Image Deblurring with Blurred/Noisy Image Pairs

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April 26, 2013

1 Abstract

Photos taken under dim lighting conditions by a handheld camera are usually either too noisy or too blurry. In our project, we implement an image deblurring method that takes advantage of a pair of blurred/noisy images. We first utilize the image pair to estimate the blur kernel using a regularized linear least-squares method. With this kernel, we apply Richardson-Lucy deconvolution to the residual image (blurred image minus denoised image) to reduce ringing artifacts of the standard RL algorithm. To further reduce the ringing artifacts, we use a gain-controlled RL deconvolution method. This method first calculates the gain map of the image from Gaussian pyramids and then uses the map to suppress the contrast in smooth area, which efficiently reduces ringing but also causes loss of the detail. At last a detail layer is extracted from residual RL result through an adaptive high pass filter and added to the gain-controlled RL result to make up the detail loss. We also examine another deblurring method that only takes a single blurred image as input. We first use Fergus’ algorithm to do the blind kernel estimation. Then we implement the joint bilateral RL method to obtain the deblurred image as well as to add more and more details progressively. The results are comparable to the first method, and both methods improve the deblurring performance in the sense that the ringing artifacts are significantly suppressed compared with the standard RL algorithm.

2 Introduction

Taking satisfactory photos under dim lighting conditions using a handheld camera can be challenging. Generally, without flash or a tripod, the photos can be either blurred or noisy. When in a dark environment, to guarantee enough amount of light entering the camera, we can slow down the shutter speed. However, if the shutter speed is lower than the safe shutter speed, which is the reciprocal of the focal length, camera shake will cause blur in the image. We can also set a higher ISO value, which will essentially increase the camera’s gain. This can be also problematic since the high camera gain amplifies the noise as well. Note that although the image is noisy, it still can be sharp due to a fast shutter speed.

For single image deblurring, in [1] natural image statistics together with a sophisticated variational Bayes inference algorithm are used to estimate the kernel. The image is then reconstructed using a
standard non-blind deconvolution algorithm. Very nice results are obtained when the kernel is small\cite{1}. However, for large blur, the result can be inaccurate and unreliable using a single image.

Deblurring can also benefit from multiple images. Images with different blurring directions\cite{2,3,4} can be used for kernel estimation. The work most related to ours is\cite{5} which also makes use of a short exposure image to help estimate the kernel and deconvolution. The kernel is estimated in the linear least-squares sense using two images. Their works has also suggested an application for defocus using large/small aperture images. However, their work does not show any results or analysis.

Our project are mostly based on the work of\cite{6}. Our iterative image deblurring algorithm consists of the following steps: estimate the kernel $K$ by Lim and Silverstein’s method, compute the residual deconvolution image by standard Richardson-Lucy (RL) algorithm, compute the gain-controlled deconvolution image by gain-controlled RL algorithm, and construct the final image by adding the detail layer, which is extracted from the standard RL deconvolution image by joint/cross bilateral filtering. We made a few changes to Yuan et al.’s work. The main difference is that we did not take any real photos. All blurred images and noisy images are produced by MATLAB from a true image and manually made kernel. We utilized a different method to estimate the kernel. The times of iterations of Richardson-Lucy in our work is much larger than their work. Parameters we used are also different from theirs.

3 Hypothesis

The central hypothesis of this project is that using a blurred/noisy image pair instead of a single blurred image will improve the deblurring result. The information contained in the noisy image will help us to get a better kernel estimation and a better deblurring result.

If we know both the true image and the blurred image, the kernel can be calculated easily and precisely. Thus we suppose that taking use of the denoised result of the noisy image, which is similar to the true image, will help us to make better kernel estimation. The denoised result is a good initial condition for kernel estimation. For deconvolution, using the blurred denoised image as background and deconvoluting the residual image can largely reduce the ringing effect of Richardson-Lucy deconvolution, since the amplitude of ringing is proportional to the amplitude of signal. Thus the mean square error (MSE) of residual deconvolution should be less than the MSE of standard deconvolution. The gain-controlled method, which suppress ringing effects in smooth regions of the image, should give a better-looking result for human eyes. But the MSE might not be less than the MSE of residual deconvolution method. The detail lost in gain-controlled method can be recovered through extracting detail from standard RL algorithm.

4 Theory and Methods

Assume we take a pair of images: a blurred image $B$ with a slow shutter speed and low ISO, and a noisy image $N$ with high shutter speed and high ISO. We then pre-multiply the noisy image by a ratio $\frac{ISO_B}{ISO_N}$ to compensate for the exposure difference between the blurred and noisy images, where
t is the exposure time. Our goal is to reconstruct a high quality image $I$ using the input images $B$ and $N$:

$$B = I ** K,$$ (1)

where $K$ is the blur kernel. For the noisy image $N$, we compute a denoised image $N_D$. Now, we can represent the high quality image as

$$I = N_D + \Delta I,$$ (2)

where $I$ is a residual image.

### 4.1 Kernel Estimation

The goal of kernel estimation is to find the blur kernel $K$ from $B = I ** K$, with the initialization $I = N_D$. According to [5], we can write the convolution in vector-matrix form $b = Ak$, where $A$ is the matrix representation of image $I$, hence $k$ can be solved by linear least-squares. However, the estimated kernel by this simple approach may be poor. To obtain a better estimation, we add a regularization term to $k$, solving:

$$\min_k \|Ak - b\|^2 + \lambda^2 \|k\|^2.$$ (3)

This has a closed-form solution

$$(A^T A + \lambda^2 I)k = A^T b.$$ (4)

Since the real kernel is unknown to us, we firstly assume the kernel size is 3, and calculate the regular linear square. Then add the kernel size by 2 and calculate the regular linear square again, until the regular linear square stop decreasing, meaning that we have found the right kernel which have the minimum regular linear square.

According to our experiment, this kernel estimation method can also work well when the kernel is large ($64 \times 64$).

### 4.2 Residual Richardson-Lucy Deconvolution

Given the kernel $K$, the true image can be reconstructed through Richardson-Lucy (RL) deconvolution. One problem of standard RL is that it produces the ringing artifacts, which result from discontinuity of the image. More iterations introduce more details but causes more obvious ringing at the same time. To reduce ringing, we implement residual RL algorithm. The true image $I$ can be written as $I = N_D + \Delta I$, so

$$I ** K = B - N_D ** K.$$ (5)

Define $\Delta B = B - N_D ** K$. We do deconvolution to $\Delta B$, which has a much smaller amplitude than $B$. The amplitude of ringing will also has a much smaller amplitude. The final reconstructed image of residual RL is $I = N_D + \Delta I$. We still use RL deconvolution to $\Delta B$. This algorithm enforces the non-negativity of pixel values. As all images are normalized to range $[0, 1]$, the range of $\Delta I$ and $\Delta B$ is $[-1, 1]$. So we add 1 to both $\Delta I$ and $\Delta B$ to make sure that values of all pixels are positive. The RL iteration equation is written as:

$$\Delta I_{n+1} = K ** \frac{\Delta B + 1}{(\Delta I_n + 1) ** K} (\Delta I_n + 1) - 1,$$ (6)

where ‘**’ is the correlation operator.
4.3 De-ringing with Gain-controlled RL

Residual RL results still have noticeable ringing effects, which are relatively obvious in smooth regions. On the other hand, ringing in highly textured regions are hard to notice. To further reduce ringing effects in the smooth regions, we implemented gain-controlled RL algorithm. We introduced gain map $I_g$ to residual RL:

$$\Delta I_{n+1} = I_g \cdot \frac{\Delta B + 1}{(\Delta I_n + 1) * * K} (\Delta I_n + 1 - 1), \quad (7)$$

The gain map is a map less than (or equal to) 1 at each pixel. Multiplying a factor less than 1 at each iteration will suppress the propagation of the ringing effects. This factor does not decrease the overall magnitude of the signal. This is because although $\Delta I_{n+1}$ decreases in one iteration, it becomes denominator in the next iteration and thus increases $\Delta I_{n+2}$. At the last iteration, we do not multiply gain map to the result.

$I_g$ smaller than 1 not only suppresses ringing effects, but also lead to loss in details. Since we want to suppress the ringing in the smooth regions while avoiding suppression of edges and details, the gain map should be small in smooth regions and large in others. We call regions where gain map has a value of 1 ‘safe regions’, which means that in these regions, gain-controlled RL has the same result as residual RL and no detail is lost. Basically, the gain map is based on the gradient of the denoised image ND. To make safe regions cover edges and textured regions thoroughly, we first filter ND with a Gaussian filter of $\sigma = 2$ and then take gradient. This procedure is repeated for 5 times for an image of $512 \times 512$. $I_g$ can be written as:

$$I_g = \min(1 - \alpha + \alpha \Sigma |N_D|, 1). \quad (8)$$

RGB images have three channels. Each channel has a different gain map. We use the maximum value of all three channels at each pixel. This is to guarantee that if some regions of the image only have detail in one color, it is still well protected by safe regions so that we do not lose detail in this color,

$$I_{g,RGB} = \max(I_{g,Red}, I_{g,Green}, I_{g,Blue}). \quad (9)$$

4.4 Extract Detail Layer

After de-ringing with gain-control RL, the details of the image are also suppressed. In order to recover the fine details in the original image, a low pass joint/cross bilateral filter is used to preserve large scale edges in $I_g$, then the result image is subtracted from the residual RL image and therefore extract the scale detail layer. This method takes advantage of the result image of Residual RL, which contains the fine details but suffers from serious ringing, and the gain control RL result image, which is spared from ringing whereas loses details.

The joint/cross bilateral filtering is designed to average adjacent pixels that have similar values while stop at the position of edges. This is implemented through attenuating the filter kernel weighting coefficients where the difference between pixels is large. The equation that is used to solve for the filtered image is as follows:

$$F(I(x); I_g) = \frac{1}{Z_x} \sum_{x' \in W(x)} G_d(x - x')G_r(I(x) - I_g(x')) \times I_{x'} \quad (10)$$

where $Z_x = \sum_{x' \in W(x)} G_d(x - x')G_r(I(x) - I_g(x'))$ is a normalization term.

The Gaussian kernel $G_d$ sets the weights in spatial domain in respect to the distance between pixels,
whereas kernel $G_r$ serves as an edge-stop function that sets weights based on intensity differences between pixels in the residual RL image and corresponding pixels in the gain control RL image. When the distance between them is large, it is very possible that there exist details that only shown in residual RL image but not in gain control RL image. At those locations, $G_r$ is attenuated and thus when the result is subtract from the base residual RL image, details are extracted. Because most ringing has low spatial frequency, it is ignored by the distance control kernel $G_d$ and preserved in the filtered image and therefore removed from the detail layer. $\sigma_d$ and $\sigma_r$ are the standard deviations that control the widths of the two Gaussian kernels.

This method is applied to each RGB color channel respectively with the same standard deviation parameters.

5 Results

5.1 Kernel estimation

5.1.1 Methods to evaluate the estimated kernel

To evaluate the accuracy of our estimated kernel, we make some blurry/noisy image pairs with some known kernels. The blurry images are acquired by convolving the true images with the known kernels, and the noisy images are acquired by adding various noise to the true images. Then we utilize the blurry and noisy image as input, and get estimated kernel as output, if the estimated kernel are mostly same in shape with the known kernel, we consider the kernel estimation system works well. The whole process are shown below:

To realize this, we utilize an image with $512 \times 512$ pixel size and a $31 \times 31$ kernel, blur the original image with the kernel to get the blurry image and add a 5% gaussian noise to the original image to get the noisy image. The kernel estimated by these two images is mostly the same with the original
kernel, with a slightly smaller pixel size, cutting the blank edges which won’t influence the kernel shape.

![kernel, with a slightly smaller pixel size, cutting the blank edges which won’t influence the kernel shape.](image)

Figure 2: (a) Blurry image. (b) Noisy image corrupted with standard deviation 0.05 Gaussian noise. (c) Real kernel of size $31 \times 31$. (d) Estimated kernel of size $29 \times 29$.

### 5.1.2 Estimated kernel and kernel size

As illustrated above, we estimate the kernel by firstly assuming a small kernel size, and then solve (4), and calculate the error in (3) (i.e., the regular linear square) and add the kernel size by 2 and calculate the regular linear square again, until the regular linear square stop decreasing, meaning that we have found the right kernel. Below are the same kernel estimations in the whole estimation process. We can see that the kernel shape is recovered from center to surroundings with increasing kernel size.

![kernel, with a slightly smaller pixel size, cutting the blank edges which won’t influence the kernel shape.](image)

Figure 3: (a) Real kernel with pixel size $31 \times 31$. (b)-(f) estimated kernel with pixel size: $15 \times 15$, $19 \times 19$, $23 \times 23$, $27 \times 27$, $31 \times 31$.

### 5.1.3 Residual Richardson-Lucy Deconvolution

To compare with residual RL, we also did standard RL to each pair of images. The results are shown in Figure [4] and Figure [5]. For standard RL and residual RL, the iteration times are 15000. Compared with standard RL, residual RL deconvolution reduces ringing effects. But ringing effects are still noticeable, as shown in the error maps. Because we know the true image, the MSE of each result can be calculated. Residual RL has a less MSE than standard RL, which is in accordance to our hypothesis.

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Interestingly, our results are much different from results in the original paper. The original paper only did 20 iteration, which is far from enough in our experience. We find that for both standard and residual RL, more than 10000 iterations are necessary to make detail clear enough. Moreover, the ringing strips in the original paper are generally very wide while ringing strips in our results are very thin. This causes some problem in the following gain-controlled method, which we will explain in the next section.

5.1.4 De-ringring with Gain-controlled Richardson-Lucy

As shown in Figure 4 and Figure 5, gain map covers regions with detail and edge very well. The times of iteration is 10000 for gain-controlled RL. We used different $\alpha$ for different images because ringing effects are strong in some images and weak in others. For images with strong ringing effects, we need a larger $\alpha$. Generally, the range of $\alpha$ is $[0.0001, 0.0005]$. The value of $\alpha$ is small because the times of iteration is very large and a small $\alpha$ is enough to control the ringing effects in smooth regions of the image. A larger $\alpha$ will lead to artifacts, which results from gain-controlled RL algorithm itself. As we can see, gain-controlled RL has weaker ringing effects and less detail than residual RL.

The final image is the sum of gain-controlled RL and the detail layer, where detail layer is extracted from standard RL instead of residual RL. We do this because ringing stripes of residual RL are always extremely thin, which is a high-frequency signal. They can be extracted together with detail, which decreases the quality of the final result. The MSE of gain-controlled RL are also calculated, which is larger than MSE of standard and residual RL results.

Compared with the original paper, our gain-controlled RL does not work that well. This is because ringing strips in our results are too thin to be considered as low-frequency signal. They will be extracted together with real detail. Fairly speaking, although ringing effects are less obvious in the final image, its performance is not as good as residual RL. This conclusion is also obviously indicated by the high MSE of final image.
Figure 4: (a) is the true image. (b) and (c) are the blurred/noisy image pair. (d) is the denoised image of (c). (e)-(g) show the results of three different methods and (i)-(k) are their corresponding error map. (h) is the sum of (g) and (n). (l) is its corresponding error map. Notice that all four error maps are multiplied by 10 to make them clear. (m) is the gain map. Its contrast has been enhanced. (n) is the detail layer multiplied by 10. (o) is the kernel.
Figure 5: (a) is the true image. (b) and (c) are the blurred/noisy image pair. (d) is the denoised image of (c). (e)-(g) show the results of three different methods and (i)-(k) are their corresponding error map. (h) is the sum of (g) and (n). (l) is its corresponding error map. Notice that all four error maps are multiplied by 10 to make them clear. (m) is the gain map. Its contrast has been enhanced. (n) is the detail layer multiplied by 10. (o) is the kernel.
5.1.5 Extract Detail layer

In a reasonable range, decreasing $\sigma_d$ and increasing $\sigma_r$ generate results with less ringing but also less details. There is a trade-off between detail preserving and ringing elimination on the selection of those parameters. $\sigma_d$ determines the width of extracted details while $\sigma_r$ decides the amount of details that is extracted. For a certain $\sigma_r$, increasing $\sigma_d$ generates more details with larger width while decreasing $d$ generates thinner and less details. For a certain $\sigma_d$, increasing $\sigma_r$ preserves more details, however, when $\sigma_r$ is small ($<0.04$), large ringing is also preserved in the detail layer because even small differences between corresponding pixels in residual RL image and gain control RL image are also attenuated by $G_r$ and extract by the subtraction operation. For most image, $\sigma_d = 0.08$ and $\sigma_r = 1.0$ works well according to our experiments.

![Residual](image1)
![Gain](image2)
![Detail](image3)
![Final](image4)

Figure 6: Using (a) residual RL image and (b) gain-controlled RL image, we get (c) detail layer, (d) final image with half window size as 5, standard deviation $\sigma_d = 0.08$ and $\sigma_r = 1.0$.

5.1.6 Extension

We also extend our work to blind deconvolution. Fergus et al. \cite{1} showed that a very accurate kernel can be estimated for blur due to camera shake by using natural image statistics together
with a sophisticated variational Bayes inference algorithm. They then recovered the image using
the standard Richardson-Lucy (RL) algorithm \[3\]. However, as we saw previously, this method will
cause noticeable ringing artifacts in the resulting images. Yuan et al. showed that these ringing
artifacts can be significantly reduced using a method called Joint Bilateral Richardson-Lucy (JBRL)
\[9\].

Bilateral Richardson-Lucy (BRL)
The RL algorithm does not preserve the edges in the design of regularization. We add a new
edge-preserving regularization term to the RL iteration:

\[
I_{t+1} = \frac{I_t}{1 + \lambda \nabla E_B(I_t)} (K * B) \frac{B}{I_t * K}.
\] (11)

The derivative \( \nabla E_B(I) \) can be computed by:

\[
\nabla E_B(I) = \sum_{y \in \Omega} (I_y - I_{y'} D_y),
\] (12)

where \( \Omega \) is a spatial support centered at each pixel, \( D_y \) is the displacement matrix (operator) which
shifts the entire image \( I_{y'} \) by \( |\vec{e}| \) pixels in the direction of \( \vec{e} \). Here, \( \vec{e} \) denotes the displacement
vector from each pixel \( x \) to its neighbor pixel \( y \). \( I_{y'} \) is a weighted long-range gradient image in the
direction of \( \vec{e} \).

For each pixel \( x \) in the image \( I_{y'} \),

\[
I_{y'}(x) = f(|x - y|) g(|I(x) - I(y)|) \frac{I(x) - I(y)}{\sigma^2_f},
\] (13)

where \( f(\cdot) \) and \( g(\cdot) \) are Gaussian kernels with variances \( \sigma^2_f = (r_\Omega / 3)^2 \) and \( \sigma^2_r = 0.01 \times |max(I) - min(I)|^2 \), respectively. We adaptively set the radius of spatial support \( r_\sigma \) to match the size of the
blur kernel: \( r_\Omega = 0.5 r_K \). Since the weight for the difference \( I(x) - I(y) \) is computed by a bilaterally
weighted filter \( f(\cdot)g(\cdot) \) in image and range, also called bilateral filter \[10, 11\].

Because a bilateral regularization term is applied in the iterative formula of the standard RL, the
above method is called bilateral Richardson-Lucy (BRL) \[9\]. Figure 7 shows that when the kernel
size is relatively small, BRL preserves image edges well, suppresses ringing artifacts, and recovers
more image details.

![Figure 7: From left to right, original image, blur image, deconvolved image using standard RL and decon-
volved image using BRL. The ringing artifacts are reduced. The kernel size is 11 × 11.](image)

However, blurred images with large blur kernels are beyond the capability of the BRL algorithm.
When the kernel size is large, details are also suppressed. In the next section, we present a progressive
deconvolution approach that is capable of effectively handling large blur kernels.
Joint Bilateral Richardson-Lucy (JBRL)

The basic idea of JBRL is that in the scale space, we use the image recovered in one scale as the guide image for the deconvolution in the next scale. Meanwhile, we modify the regularization term in BRL so that it takes both the image and the guide image into account:

\[
I_g^d(x) = f(|x - y|)g(|I(x) - I(y)|)g'(|I_g(x) - I_g(y)|) \frac{I(x) - I(y)}{\sigma_r^2},
\]

(14)

where \(I_g\) is the guide image, and \(g'(\cdot)\) is also a Gaussian kernel with variance \(\sigma_g^2 = 0.01 \times |\text{max}(I_g) - \text{min}(I_g)|^2\). As in Figure 8, much more details are discovered by JBRL, while ringing artifacts are significantly suppressed compared with standard RL. Figure 9 also illustrates a comparison in the Fourier domain.

Figure 8: Upper left: blurred image. Upper right: standard RL, ringing artifacts are noticeable. Lower left: BRL, details are also suppressed. Lower right: JBRL, much more details are recovered, while ringing artifacts are significantly reduced. Kernel size: 23 × 23.
6 Conclusion

In this project, we implemented a deblurring algorithm proposed by Yuan et al. This method takes advantage of a pair of blurred/noisy images and consists of four steps, each using the result of its previous step. The step of kernel estimation uses both blurred and noisy images to estimate the kernel in a linear least-square manner. The results match our hypothesis very well: estimated kernel is almost the same as real kernel. In the second step of residual RL deconvolution, our results reduces ringing effects and MSE. Actually, our residual RL already gives very good results, since the ringing stripes are very thin and weak. In the third step of gain-controlled RL deconvolution, the ringing effects are further reduced. Finally, we performed a detail extraction step and then combined the detail layer with the gain control RL image. The detail layer extracted most details and successfully suppressed ringing artifacts. Generally, although median results were slightly different from results of the original paper, our overall results are satisfactory and match hypothesis. We implemented the algorithm presented in the paper and achieved most objectives. We also did some extensive works and implemented another method proposed by Yuan et al. to compare with algorithms mentioned above.
References


