A Novel Type of Phase Grating for THz Beam Multiplexing

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Introduction

Phase gratings have become popular as local oscillator beam multiplexers for array receivers in the submillimeter wavelength domain. In general, binary or multilevel gratings, also known as Dammann gratings, are used [1][2][3][4]. Dammann gratings can be designed and manufactured easily and perform satisfactorily for most one–dimensional applications. Two–dimensional dispersion, however, is much more difficult to achieve. This is mainly due to the fact that a two–dimensional multilevel structure can not be machined easily.

We introduce a new type of phase grating, the Fourier grating, which replaces the sharp edges of the multilevel gratings by a smooth grating structure. These gratings can be machined easily with standard machine shop equipment. The grating design is relatively simple and the diffraction efficiency is usually significantly higher than the efficiency of Dammann gratings. Typical efficiencies for two–dimensional gratings are above 80%. For one–dimensional dispersion, efficiencies beyond 90% are typical.

In this paper we describe the design and the manufacturing of one– and two–dimensional Fourier gratings, and present beam measurements performed with reflection gratings at a frequency of 0.5 THz.

The Grating Concept

The Fourier grating concept is an extension of the well known sinusoidal phase grating. In the following we describe the design of symmetric one–dimensional gratings. The general case is an obvious extension of this.

The spatial phase modulation within the grating unit cell is modeled by a finite Fourier series:

$$\Delta\phi(x) = \sum_{n=1}^{N} a_n \cos(n \cdot \frac{2\pi x}{D}), \tag{1}$$

where the unit cell extends from $-D/2$ to $D/2$. Every member in this sum corresponds to a sinusoidal phase grating whose far field diffraction pattern is given by the Fourier transform of $\exp[a_n \cos(n \cdot 2\pi x/D)]$, the electric field in the grating plane [5]:

$$U_n(\theta) = U_0 \times \sum_{q=-\infty}^{\infty} J_q(a_n)\delta(\theta - nq\frac{\lambda}{D}). \tag{2}$$

J_q denotes the Bessel function of the first kind of order q.

Taking the complete Fourier series of eq. (1) corresponds to multiplying the fields of many sinusoidal gratings. This results in a diffraction pattern consisting of a multiple convolution of the diffraction fields of the individual Fourier components:

$$U(\theta) = U_0 \times \bigotimes_{n=1}^{N} \left[\sum_{q=-\infty}^{\infty} J_q(a_n)\delta(\theta - nq\frac{\lambda}{D}) \right]. \tag{3}$$

This expression is mathematically fairly elegant, but for computational purposes it is usually much more efficient, to calculate the Fourier transform of the aperture field using a numerical FFT algorithm.

Thus, the set of Fourier components a_n of the phase modulation defines a set of complex coefficients b_i, each of which describes the field in one diffraction order of the grating. Since we are only interested in the intensity distribution within the diffraction pattern, our task consists of finding a set of a_n, which produces the desired set of $b_i b_i^*$. For our standard example — a one–dimensional grating producing a symmetric pattern of four beams — this set is $b_i b_i^* = \{\ldots, 0, 0, \frac{1}{4}, 0, \frac{1}{4}, 0, \frac{1}{4}, 0, \frac{1}{4}, 0, 0, \ldots\}$.

We cannot calculate the set of a_n directly, and therefore have to resort to a numerical optimization to find the best values. This optimization is made difficult by the fact that the parameter space contains an extremely large number of local optima, and only for a very small fraction of all possible starting values the optimization converges to the global optimum.

We have developed two strategies to obtain good starting values. The first method is to randomly choose a large number of starting values. Although this is very crude, it proved to be rather efficient for gratings using a relatively small number of Fourier components (i.e. $N \lesssim 10$). This range of N includes most common one–dimensional gratings and a large number of simple two–dimensional gratings.

For larger numbers of Fourier components, we perform an inverse Fourier transform on the desired diffraction field. By varying the relative phases between the diffraction orders, we try to obtain a flat amplitude distribution in the grating plane. The resulting phase distribution is expanded into a Fourier series, which then serves as starting values for the optimization. This technique is faster but less complete than the first one. Although it may miss the global optimum, it usually still gives very good results.

For a large enough number of randomly chosen starting values the first method necessarily finds the global optimum for a given number of Fourier components. As long as the diffraction pattern is not too complex, we find that the optimum grating structure only requires a few non–zero Fourier components. Due to the completeness of the Fourier series expansion, this means that our approach converges rapidly to *the best possible phase grating* for the desired diffraction pattern.

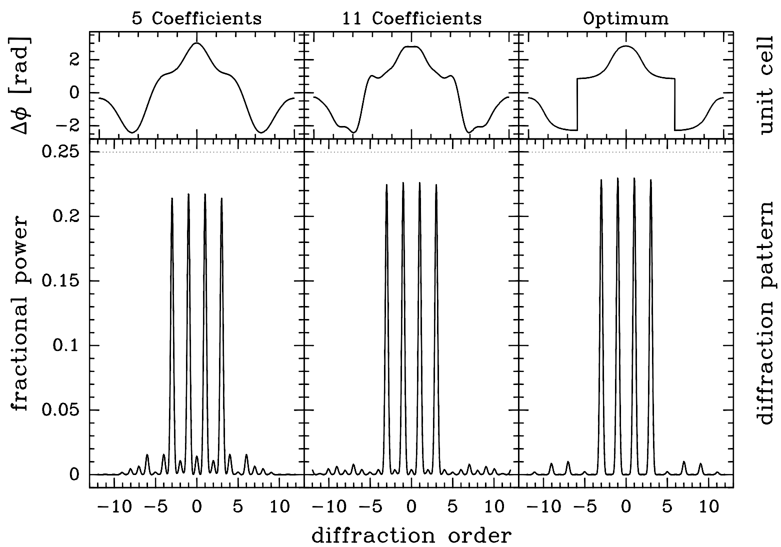

Figure 1: Comparison of Fourier gratings using different numbers of Fourier coefficients. The lower panels show the diffraction patterns produced by the grating profiles in the upper panels.

Grating Performance

To illustrate the potential of the Fourier grating concept, we now take a closer look at one example, a grating that splits an incoming beam into four equally spaced beams of equal intensity. In the symmetric case the power is then distributed between the diffraction orders -3, -1, $+1$, and $+3$. Fig. 1 shows the diffraction patterns obtained for different numbers of Fourier components together with the corresponding grating unit cell structure. It is obvious that already with a very smooth structure composed of 5 components we get a very good grating with $\approx 87\%$ efficiency. The power lost into parasitic diffraction orders decreases rapidly, as we increase the number of coefficients. Correspondingly, the grating efficiency rapidly approaches the limiting efficiency of 92% (Fig. 2).

In Fig. 3 we compare grating efficiencies for different one–dimensional gratings producing a certain number of identical beams. Typical efficiencies that can be reached are higher than 90%. Gratings for an even number of beams require a larger number of Fourier components and usually have a somewhat lower limiting efficiency than

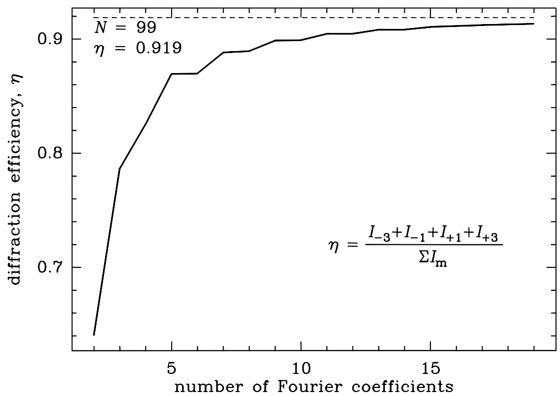

Figure 2: Diffraction efficiency of a Fourier grating plotted against the number of coefficients used in the optimization. The grating was optimized to produce four identical beams in a symmetric pattern.

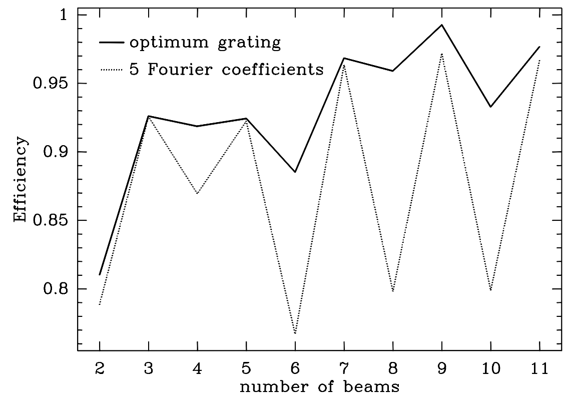

Figure 3: Diffraction efficiency of Fourier gratings producing a given number of identical beams, using $N = 5$ Fourier coefficients (dotted line) or $N \to \infty$ (solid line).

gratings for odd numbers. The reason for this is that, in order to produce a symmetric pattern of an even number of beams, we need to suppress all the even diffraction orders. This is achieved by the sharp steps in the grating unit cell (right hand panel in Fig. 1) which splits the unit cell into two identical sub–cells with a relative phase shift of π. To closely approximate this phase step, a relatively large number of Fourier components would be required. However, as we have seen, excellent grating efficiencies can be achieved with surprisingly crude approximations of the phase step.

The question arises, whether an asymmetric diffraction pattern could yield higher grating efficiencies in this case. For instance, we could produce an asymmetric four beam pattern consisting of the diffraction orders -2, -1, 0, and $+1$. The optimum grating we obtained for this beam arrangement has the same efficiency as the symmetric grating, and its unit cell is basically identical (Fig. 4). The only difference in the grating structure is that the above mentioned phase step is replaced by a sawtooth pattern with a grating blaze angle corresponding to the -0.5^{th} diffraction order.

Two–dimensional Gratings

For the common case of a rectangular beam pattern, the corresponding grating is just an orthogonal overlay of two one–dimensional gratings. Accordingly, the grating efficiency is typically about 80 to 90%, the product of the efficiencies of the one dimensional gratings. Since the two orthogonal patterns can be optimized independently, this case is not fundamentally different from the one–dimensional problem.

In the general two–dimensional case similarly high efficiencies are obtained as long as the grating structure is sufficiently simple that our method of choosing the starting values for the optimization works well enough. Very complex beam patterns requiring

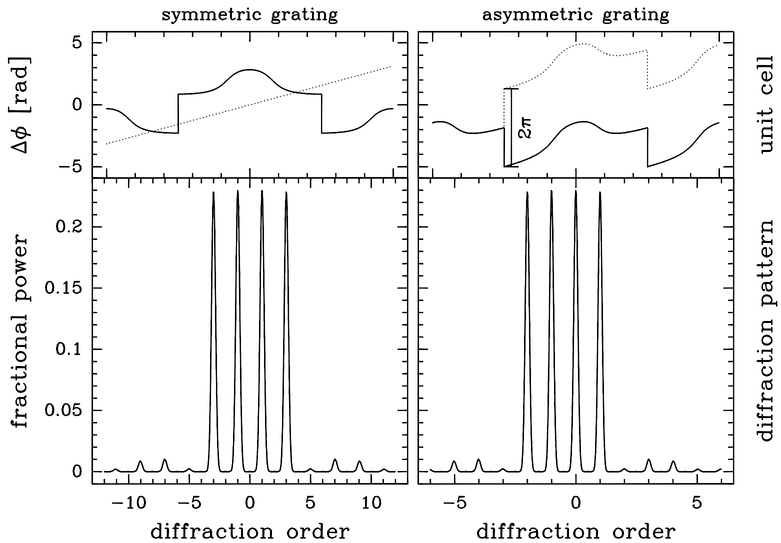

Figure 4: Transition from a symmetric grating to an equivalent asymmetric grating. Adding a phase gradient to the symmetric structure and flipping the sign of every other phase step yields the optimum asymmetric structure. Effectively, the step function in the symmetric grating thus changes into a blaze function to the -0.5^{th} diffraction order.

a highly structured grating unit cell may end up with somewhat lower efficiencies. Since the number of Fourier components to be optimized is now N^2, it is much more difficult to find the global optimum. However, for the application we have in mind, namely the distribution of LO power to an array of heterodyne mixers, the beam patterns are usually sufficiently simple that this is not a real concern.

Bandwidth

The bandwidth of phase gratings is limited by two effects. First, the intensity balance between the diffraction orders is wavelength dependent and, second, due to the frequency dispersion of the grating, the spacing between beams also varies with wavelength. Usually the latter effect is dominating. However, it can be compensated for by some sort of zoom optics. The frequency dependent intensity balance is intrinsic to the phase grating concept and can not be influenced by external optics.

Since the gratings are so easily manufactured, we deal with this problem by making several gratings that cover different parts of the required total receiver band. In our application, we can cover a relative bandwidth of approximately 10 to 20% with each grating. Thus, the need to change the grating only arises when the receiver frequency is changed by a considerable amount.

Having exchangeable gratings requires a means of relocating the gratings with high accuracy in order to maintain the optical alignment when gratings are swapped. To achieve this, we machine the mounting surfaces of the gratings together with the

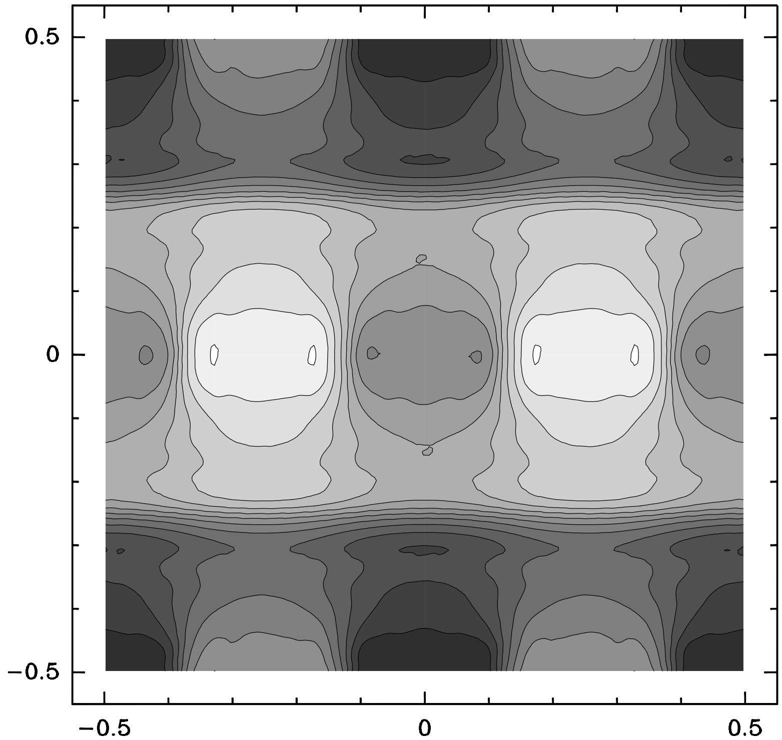

Figure 5: Surface topology of the grating used in the beam measurements shown in Fig. 6. 13×13 coefficients have been used in the optimization. The theoretical efficiency is 84%

grating structure. In our measurements we could not detect any alignment changes, when these gratings were exchanged.

Manufacturing

The main advantage of the Fourier grating is that high diffraction efficiencies are reached with very smooth grating surfaces. This allows us to manufacture reflection gratings by directly machining them with a relatively large diameter cutting tool. We produced a number of gratings with a unit cell size of 30×30 mm^2 for a frequency of 492 GHz. The grating structures used 13 Fourier components per dimension, resulting in a minimum radius of curvature of approximately 7 mm, which is comfortably larger than the 3 mm tool radius we used for machining.

Our Fourier gratings are manufactured by directly milling the structure into a block of brass, using a spherical end mill on a numerically controlled milling machine. Measurements with a dial indicator show that the resulting surface accuracy is approximately 2 μm RMS. This accuracy corresponds to $\lambda/300$ at the design frequency and should be good enough for gratings operating well into the THz region.

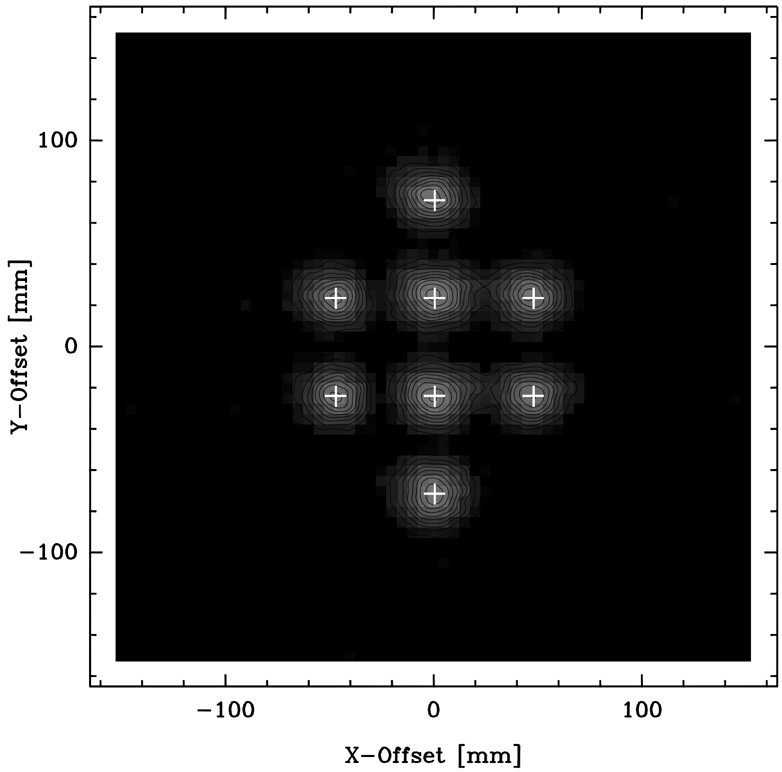

Figure 6: Measured 490 GHz diffraction pattern of the grating structure shown in Fig. 5. From the extremely low side lobe level it is obvious that the grating efficiency is very high.

Measurements

As an example for the practical results obtained with the Fourier gratings, we present the measurements made with a grating producing a two–dimensional pattern of 2, 4 and 2 beam in 3 parallel rows (Figs. 5 and 6). This pattern is important for the use in square 4×4 arrays that are split into two interleaved sub–arrays. Two identical versions of this beam pattern, rotated by 90 degrees with respect to each other combine to a 16 pixel square array.

For the measurements, the beam of a 492 GHz SIS receiver [6] was re-imaged to form a waist at the plane of the grating. The diffracted beam pattern was then scanned with a chopped liquid nitrogen load mounted on an xy–translation stage. The clean beam pattern and the low side lobe level (Fig. 6) show immediately that the concept works well and that the grating efficiency is very high. In fact, within the measurement accuracy there is no deviation from the predicted beam pattern and the measured diffraction efficiency is within 1% of the theoretical value.

Conclusion

We have shown that the novel concept of designing submillimeter phase gratings as Fourier gratings works very well. The gratings are relatively easy to design and have very high diffraction efficiency. Due to their smooth surface structure, they can be machined directly, even as reflection gratings with two–dimensional dispersion.

We have manufactured a series of gratings and tested them at 490 GHz showing that they can be produced sufficiently accurate with standard machine shop equipment, and that they perform as predicted.

Acknowledgments

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