# CANCELLATION OF JOSEPHSON CURRENTS IN SUPERCONDUCTING JUNCTION ARRAYS

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## Abstract

We propose a method to investigate the possibility of a total suppression of the Josephson currents in junction arrays. Using a tunable external magnetic field and correctly interpreting the Josephson peaks in the I-V curve, it is possible to separate the individual behavior of the junctions and to study the magnetic dependence of their critical Josephson current. This way we can investigate some intrinsic properties of the junctions like the difference between their barrier thicknesses and between their surfaces and the polluting effect of the magnetic environment.

## 1 Introduction

Suppression of Josephson current is one of the main problems in SIS heterodyne receivers working at frequencies greater than 200 GHz. It is responsible for excess of noise and bias instabilities [1]. For these reasons most of the radiometers use a tunable external magnetic field generally produced by a couple of superconducting coils.

The aim of this paper is to suggest a method that uses this facility to study the evolution of the critical Josephson currents in SIS junction arrays as a function of the field produced by the coils [2].

Separating the individual behavior of each junction it is possible to investigate their intrinsic properties and the influence of the magnetic environment.

In section 2 we describe the DC I-V curve of an SIS array emphasizing the interpretation of Josephson peaks; in section 3 we recall the theoretical behavior of Josephson current under a variable magnetic field and we extend it to the case of an SIS array; in section 4 we apply our method to study an array of four Nb/Al-AlOx/Nb junctions.

## 2 I-V curve

Two different kinds of charge carriers are present in a superconductor: Cooper pairs and quasiparticles. As a consequence two different current regimes can be distinguished in the I-V curve of an SIS junction: the Josephson current branch at V=0 and the quasiparticle current branch for $V \geq 2\Delta/e$, where $\Delta$ is the energy gap of the superconductor and e is the electron charge.

When varying the bias current from zero to above the gap current, the junction describes an hysteretic cycle (see figure 1a). It starts in the Josephson branch and switches to the quasiparticle regime for a transition value given by

$$I_{jos}=I_{max} \sin (\varphi_0) \qquad (1)$$

where $I_{max}$ is the critical Josephson current and $\varphi_0$ is the difference of phase between the two superconductors.

As a statistical rule (see figure 2) for small variation of the bias current the junction in the pair regime prefers to accord its phase difference rather than changing its physical state. As a consequence the transition happens almost every time for a bias value equal to the critical Josephson current that is for $\varphi_0=\pi/2$.

In an SIS array (see figure 1b) the junctions have different values of the critical Josephson current. Then the transition between the

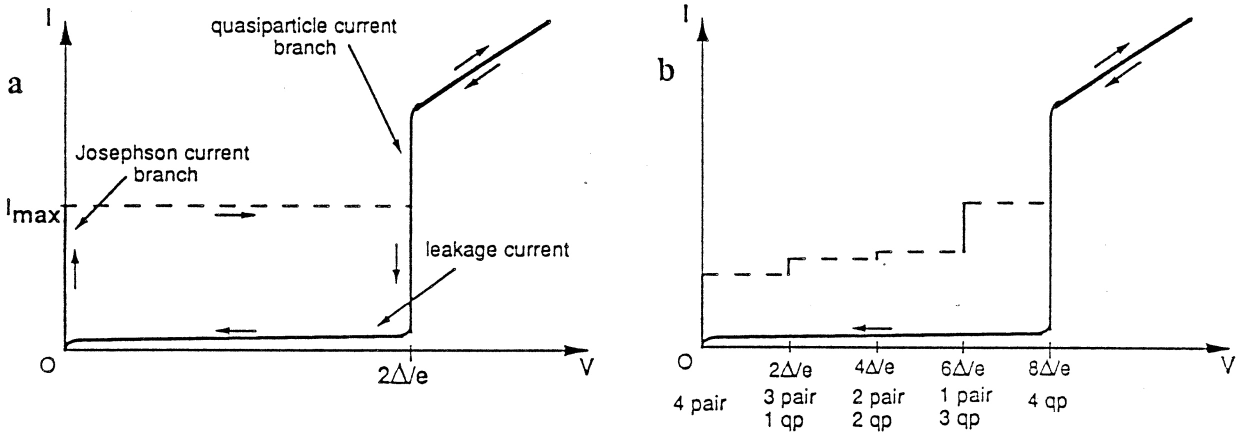

Figure 1: (a) Typical DC I-V curve for a current biased SIS single junction showing the hysteretic behavior between the two current regimes. (b) Typical DC I-V curve for a four junction array. Each peak is associated to a different physical state of the array where some junctions are in the pair regime and the others are in the quasiparticle one.

pair regime and the quasiparticle regime occurs in the different junctions for different values of the bias current. There are $(n+1)$ regimes for the array and $n$ peaks in the I-V curve before the gap at $V=2n\Delta/e$ where all the junctions are in their quasiparticle regime.

The $i^{th}$ peak is associated to a physical state with $(i-1)$ junctions in the quasiparticle regime and $(n-i+1)$ junctions in the pair regime; its height corresponds to the critical Josephson current of the $i^{th}$ junction where the junctions are sorted for growing values of their $I_{max}$.

The absence of some intermediate peak can be interpreted as the simultaneous transition to the quasiparticle regime of more than one junction, due to the almost equal value of their critical currents.

## 3 Josephson current and magnetic environment

The theoretical magnetic field dependence of the critical Josephson current is well known [3]. Assuming that the magnetic field produced by the coils is in the x direction we have

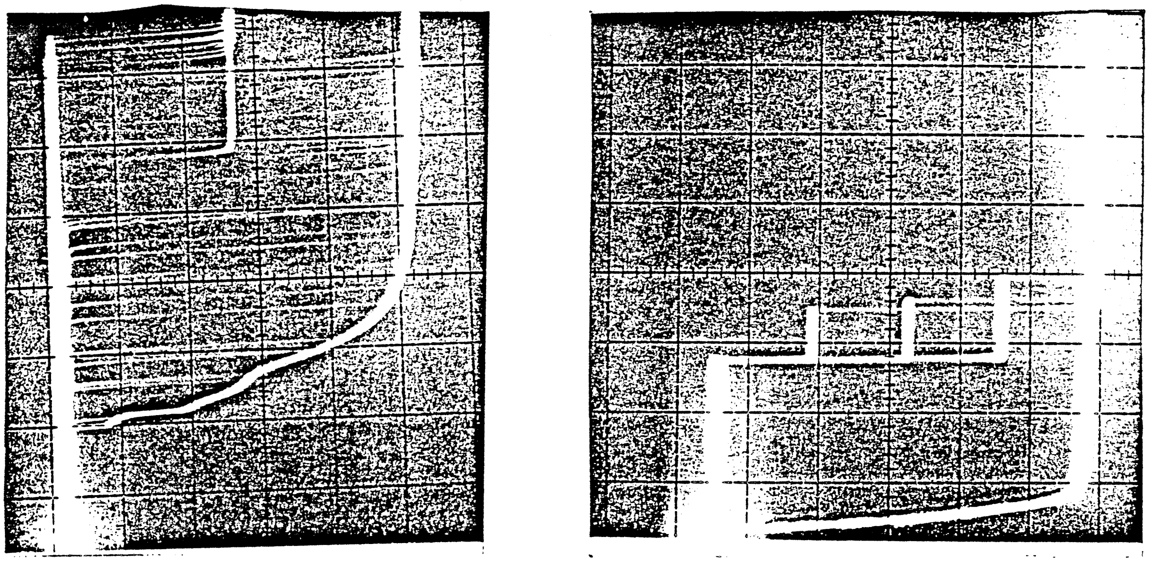

Figure 2: Transitions between different regimes of an array. (Left) I-V curve of a two junction array: even if statistically unlikely transitions between the two regimes of a junction can happen also for a bias current smaller then the critical Josephson current, as shown by the presence of numerous horizontal lines. (Right) I-V curve for a four junction array: the transition of a junction to its quasiparticle regime can induce a premature switch of the other junctions.

$$I_{max} = I_0 \left| \frac{\sin\left(\pi \frac{\phi_x}{\phi_0}\right)}{\pi \frac{\phi_x}{\phi_0}} \right| \left| \frac{\sin\left(\pi \frac{\phi_y}{\phi_0}\right)}{\pi \frac{\phi_y}{\phi_0}} \right| \qquad (2)$$

for rectangular junctions, and

$$I_{max} = I_0 \left| \frac{J_1\left(\pi \frac{\sqrt{\phi_x^2+\phi_y^2}}{\phi_0}\right)}{\frac{\pi}{2} \frac{\sqrt{\phi_x^2+\phi_y^2}}{\phi_0}} \right| \qquad (3)$$

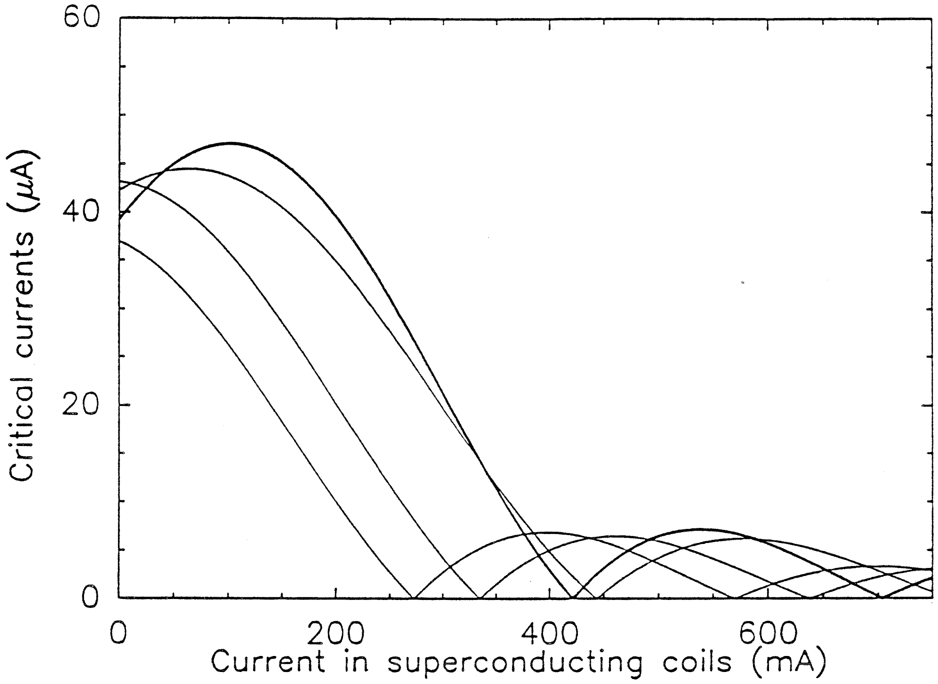

Figure 3: Simulation of the magnetic dependence of the critical Josephson currents for a four junction array. We allowed for each junction a variation of 10% for the $I_0$ and $\phi_0$ values as well as trapped fluxes between 0 and $\phi_0/2$ in the x and y direction.

for circular junctions. In these formulas, $\phi_x = \phi_{coil} + \phi_{ext,x}$ is the flux generated by the superconducting coils plus the external flux trapped along the coils axis, $\phi_y = \phi_{ext,y}$ is the flux trapped along the perpendicular direction in the junction plane, $\phi_0$=hc/2e is the flux quantum (2.07 $10^{-7}$ G $cm^2$), $I_0$ is the critical Josephson current in absence of magnetic fields, $J_1$ is the Bessel function of the first kind.

In figure 3 we plotted the theoretic behavior of a four circular junction array. In this simulation we took different values of $I_0$, $\phi_0$, $\phi_{ext,x}$ and $\phi_{ext,y}$ for the different junctions.

Following one junction, for example that one plotted in bold line and keeping in mind that the peaks in the I-V curve are sorted for growing value of the critical current, we can see that the junction is not associated to the same peak for all the values of the current producing the tunable magnetic field. For example it is associated to

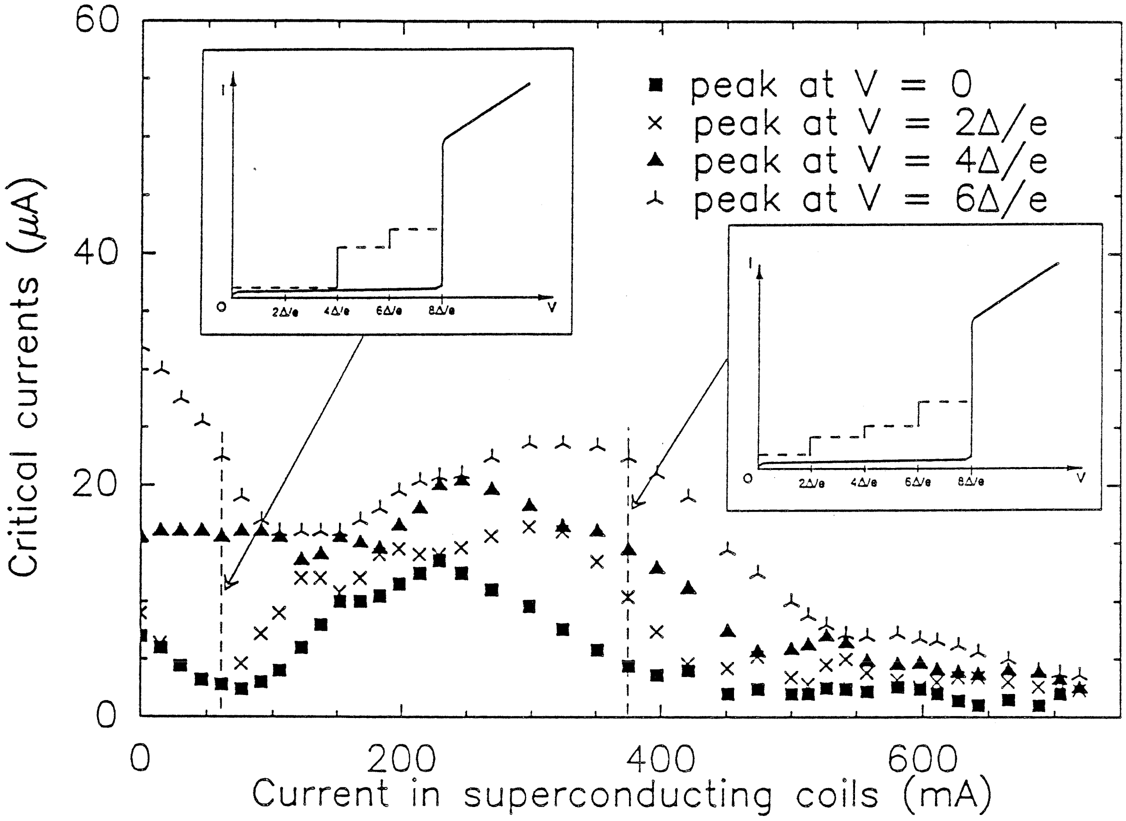

Figure 4: Critical Josephson currents vs tunable magnetic field for a four junction array in the case of a very polluting magnetic environment. The insets show the I-V curve associated to two different fixed values of the coils field.

the second peak for $I_{coil} \leq 20$ mA, to the fourth peak for 40 mA$\leq I_{coil} \leq$330 mA and to the first one for 390 mA$\leq I_{coil} \leq$430 mA.

As a general rule, when changing the value of the tunable magnetic field, the relative order of the critical currents changes too and it is not possible to associate a given peak always to the same junction.

Then to reconstruct the individual behavior of each junction in an external magnetic field we have to search for continuity of the theoretical function in the adjacent peaks.

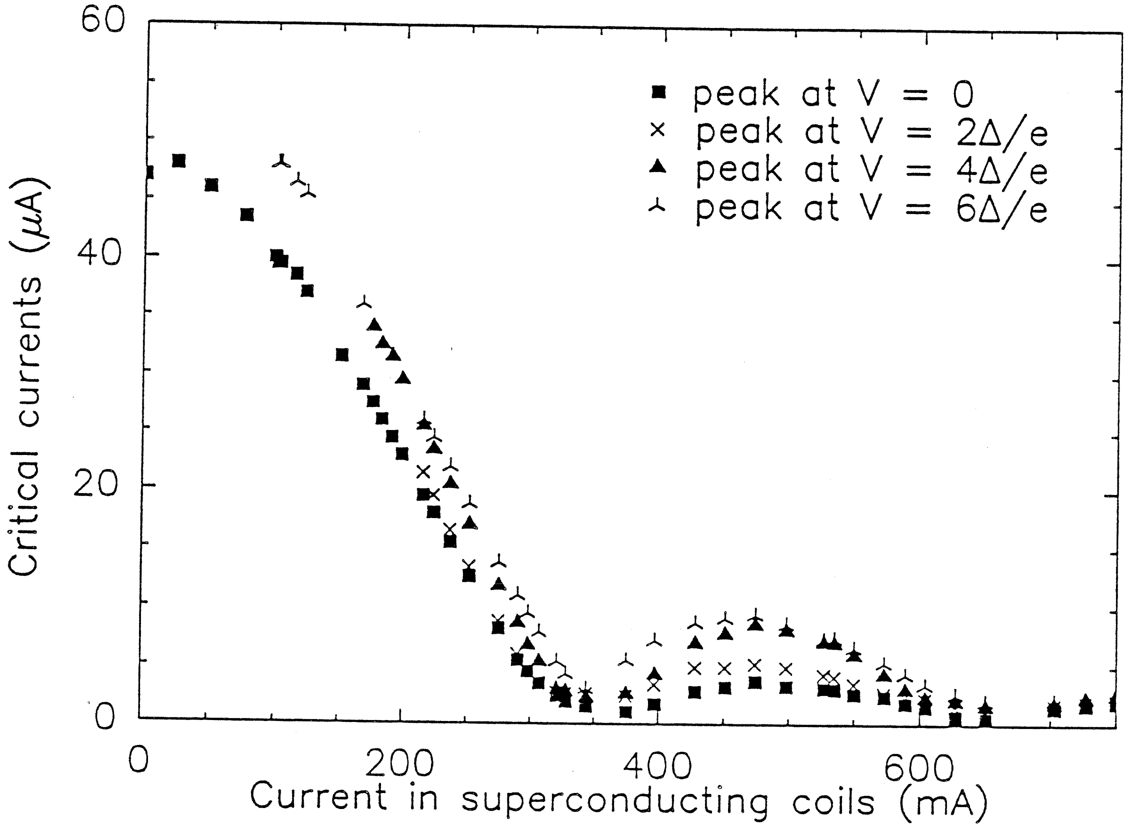

Figure 5: Critical Josephson currents vs tunable magnetic field for a four junction array in a clean magnetic environment.

## 4 Experimental results

In order to apply this method to a practical case we tested an array of four Nb/Al-AlOx/Nb junctions [4]. The junctions have circular shape with a diameter of 1.9 $\mu$m and a normal resistance for the array of 115 $\Omega$. The array was placed in a cryogenerator at 4.2 K and a couple of superconducting coils was used to produce the tunable magnetic field.

The method consists in plotting the height of the I-V curve peaks for different values of the magnetic field produced by the coils (see figure 4).

When we measure the dependence of the critical Josephson current from the current in the superconducting coils the formula (3) becomes

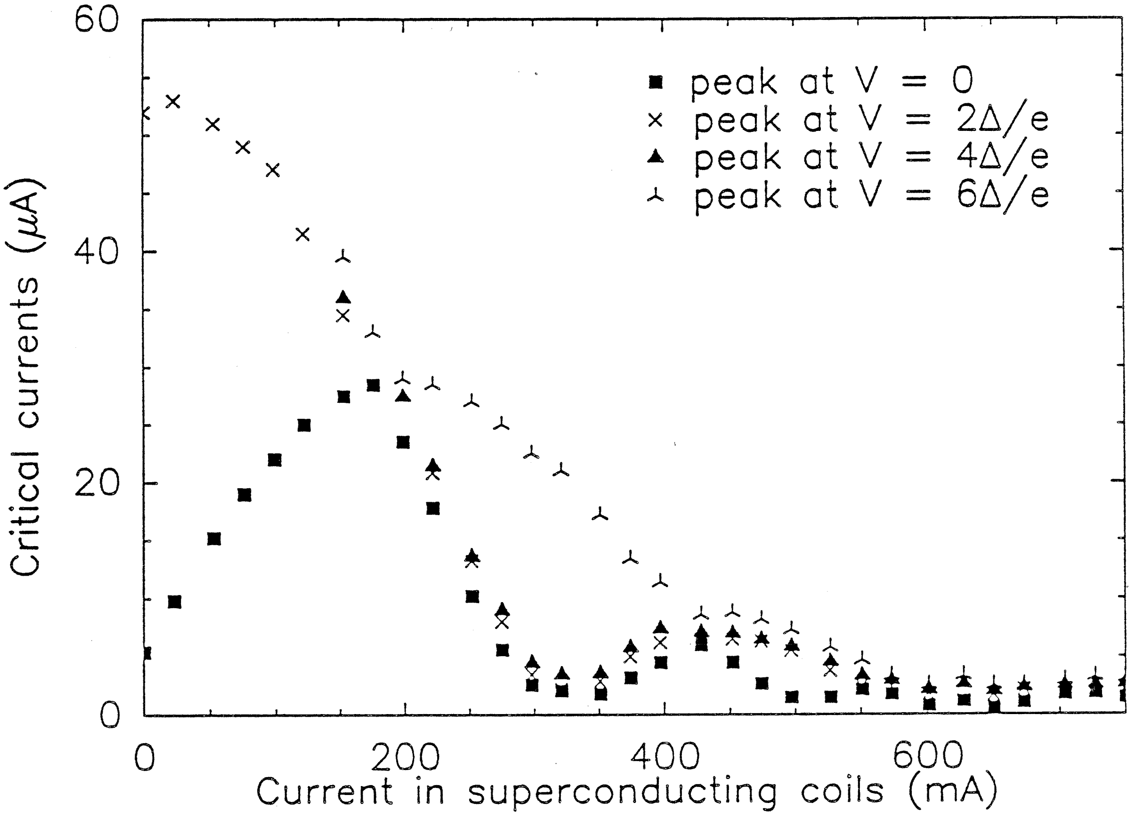

Figure 6: Critical Josephson currents vs tunable magnetic field for a four junction array. One of the junction has some trapped flux. Its maximum critical current is lower and its behavior is shifted along the x axis compared to the other junctions.

$$I_{max} = I_0 \left| \frac{J_1\left(k\,\pi\,\sqrt{\left(I_{ext,x}+I_{coil}\right)^2 + I^2_{ext,y}}\right)}{k\,\frac{\pi}{2}\,\sqrt{\left(I_{ext,x}+I_{coil}\right)^2 + I^2_{ext,y}}} \right| \qquad (4)$$

where k is the inverse of the coils current that produces a flux in the junction equal to $\phi_0$ while $I_{ext,x}$ and $I_{ext,y}$ are the coils current values that produce a flux equal to the trapped one.

In figure 4, 5 and 6 we represent some examples of the peaks evolution in different conditions of the magnetic environment.

| $\Delta k$ $\Delta I_0$ | -20 | -10 | -5 | 0 | 5 | 10 | 20 |
|---|---|---|---|---|---|---|---|
| 0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 1 | 0.4 | 0.4 | 0.4 | 0.4 | 0.4 | 0.4 | 0.4 |
| 2 | 0.7 | 0.8 | 0.8 | 0.8 | 0.8 | 0.8 | 0.9 |
| 3 | 1.0 | 1.1 | 1.2 | 1.2 | 1.2 | 1.2 | 1.3 |
| 5 | 1.7 | 1.8 | 1.9 | 1.9 | 2.0 | 2.0 | 2.1 |
| 7 | 2.4 | 2.5 | 2.6 | 2.6 | 2.7 | 2.8 | 2.9 |
| 10 | 3.3 | 3.5 | 3.6 | 3.6 | 3.7 | 3.8 | 4.0 |
| 15 | 4.7 | 5.0 | 5.1 | 5.2 | 5.3 | 5.4 | 5.6 |
| 20 | 5.9 | 6.3 | 6.4 | 6.6 | 6.7 | 6.9 | 7.1 |
| 25 | 7.0 | 7.5 | 7.6 | 7.8 | 8.0 | 8.1 | 8.4 |
| 30 | 8.0 | 8.5 | 8.7 | 8.9 | 9.1 | 9.2 | 9.5 |
| 40 | 9.7 | 10.3 | 10.5 | 10.7 | 10.9 | 11.1 | 11.4 |

Table 1: Residual Josephson current as a function of the k and $I_0$ mismatches between the junctions. The values are given in percentage of the critical Josephson current in a zero magnetic field.

## 4.1 Intrinsic properties

In figure 5 the array is in a clean magnetic environment. For $I_{coil} < 100$ mA only one peak is present due to the quite identical value of the critical currents. The junctions reach the first minimum for almost the same magnetic flux.

This situation is the most favourable for the receiver to work. It is possible to strongly suppress the Josephson currents in all the junctions of the array at the same time and with a small magnetic field.

In this situation we can investigate about some intrinsic properties of the junctions. Using a $\chi^2$ test with $I_0$ and k as free parameters to fit the theoretical behavior we found a maximum mismatch of 20% for $I_0$ and 8% for k. From the former we get qualitative information about the barrier thicknesses of the junctions while from the latter we directly measure the mismatch between their linear dimensions.

The k mismatch gives the intrinsic limit for a specific array concerning the Josephson current suppression (see table 1). This limit is important in the choice between single junctions or arrays as non linear elements for high-frequency operating SIS mixers [5]. Approaching the gap frequency (~700 GHz for Niobium) the receiver performances become more sensitive to the presence of a residual Josephson current [6] and single junctions should be preferred to junction arrays.

## 4.2 Magnetic environment

In figure 6 one of the junctions has some trapped flux. Performing a $\chi^2$ test with two more free parameters, $I_{ext,x}$ and $I_{ext,y}$, we estimated a trapped flux equal to $\phi_0$ due to a magnetic field at 45° with respect to the coils axis.

An example of important magnetic pollution is shown in figure 4. All the junctions have some trapped flux, consequently their maximum critical currents are decreased and shifted along the x axis.

In both situations, the minima are displaced as well, hence, it is not possible any more to find a value of the current in the coils for which the Josephson current is suppressed in all the junctions.

If the Josephson currents remain too high for acceptable values of the magnetic field there is no way to bias the junction properly because of the drop-back effect.

Using large magnetic fields can generally reduce the Josephson currents but degrades the quality of the I-V curve and therefore the mixing performances.

The only solutions are to warm up the junction in order to expel the flux and to shield the mixer block from external magnetic pollution.

## 5 Conclusion

Studying the evolution of the Josephson peaks of SIS junction arrays as a function of an external magnetic field it is possible to investigate the impediments to a total suppression of the Josephson current.

We can distinguish an intrinsic impossibility due to area discrepancies between the junctions and an external impossibility due to the sensitivity of the array to the magnetic environment.

The first decides the limit where the technology becomes not good enough to continue using junction arrays and suggests the choice of single junctions for SIS receiver working above 500 GHz.

The second one may be important for studying the efficiency of mixer shielding especially in view of space qualified receivers where heating the system might not be possible and generally human interventions are limited to remote control systems.

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