that appears in the Linux Kernel [23] [18]. VMS’s DLM has 6 permissions—Null (NL), Concurrent Read (CR), Concurrent Write (CW), Protected Read (PR), Protected Write (PW), and Exclusive (EX). The following combinations of permissions are prohibited: (CR,EX), (CW,EX), (PW,EX), (PW,PR), (PR,EX), (PR,PW), (PW,EX), (PW,PW), (EX,EX).

For scalable resource management, one can organize nodes in a cluster as a hierarchy according to the Neo framework. This would facilitate verification, for arbitrary system sizes, that no two nodes hold a prohibited combination of permissions. We could set $\text{Sum} = \{NL, CR, CW, PR, PW, EX, bad\}$ and define a partial order $<$ such that $NL < CR < PR < PW < EX < \text{bad}$ and $NL < CR < CW < EX < \text{bad}$; $<$ does not order $PW$ and $CW$. Then, imposing similar constraints to the $\text{sum}$ functions of Sect. III-A, one can show that $\text{sum}$ not evaluating to $\text{bad}$ implies that the system never violates DLM safety.

IV. CASE STUDY: THE NEOGERMAN PROTOCOL

To illustrate our verification methodology, we design and verify a hierarchical cache coherence protocol called NeoGerman. Using a parametric model checker, we verify that NeoGerman is a Neo System and, consequently, satisfies the coherence invariant for arbitrary configurations.

A. NeoGerman Description

German’s protocol is a simple, directory-based caching protocol proposed as a challenge for parameterized verification [10]. To make the protocol hierarchical, we made significant modifications. In particular, the directory was modified to communicate with a parent, hence serving as an internal node.

7The directory was used unmodified to create the root node.
1) The German Protocol: The German protocol is a flat cache coherence protocol. We use the version specified in [6], which is parameterized to have a single directory connected to an arbitrary number of private caches. Each cache block is in one of three states: I (invalid), S (shared), or E (exclusive). The protocol uses the directory to maintain the invariant that no two caches are simultaneously in (S, E) or (E, E). The directory maintains a list of all nodes in S or E, called sharers.

   If a cache sends a message to the directory to request S (GetS) when there is a cache in E, the cache in E gets sent an Invalidate message, and the directory collects an invalidation acknowledgement (InvAck) from it. The directory then sends a GrantS message to the requesting cache to grant it S permissions. If the directory receives a GetE, it invalidates all sharers and collects all their InvAck’s before sending a GrantE message to the requesting cache.

2) Modifications to German: To turn German into an open Neo system, we modify the directory so that it behaves like a private cache along a (previously non-existent) communication channel shared with a parent. Upon receiving requests from its children, the directory now has the ability to seek permissions from its parent. We will refer to this modified directory as the internal directory, to distinguish it from the original German directory that we use as a root node to close the Neo hierarchy.

The internal directory maintains a variable called Permissions_O, which summarizes the permissions of the open Neo system it heads as that of a single private cache. The intent is that if, for example, Permissions_O is in I and the internal directory receives a GetS from a child, the internal directory forwards the request to its parent. Upon receiving a subsequent GrantS from its parent, Permissions_O changes to S and the internal directory sends a GrantS to the requesting child and makes it a sharer. If the internal directory receives an Invalidate from its parent, it invalidates all sharing children and collects all InvAcks. Finally, the internal directory sends an InvAck to its parent and updates Permissions_O to I.

B. Tying NeoGerman to the Neo Framework

In NeoGerman, we have $U = \{ \text{GetS, GetE, InvAck} \}$, $D = \{ \text{GrantS, GrantE, Invalidate} \}$ and $P = \emptyset$. The private caches correspond to tagged leaf processes of the form $\phi_i(L)$, where $L$ is a leaf node. Each individual leaf is tagged with a parameter that enables unique communication with its directory. The directory/memory $R(n)$ of the original German protocol constitutes an $n$-child root node of NeoGerman. The directory $I(n)$ of the NeoGerman protocol constitutes an $n$-child internal node. Armed with $R(n)$, $I(n)$, $L$ and the Cubicle process composition methods we discussed above, we have all the ingredients to build NeoGerman as a Neo system.

C. Modeling NeoGerman in a Model Checker

We modeled NeoGerman in Cubicle [8]. Cubicle is a symbolic model checker used to verify parameterized array-based systems by using a backwards reachability algorithm and an SMT solver. Its support for parametric verification allows us to verify safety properties for arbitrary configurations of a Neo hierarchy. Cubicle’s processes are parameterized by indices of a built-in type proc. The state of an arbitrary number of processes is represented by arrays indexed by proc.

Even though neither Cubicle nor our framework impose size restrictions on communication buffers, we model NeoGerman with a single-entry communication buffers for simplicity.

1) Representing a Process: To illustrate how we model processes in Cubicle, let set $B = \{ \phi_i(A) : i \in \mathbb{N}_n \}$, where $A$ is a Neo leaf node with steps $\phi_i(A) =$ \{(s0, (a0, i)), (s1, (a1, i)), (s2, (a2, i), s0)\}. Let $start(A_i) = \{(s0)\}$, $in(A_i) = \{(a0, i)\}$, $out(A_i) = \{(a1, i)\}$, and $int(A_i) = \{(a2, i)\}$. We would model B in Cubicle as follows:

2) Representing Composition: For IOA $B$, let $steps(B) = \bigcup_{i=0}^{n-1} \{(s0, (a0, i), s1), (a1, i), s2, (a2, s0)\}$. Let $start(B) = \{(s0)\}$, $out(B) = \{(a0, i)\}$, and $int(B) = \{(s1)\}$. By combining the guards and state updates of transitions with identical names, we represent the composition $C = B \prod_{i=0}^{n-1} A_i$ as follows:

For all Cubicle code in this paper, we deviate slightly from Cubicle syntax for conciseness.
D. Proving the NeoGerman Hierarchy is Coherent

We leverage the Neo framework and Cubicle’s parametric verification to prove that any NeoGerman configuration is coherent. Our strategy is to first define \( \text{sum}_{A(n)} \) for each \( A \in \{ R, I \} \) and \( n \geq 1 \) such that it satisfies the constraints of Section III-A. Then, we prove the conditions of Theorem 1 and Lemma 8 from which coherency of \( \Omega \) follows. Let \( \Omega_{A(n)} = A(n) \cdot \prod_{i=0}^{n-1} \phi_i(L) \), and let \( \text{Sum} = \{ I, S, E, \text{bad} \} \), ordered \( I < S < E < \text{bad} \). To leverage Theorem 1 and Lemma 8 we model \( \Omega_{R(n)} \) and \( \Omega_{I(n)} \) in Cubicle and prove the following for all \( n \) and each \( A \in \{ R, I \} \):

\[
\Omega_{A(n)} \text{ is safe}
\]

\[
\forall i : s_i \leq \text{sum}_{A(n)}(s, s_0, \ldots, s_{n-1})
\]

\[
\forall i \neq j : s_i \in \{ M, E \} \land s_j \neq I \Rightarrow \text{sum}_{A(n)}(s, s_0, \ldots, s_{n-1}) = \text{bad}
\]

\[
\Omega_{I(n)} \preceq L
\]

First, we define \( \text{sum}_{R(n)} \) and \( \text{sum}_{I(n)} \). Unless the cache coherence constraints (Sec. III-A) require \( \text{bad} \), \( \text{sum}_{R(n)}(s, s_0, \ldots, s_{n-1}) = E \). Likewise, unless the cache coherence constraints require \( \text{bad} \), \( \text{sum}_{I(n)}(s, s_0, \ldots, s_{n-1}) = \text{Permissions}_O \), where \( \text{Permissions}_O \) is a Cubicle variable of \( I(n) \). Hence, \( \text{Permissions}_O \) is a function of states \( I(n) \).

1) Safety and Monotonicity of \( \text{sum} \): To prove (6), we parametrically model check \( \Omega_{R(n)} \) and \( \Omega_{I(n)} \); after each \( \Omega_{I(n)} \) transition, a variable \( \text{Sum}_{\text{Output}_O} \) representing the output of \( \text{sum}_A \) is updated to \( \text{Permissions}_O \). The following is specified as a safety violation:

1 unsafe(i,j) CacheState[i]=Bad || (CacheState[i]=E & CacheState[j]!=I)

where, for all \( i \), \( \text{CacheState}[i] \equiv \text{sum}_{L}(\phi_i(L)) \).

For \( I(n) \), \( \text{sum}_{R(n)} \) is monotonically increasing, and we model check \( \Omega_{I(n)} \), specifying the following as safety violations:

1 unsafe(i) {Sum_{Output[O]}'=E & CacheState[i]=E}
2 unsafe(i) {Sum_{Output[O]=I & CacheState[i]=S}

To prove (8), we model check \( \Omega_{I(n)} \) with the following:

1 unsafe(i,j) {not ((CacheState[i]=E & CacheState[j]!=Invalid) \Rightarrow Sum_{Output[O]=Bad})

2) Observational Process Pre-order: To prove (9), we employ a similar approach to Park et al. [24], with an important difference that we generalize to a parametric setting to verify our pre-order. Park et al. show how to prove that a process \( A \) implements a process \( B \) in a model checker by expressing \( B \) as a function. \( A \) is model-checked and, on each transition \( t \), \( B \)’s function is called to give \( B \)’s next state, given \( A \)’s state at the start of \( t \). An assertion checks that a simulation relation holds, given \( t \)’s action and the states of \( A \) and \( B \) at the start and end of \( t \). Cubicle does not support functions and in-line assertions due to its underlying algorithm, so we must rely only on safety properties. As a result of these limitations, we must prove a stricter pre-order \( \preceq_c \) based on a slightly different function \( \text{sum}_c \) defined below.

Let IOA \( A \) execution \( e = s_0a_1s_1 \ldots a_is_k \). Then, \( \text{sum}_c(e) \) is a sequence derived as follows. Replace each \( a_i \) with \( \text{sum}_{A(s_i)} \). Replace each \( a_i \in \text{int}(A) \) with the symbol \( \lambda \). For IOA \( A_1 \) and \( A_2 \), \( A_1 \preceq_c A_2 \) implies for any execution \( e_1 \) of \( P_1 \), there exists an execution \( e_2 \) of \( A_2 \) such that \( \text{sum}_c(e_1) = \text{sum}_c(e_2) \).

Lemma 9. \( \Theta \preceq_c \Omega \) implies \( \Theta \preceq \Omega \).

Proof. Assume \( \Theta \preceq_c \Omega \). Let arbitrary \( \Theta \) execution \( e_1 = s_0a_1s_1 \ldots a_is_k \), and let \( e_2 = S_0\pi_1S_1 \ldots \pi_kS_k \) be some execution of \( \Omega \) such that \( \text{sum}_c(e_1) = \text{sum}_c(e_2) \). Let \( \text{sum}_c(e_1) = \text{sum}_{\text{oa}}(s_0, a_1', \ldots, a_k', \text{sum}_e(s_k)) \) and \( \text{sum}_c(e_2) = \text{sum}_{\text{oa}}(S_0, \pi_1, \ldots, \pi_k, \text{sum}_{\text{oa}}(S_k)) \). Observe that \( a_i' = \lambda \), \( \text{sum}_e(s_k) = \text{sum}_e(s_{k-1}) \) implies \( \pi_i' = \lambda \) and \( \text{sum}_{\text{oa}}(S_i) = \text{sum}_{\text{oa}}(S_{i-1}) \), by \( \text{sum}_{\text{oa}}(e_1) = \text{sum}_{\text{oa}}(e_2) \). Then, by definition of \( \text{sum}, (\text{sum}_{\text{oa}}(e_1)) = (\text{sum}_{\text{oa}}(e_2)) \).

The definition of \( \text{sum}_c \) implies that, to prove \( \Omega_{I(n)} \preceq_c L \), we must match every \( \Omega_{I(n)} \) execution with an equal-length execution of \( L \). Hence, we make a trivial modification to the \( L \) IOA in NeoGerman by adding \( \lambda \) to \( \text{int}(L) \) such that, for all \( s \in \text{states}(L) \), \( (s, \lambda, s) \in \text{states}(L) \). This allows \( L \) to make as many stuttering steps as needed to match execution fragments of \( \Omega_{I(n)} \) that have only internal steps with no change in summary state.

The key to our approach in proving the pre-order is that in the same Cubicle file, we model both \( \Omega_{I(n)} \) and \( L \) and instrument the code of both processes such that they transition in lockstep, starting with \( \Omega_{I(n)} \). Our instrumentation also guides \( L \) to pick transitions that match each \( \Omega_{I(n)} \) transition. We use a safety property to check that, after each \( L \) transition, the states and actions of \( L \) and \( \Omega_{I(n)} \) correspond as required for \( \text{sum}_c \) to be equal. We also use a safety property to check that, after each \( \Omega_{I(n)} \) transition, there always exists an \( L \) transition that can fire. If both safety checks pass, then we know that \( \Omega_{I(n)} \preceq_c L \) and, thus, \( \Omega_{I(n)} \preceq \Omega \) (Lemma 9).

Matching Executions:

1) To force \( \Omega_{I(n)} \) and \( L \) to transition in lockstep, a variable \( L_{\text{to\_run}} \) is initialized to false. It is set to true after each \( \Omega_{I(n)} \) transition and set to false after each \( L \) transition. Then, the expression \( L_{\text{to\_run}} = \text{false} \) is conjuncted to the guard of each \( \Omega_{I(n)} \) transition and \( L_{\text{to\_run}} = \text{false} \) is conjuncted to the guard of each \( L \) transition.

2) To access the most recent actions of \( L \) and \( \Omega_{I(n)} \), we update a variable \( O_{\text{action}} \) only at the end of each \( \Omega_{I(n)} \) transition and variable \( L_{\text{action}} \) only at the end of each \( L \) transition. For external transitions, \( O_{\text{action}} \) and \( L_{\text{action}} \) are updated to the transition’s name. Otherwise, they are updated to \( \text{lambda} \).

\[ \text{As a result of the limitations of Cubicle, we need to prove (8) and (9) for reachable states, rather than writing code that clearly satisfies these constraints for any state.} \]
3) To guide \( L \) to make a matching external step to each external step of \( \Omega_{(n)} \)'s, we conjunct to the guard of each \( L \) external transition named \( \text{trans} \_\text{name} \) the expression \( O\_\text{action}=\text{trans} \_\text{name} \).

4) To guide \( L \) to make a matching step to each internal step of \( \Omega_{(n)} \)'s, a variable \( \text{Forced} \_\text{Transition} \) is updated to some value \( \text{int} \_\text{name} \) after each \( \Omega_{(n)} \) internal transition. Then, the guard of the desired \( L \) internal transition is conjuncted with the expressions \( \text{Forced} \_\text{Transition}=\text{int} \_\text{name} \) and \( O\_\text{action}=\lambda \).

Note that the above modifications maintain the integrity of the pre-order check. All modifications to \( L \)'s guards involve logical conjunctions, which could only restrict \( L \)'s transitions. And the only modification to \( \Omega_{(n)} \)'s guards is conjuncting \( L\_\text{to-run}=\text{False} \), which holds after every \( L \) transition.

Safety Checks:

We must check that, after each \( L \) transition, the actions and summaries of states of \( L \) and \( \Omega_{(n)} \) match. Where \( \text{sum} \_L(S_L) \equiv \text{Cache} \_\text{State} \_L \), the following illustrates our safety checks.

Finally, we must check that after each \( \Omega_{(n)} \) transition, there exists an \( L \) transition that can fire. To do that, we express a safety property that says that if \( L\_\text{to-run}=\text{True} \), the conjunction of the guards of all \( L \) transitions must not evaluate to \( \text{False} \). With both safety checks passing, we can conclude that \( \Omega_{(n)} \subseteq L \), and, consequently, \( \Omega_{(n)} \leq L \) (Lemma 9).

This completes our proof that NeoGerman is a Neo hierarchy and that CC satisfies coherence for any arbitrary configuration. The full NeoGerman Cubicle model and proofs can be viewed at: [http://people.duke.edu/~om26/papers/FMCAD16](http://people.duke.edu/~om26/papers/FMCAD16).

V. CHARACTERIZING THE SCOPE OF OUR FRAMEWORK

To characterize the scope of our framework, we define a fragment of first order formulas over leaf states that we can verify using our approach and define summary functions that are guaranteed to verify a given property. Let \( LP = \{x_1, \ldots, x_m\} \) be a set of predicates over the leaf states \( \text{states}(L) \). We show how we can verify any invariant of the form

\[ \forall x_1, \ldots, x_k. \text{Distinct}(x_1, \ldots, x_k) \Rightarrow P(x_1, \ldots, x_k) \quad (10) \]

where the \( x_i \)'s range over leaves, \( \text{Distinct}(x_1, \ldots, x_k) \) indicates that the \( x_i \)'s are pairwise not equal, and \( P(x_1, \ldots, x_k) \) is a propositional formula over the atoms \( \{f_j(x_i)\}_{1 \leq j \leq m \land 1 \leq i \leq k} \). For example, where \( LP = \{E, S, f\} \), the cache coherence invariant we verified for NeoGerman could be expressed as \( \forall x_1, x_2. \text{Distinct}(x_1, x_2) \Rightarrow (E(x_1) \Rightarrow I(x_2)) \).

To verify that \( (10) \) is invariant, we construct summary functions such that the state of a NEO hierarchy summarizes to \( \text{bad} \) if and only if \( (10) \) is false of the system's state. The summary functions have co-domain \( \text{Sum} \), where \( \text{Sum} \) is the (finite) set

\[ \text{Sum} = \{2^{LP} \rightarrow \{0, \ldots, k\} \} \cup \{\text{bad}\} \]

When \( \text{bad} \) is returned to node \( A \)'s summary function, this indicates that \( (10) \) fails to hold of the sub-hierarchy rooted at \( A \). Otherwise, a function \( f \) is returned, with the interpretation that \( f(LP') \) is the number of distinct leaves under \( A \) with states satisfying exactly the predicates \( LP' \subseteq LP \); if there are \( k \) or more such leaves, \( f(LP') = k \).

The leaf summary function \( \text{sum} \_L \) simply returns the function that maps all sets to \( 0 \), except the exact subset of \( LP \) that holds of the leaf's state, which is mapped to \( 1 \). However, if \( k = 1 \) and \( P \) does not hold of the leaf's state, \( \text{bad} \) is returned. Where \( A \) is an n-child internal or root node, it is relatively straightforward to define how \( \text{sum} \_A \) (which is independent of its first argument \( s_A \in \text{states}(A) \)) depends on arguments \( (g_0, \ldots, g_{n-1}) \in \text{Sum}^n \) and under what conditions it should return \( \text{bad} \). \( \text{sum} \_A \) returns the function that maps each \( LP' \) to \( g_0(LP') + \cdots + g_{n-1}(LP') \), saturating at \( k \), unless any \( g_i \) is \( \text{bad} \) or the counts of \( (g_0, \ldots, g_{n-1}) \) for each \( LP' \) indicate that some \( x_i \)'s below \( A \) violate \( P(x_1, \ldots, x_k) \), in which case \( \text{bad} \) is returned.

VI. CONCLUSION

We present the Neo framework that leverages network invariants and parameterized model checking together to enable the design and automated verification of hierarchical (tree) protocols that, for any size or configuration of the hierarchy, satisfy a safety property. We use our framework to design and verify a hierarchical cache coherence protocol called NeoGerman, using Cubicle as our parametric model checker. Significantly, we prove an observational pre-order in a parametric setting. We believe there are no fundamental limitations that prevent our framework from being used to design and verify more complex, industrial-strength hierarchical protocols, especially given that model checkers like Cubicle have already been used to parametrically verify several industrial-strength flat protocols.

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