INVESTIGATION OF A SUPERCONDUCTING HOT ELECTRON MIXER

by

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ABSTRACT

Mixing at 20 GHz in niobium superconducting thin film strips in the resistive state is studied. Experiments give evidence that electron-heating is the main cause of the non-linear phenomena. The requirements on the mode of operation and on the film parameters for small conversion loss and the possibility of conversion gain are discussed. Measurements indicate a minimum intrinsic conversion loss around 1 dB with a sharp drop for the lowest voltage bias-points, and a DSB mixer noise temperature between 100 and 450 K at 20 GHz. The device output noise temperature at the mixer operating point can be as low as 30-50 K. A simple theory is presented, which is based on the assumption that the small signal resistance is linearly dependent on power. This type of mixer is considered very promising for use in low-noise heterodyne receivers at THz frequencies.

1. INTRODUCTION

Recently proposed hot-electron bolometer (HEB) mixers utilising thin superconducting films in the resistive state [1-3], have a potential to be competitive with traditional mixers in heterodyne receivers at Terahertz frequencies for radio astronomy and remote sensing applications. These new types of mixers have the advantage of simplicity over SIS and Schottky mixers both from the circuit and the technological point of view. This is particularly obvious in comparison with SIS trilayer mixers, which show excellent performance below the bandgap frequency, approaching 650-700 GHz [4,5]. However, for frequencies above the gap frequency of the superconductors, the SIS mixers are not expected to perform as well. Schottky-diodes are known to work near and above 1 THz, but with lower sensitivity due to an increased noise temperature [4,6]. The well-known InSb hot electron mixers are sensitive enough in the range 0.5-1 THz but the bandwidth is too narrow for many interesting applications [7-9]. A more recent development where a 2D electron gas in heterostructures are used for hot electron mixing may possibly be competitive at THz frequencies [10,11]. Early experiments at 20 GHz with thin and narrow strips of non linear niobium elements, indicate good mixer performance, i.e. low conversion loss and noise temperature [3]. We anticipate that these results can be translated to the THz frequencies, based on the frequency insensitive nature of the electromagnetic interaction and non-linearity of these devices.

The purpose of the effort presented in this paper has been to carefully measure and model conversion loss and noise properties of the HEB at a low millimeter wave frequency, in order to establish a firm basis for the design of future THz HEB mixers.

2. THE DEVICE

A superconducting Hot Electron Bolometer consists of one or several thin superconducting strips, deposited on a substrate of for example silicon, single crystalline quartz or sapphire. The strips are cooled to the superconducting state and then heated by DC and microwave power to the neighbourhood of the superconducting-normal transition temperature, Tc, the resistive state, where the superconductor will gradually become normal (Fig. 1)
The resistance of the device in the resistive state, may be explained by several possible physical phenomena, such as formation of normal domains, phase slip centers, and moving magnetic vortices. Let us adopt a simplistic model\textsuperscript{1}; assume one resistive region (or several that may be combined into one). This situation is illustrated in Fig. 2, where the electron temperature in the "resistive transition regions" is in the interval $\Delta T_c$, marked in Fig. 1.

The main power dependence of the resistance may be due to creation and annihilation of "hot spots" and heating of electrons in the resistive state. Here the gradient in the electron temperature is also steepest. The electron temperature as well as the lattice temperature of the superconductor outside these areas is close to $T_c$, but low enough to allow zero resistance.

\textsuperscript{1}Note that the validity of the circuit-based model which we will present in the next section does not depend on the details of the microscopic model, such as the one suggested here.
Note the quasi particle bandgap, $2\Delta$. If the signal and LO frequency are high so that $f_{LO}, f_s > 2\Delta/h$, the full length of the superconducting strip will be seen as a normal conductor strip. For $f_{LO}, f_s < 2\Delta/h$ the RF-resistance will depend on the bias point and the amount of normal regions in the strip. If the IF is such that $f_{IF} < 2\Delta/h$, it will only develop a voltage across the normal regions while the superconducting part is simply a short-circuit. These facts have important consequences on the functions of the mixer.

The response time of the device is related to a time constant $\tau_{e-e}$ which determines the rate at which the excess energy absorbed by the electrons relaxes. Basically the maximum IF is determined by this energy relaxation or response time of the electrons, i.e. $f_{IF} < 1/(2\pi \tau_{e-e})$. If the strips are made sufficiently thin, the time constant for phonons in the superconductor to escape to the substrate, $\tau_{e-ph}$, may become shorter than the time constant for phonon electron collisions in the superconductor, $\tau_{e-ph}$ for Nb $\tau_{ph-e}$ is shorter than $\tau_{e-ph}$ if the strips are made narrow, then the back flow of phonons from substrate to the superconductor will be reduced, and thus $\tau_{e-ph}$ will be long. Consequently it is possible to heat the electrons above the temperature of the lattice and $\tau_e$ will be dominated by $\tau_{e-ph}$, which is not the case for ordinary bolometers where the electrons and the lattice are heated to the same temperature. (compare Fig. 3).

![Diagram of time constants for hot electron bolometer]

In practice, to avoid lattice heating effects in the superconductor, the bolometer should be less than or about 100 Å thick and about 1 μm wide. For Nb the resulting bandwidth corresponding to $(2\pi \tau_{e-ph})^{-1}$ is of the order 100-200 MHz, and for NbN several GHz [1,2]. With higher $T_c$, a higher operating temperature can be used, yielding shorter $\tau_{e-ph}$. For Nb it may be possible to reach 500 MHz IF bandwidth.

The bolometer resistance may be adjusted by adding parallel strips and by changing the strip length. For $f_{LO}, f_s > 2\Delta/h$ the absorption of microwave power by the normal electrons in the HEB is essentially frequency independent. Therefore these mixers will operate to RF frequencies well into the THz range, i.e. much higher than the bandgap frequency of the superconductor. Previous work has in fact shown mixing at 1.5 THz [1].

### 3. MIXER THEORY

In Fig. 4 is shown an equivalent circuit of the mixer, where the device is biased by a constant DC current. The heating of the electrons by RF or DC power will increase the device resistance. The signal, when beating with the LO, will cause a modulation of the device resistance at the IF. An IF
voltage will appear across the device, causing a current through the IF load resistance. This current will go in the opposite direction to the DC bias current in the mixer device, and will create feed-back.

Note that the resistance of the device is time dependent only for the IF. For the signal the device impedance is assumed constant since these frequencies are higher than the inverse response time of the electrons. If this statement were not true, an image current would be created, and a more complicated theory will be necessary.

![Fig. 4. Equivalent circuit of bolometer with load.](image)

The same expression for the conversion gain has been derived for three different types of HEB mixers: (1) the superconducting HEB by employing an energy balance equation [1]; (2) the hot electron InSb [7] and (3) 2DEG mixers [10,11]. A basic assumption is that the DC and time-average RF power dependence of the bolometer resistance are equal. Assume that in the bias point of the device we have $V=V_0$ and $I=I_0$. Defining the device resistance as $R_0=V_0/I_0$, one obtains

$$G = \frac{P_{IF}}{P_t} = 2C_0^2 \frac{P_{LO}}{P_{DC}} \frac{R_I}{R_0} \left(1 - C_0 \frac{P_{DC}}{R_0} \frac{R_L - R_0}{R_L + R_0}\right)^{-2}$$

(1)

where $C_0=\frac{dR_0}{dP_{device}}$. $R_I$ is the IF load resistance, $P_{IF}$, $P_t$, $P_{LO}$ and $P_{DC}$, are the IF-, signal-, LO- and DC-power respectively.

For the types of IV-characteristics which apply to InSb and 2DEG mixers, an optimum conversion gain of -6 dB can be derived from (1), see [3]. The superconducting HEB has a different IV-characteristic, and a re-interpretation of (1) leads to a prediction that $G>0$ dB is possible, as shown in the next three sections. Conversion gain of >-6 dB for superconducting HEBs was originally derived in [1], but the present treatment is more extensive.

### 4. RELATING THE CONVERSION GAIN TO THE IV-CHARACTERISTIC

A basic assumption in our model is that the low frequency resistance of the device is dependent on electron temperature. Hence, if the temperature increases linearly with dissipated power, we have $\Delta R = C_0 \Delta P$. This assumption has certain interesting implications: Consider a given IV-characteristic as shown in Fig. 5. For a given device resistance $R_0=V_0/I_0$, the device temperature must be constant, i.e. the total power dissipated must be the same. When the device is pumped with a given LO-power, the IV will change, as shown in Fig. 5. The above argument indicates that in points $A$ and $a$ we have the same dissipated power: in $A$ only DC power, and in $a$ LO power plus DC power. If we increase either the LO power or the DC power, or both, the resistance will increase, and become $R_0 + \Delta R$. 
We may now calculate $\Delta R$ and $\Delta P$. Since it makes no difference whether $\Delta P$ is DC power or a combination of DC and LO power, we may use

$$\Delta R = \frac{V_o + \Delta V}{I_o + \Delta I} - \frac{V_o}{I_o} = \frac{V_o}{I_o} \left( \frac{\Delta V}{\Delta I} \frac{I_o}{V_o} - 1 \right)$$

(2)

$$\Delta P = (V_o + \Delta V)(I_o + \Delta I) - V_oI_o = V_o \Delta I \left( \frac{\Delta V}{\Delta I} \frac{I_o}{V_o} + 1 \right)$$

(3)

From Eqs. (2) and (3) we get

$$C_o = \frac{dR}{dP} = \frac{1}{I_o} \frac{\Delta V}{\Delta I} \frac{I_o}{V_o} - 1 = \frac{R_o}{P_{DC}} \left( \frac{dV}{dl} \right)_{DC} - \frac{R_o}{P_{DC}} \left( \frac{dV}{dl} \right)_{DC} + \frac{R_o}{P_{DC}}$$

(4)

where $(dV/dl)_{DC}$ is the differential resistance of the pumped IV in the bias point determined by DC bias and LO power.

With Eq. (1) this leads to

$$G = \frac{1}{2} \frac{P_{LO}}{P_{DC}} \frac{R_L}{R_o} \left( \frac{R_o}{(dV/dl)_{DC}} - 1 \right)^2 \left( \frac{R_o}{(dV/dl)_{DC}} + 1 \right)^{-2}$$

(5)

There is still another relationship between the $P_{LO}$, $P_{DC}$ and $I_o$ that can be used to rewrite the expression for conversion gain. For constant $R_o = V_o/I_o$, the dissipated power in the device must be constant. This means that if we have a bias current $I_{oo}$ for the unpumped IV and $I_o$ for the pumped IV, the following equality must be true:

$$R_oI_{oo}^2 = R_oI_o^2 + P_{LO}$$

(6)

Note that $P_{DC} = R_oI_o^2$. Moreover, since $C_o$ is invariant for constant $R_o$, Eq. (4) yields

$$C_o = \frac{1}{I_o^2} \left( \frac{\Delta V}{\Delta I} \right)_{DC} - R_o = \frac{1}{I_{oo}^2} \left( \frac{\Delta V}{\Delta I} \right)_{noLO} - R_o$$

(7)

With Eq. (1) (6) and (7) we get

$$G = 2 \left( C_oI_{oo}^2 \right)^2 \left( \frac{I_o^2}{I_{oo}^2} \right) \left( 1 \right)^2 \left( \frac{R_L}{R_o} \right) \frac{1}{(R_L/R_o + 1)^2} \frac{1}{\left( 1 - C_oI_{oo}^2 \frac{I_o^2}{I_{oo}^2} \frac{R_L}{R_o} - 1 \right)^2}$$

(8)
This equation can be rewritten as

\[ G = 2 \left(1 - \frac{I_2^2}{I_{\infty}^2} \right) \frac{\left(C_o I_0^2\right)^2}{\left(1 - C_o I_0^2\right)^2} \frac{r_s \cdot r_t}{R_L + \left(\frac{dV}{dI}\right)_{DC}} \]  

from which it is seen that \( R_L = (dV/dI)_{DC} \) yields maximum gain.

These equations for the conversion gain are valid for low intermediate frequencies, where the finite relaxation time of the device has no influence on the conversion gain.

5. MODELLING OF THE IV-CARACTERISTICS

In Fig. 6 is shown the expected shapes of the IV under the assumptions of the model described above. (see Appendix). In this figure we have used a parameter, \( v \), which defines a constant, normalised, LO power:

\[ \frac{P_{LO}}{R_o I_{\infty}^2} = \frac{v}{R_o I_{\infty}^2} = \frac{v}{V_{oo}} \]  

where \( V_{oo} \) is the voltage where \( R_o \) intersects \( I_o = I_{oo} \). As a special case, we have chosen \( dV/dI = \infty \) for \( P_{LO} = 0 \) (the horizontal curve, \( I_{oo} = 1 \) in Fig. 6). If the unpumped IV curve corresponds instead to e.g. the curve with \( v = 1 \), we may treat this case as if an equivalent heating brings us from the \( v = 0 \) curve to \( v = 1 \) curve. Then the \( v = 2 \) case corresponds to a normalised LO power of \( v = 1 \). The same argument can be applied if the unpumped IV has a shape corresponding to \( v = -1 \). Consider e.g. the isotherm represented by \( R_{o1} \) and assume that the \( v = -1 \) curve represents the "unpumped" IV. The curve \( v = +2 \) then represents the pumped IV-characteristic for a normalised LO power of \( (3/V_{oo}) \times (R_o I_{oo}^2) \).

![Graph showing IV-curves](image)

Fig. 6. Calculated IV-curves assuming that for a particular isotherm \( V/I \) (e.g. \( = R_{o1} \)) \( dR/dP \) is constant. The parameter \( v = P_{LO} I_o \), where \( I_o = 1 \) in this diagram. The curves for \( v < 0 \) show negative resistance. See also the text.

The interesting observation in Fig. 6 is of course that negative differential resistance is allowed and included in the model. Hence, our conclusion is that the model as such allows for negative differential resistance, a feature which is necessary to achieve a conversion loss less than 6 dB, or even conversion gain.
To illustrate the negative resistance property further, let us calculate the differential resistance vs. current along an isotherm, i.e. for constant $R_0$. Let us assume that we have $(dV/dl)_{I=I_0}$ for $I=I_0$ and $(dV/dl)_{I=I_0}$ for $I=I_0$. Then

$$
(dV/dl)_{I_0} = R_0 \left( \frac{I_0^2}{I_{\infty}^2} \right) \frac{(dV/dl)_{I_0} + R_0}{(dV/dl)_{I_0} + R_0} - \frac{(dV/dl)_{I_0} - R_0}{(dV/dl)_{I_0} - R_0}
$$

(11)

This equation is illustrated in Fig. 7.

![Graph of differential resistance vs. current.](image)

**Fig. 7.** Differential resistance of pumped IV-curve $(dV/dl)_{I=I_0}$ along an isotherm as function of bias current. $R_0 = 20 \, \Omega$ and $(dV/dl)_{I=I_0} = -40 \, \Omega$ are typical values from experiments.

### 6. PREDICTING THE CONVERSION GAIN

The negative resistance can be predicted if $C_0I_{\infty}^2$ is known. From Eq. (7) the differential resistance can be expressed as:

$$
\frac{dV}{dl} = R_0 \cdot \frac{1 + C_0I_{\infty}^2}{1 - C_0I_{\infty}^2}
$$

(12)

Hence a negative resistance corresponds to $C_0I_{\infty}^2 > 1$. Using Eq. (8) we have calculated the conversion gain $G_{\text{DC}} = P_{\text{DC}}(P_{\text{DC}} + P_{\text{LO}})$. 

Fig. 8. Conversion Gain vs. $P_{DC}/(P_{DC} + P_{LO})$ for different IF load resistances. $R_L > R_O$. The differential resistance for the unpumped curve: $dV/dI = \infty$, or $C_0 I_{00}^2 = 1$.

Fig. 9. Conversion Gain vs. $P_{DC}/(P_{DC} + P_{LO})$ for different IF load resistances. $R_L < R_O$. The differential resistance for the unpumped curve: $dV/dI = \infty$, or $C_0 I_{00}^2 = 1$.
Fig. 10 Conversion Gain vs. $P_{DC}/(P_{DC}+P_{LO})$ for different IF load resistances, $R_L>R_O$. The differential resistance for the unpumped curve is negative: $dV/dI=-5R_O$, or $C_0^2I_{100}^2=1.5$.

Fig. 11. Conversion Gain vs. $P_{DC}/(P_{DC}+P_{LO})$ for different IF load resistances, $R_L<R_O$. The differential resistance for the unpumped curve is negative: $dV/dI=-5R_O$, or $C_0^2I_{100}^2=1.5$.

The fundamental limit of -6 dB gain for InSb mixers [7] is obviously not valid for the superconducting HEB mixers since it is possible to achieve positive conversion gain with negative differential resistance of the unpumped IV-curve. From Fig. 8-11 obviously larger conversion gain is available for $R_L/R_O>1$ rather than $R_L/R_O<1$.

As mentioned above the load resistance for maximum gain is equal to the differential resistance of the IV-curve in the bias point of the pumped mixer. It is as well possible to find optimum LO power for a given $R_L/R_O$ or with $R_L/R_O$ equal to the differential resistance of the IV-curve in the bias point of the pumped mixer by determining $(I_o/I_{100})^2$ yielding maximum gain.
7. CONVERSION GAIN AND IF IMPEDANCE VS INTERMEDIATE FREQUENCY

The coefficient \( C_\sigma = \frac{dR}{dP} \) can be expressed as \( C_\sigma = \frac{[dR/dT] - [dP/dT]}{dP} \). Assuming that the factor \( dT/dP \) has a frequency dependence determined by the factor \( 1/(1 + j\omega\tau_e) \), and that the term \( dR/dT \) is simply a constant, we may draw the conclusion that

\[
C_\sigma(\omega) = \frac{C_\sigma(\omega = 0)}{1 + j\omega\tau_e} \tag{13}
\]

Using this expression for \( C_\sigma \) in the equations for \( G \) above, the IF dependence of the conversion gain of the mixer can be expressed as:

\[
G = G(\omega = 0) \cdot \frac{1}{1 + \left(\omega\tau_{\text{mix}}\right)^2} \tag{14}
\]

where

\[
\tau_{\text{mix}} = \frac{\tau_e}{1 - C_\sigma I_o^2} \frac{R_L - R_o}{R_L + R_o} \tag{15}
\]

Evidently, it is possible to make the relaxation time shorter than \( \tau_e \) (and the bandwidth wider), by making the factor \( C_\sigma I_o^2(R_o - R_L)/(R_o + R_L) \) large \( (R_o > R_L) \). It is obviously possible to look for a compromise between IF bandwidth and conversion gain. We can also conclude that it is undesirable to attempt to utilise a large conversion gain, a feature which the HEB mixer shares with other negative resistance devices.

The frequency dependence of the IF impedance can be derived in the same way by using expression (13) and Eq. (4). We get

\[
Z(\omega) = R_o \left(1 + \frac{2C_\sigma I_o^2}{1 - C_\sigma I_o^2} \frac{1}{1 + j\omega\tau_{\text{imp}}}\right) \tag{16}
\]

where

\[
\tau_{\text{imp}} = \frac{\tau_e}{1 - C_\sigma I_o^2} \tag{17}
\]

It is possible to rewrite the equation for conversion gain expressing the frequency dependence in terms of the electron energy relaxation time \( \tau_e \). Using a complex load impedance viz. \( R_L \rightarrow Z_L = R_L + jX_L \) we obtain

\[
G = \left[ \frac{\tau_e}{2} \left( 1 - \frac{I_o^2}{I_{\text{sat}}^2} \right) \frac{(C_\sigma I_o^2)^2}{1 - C_\sigma I_o^2} \right] \frac{1}{R_o \cdot R_L}
\]

\[
\left[ R_L + (dV/dt)_{\text{pumped}} - \omega\tau_e \cdot \frac{X_L}{1 - C_\sigma I_o^2} + j \left( X_L + \omega\tau_e \cdot \frac{R_o + R_L}{1 - C_\sigma I_o^2} \right) \right]
\]

Hence it should be possible to improve the bandwidth somewhat by a proper design of the load circuit \( Z_L \). Fig. 12.
Fig. 12. IF Bandwidth for different load reactances.

8. THEORY OF NOISE IN HEBs

The main noise of concern for the superconducting HEB is that directly emitted by the device at the IF frequency. The noise theory does not need to take into account correlation of noise contributions at a number of frequencies, as is the case for the Schottky-barrier mixer, for example. The IF output noise is primarily of two types [12]: (1) Nyquist noise at the electron temperature, and (2) temperature fluctuation noise. The latter contribution is not usually important for microwave or millimeter wave mixers, but is likely to be the dominant one in superconducting HEB mixers. Temperature fluctuation noise is well-known from the theory of conventional (lattice) bolometers [13]. The electrons are assumed to form a subsystem in thermal equilibrium at the electron temperature, $T_e$, while conducting heat to the lattice heat reservoir at temperature $T_L$. The corresponding thermal conductance is $G$ [W/K]. It is known from thermodynamics that the average temperature of the electron subsystem will show RMS fluctuations given by [13]:

$$\Delta T_e = \frac{4k_BT_e^2}{G}$$  \hspace{1cm} (19)

The thermal conductance is

$$G = \frac{C_e V}{\tau_e}$$  \hspace{1cm} (20)

where $C_e$ is the electron specific heat and $V$ the volume of the superconducting strip. Since the resistance of the HEB depends on $T_e$, and the device is biased with constant DC current, $I_0$, there will be a noise voltage developed across the device. We can derive the equivalent device output noise temperature, $T_{d,t}$, by equating the noise power due to temperature fluctuations to that from a Nyquist source at $T_{d,t}$:

$$T_{d,t} = \frac{I_0^2}{R_e G} \left(\frac{dR}{dT_e}\right)^2 T_e^3$$  \hspace{1cm} (21)

Here $\left(\frac{dR}{dT_e}\right)$ must be evaluated for actual mixer operating conditions. The total device output noise temperature will be

$$T_d = T_e + T_{d,t}$$

This noise temperature can now be used in standard expressions for the mixer noise temperature, given the conversion loss, $L$.

If typical values are inserted in (21), one finds that this term is likely to dominate. Note that $T_{d,t}$ is proportional to the electron temperature squared. Further, $T_e$ is expected to be close to $T_c$. HEBs based on superconductors with higher $T_c$ then should have higher noise. The $T_e$-dependence may be
compensated by the appearance of $G$ in the denominator, however. As a rule, higher $T_c$ materials have shorter relaxation times, resulting in larger $G$ (See Eq. 20). An optimum material in terms of device output noise temperature can therefore not be identified. One must also consider the conversion loss which can be achieved, in order to minimise the receiver noise temperature. These matters are under investigation, but have not yet been resolved in detail. Another noteworthy consequence of the dominance of $T_{d,t}$ is that the output noise temperature is predicted to fall with a time constant similar to $\tau_{\text{mix}}$ (See Eq. 14). If this can be verified, then one concludes that the bandwidth over which a given mixer noise temperature can be maintained is actually wider than $1/2\pi\tau_{\text{mix}}$, since the device output noise decreases at the same time that the mixer conversion gain decreases. Measurements of the output noise versus IF-frequency have so far given inconclusive results.

In conclusion, one can roughly estimate a device output noise temperature for Nb in the range 25-100 K. This is consistent with measurements, as described in a later section.

9. EXPERIMENT

HEB devices consisting of 2 parallel niobium strips, 100-150 Å thick, 1.5 μm wide and 7 μm long with normal resistance between 40 and 150 Ω have been manufactured on silicon substrates and measured at 20 GHz signal and 1-1000 MHz intermediate frequency. The HEB mixer is biased in the lower part of the resistive region (Fig. 13 and 16) for best conversion. The unstable part of the IV-curve between the pure superconductive state for zero bias and the resistive state indicates that a negative differential resistance region exists where conversion gain larger than one may be achieved. For no LO and low bias we have indeed observed a part of the resistive region with negative differential resistance. Compare Fig. 13 and 16 for no LO.

A) DETERMINATION OF RF COUPLING LOSS

The measured conversion loss was obtained by determining the RF coupling to the mixer from a set of IV-curves recorded for different LO power. It is again assumed that the response to time-average RF power is equal to the response to DC power taken as I-V from the IV-curve. The points on the IV-curves from which the DC powers are derived are along the isotherm which passes through the bias point A in Fig. 13.

Fig. 13. IV curves for two different applied LO powers. To the left for temperatures very close to $T_c$, where there is no super-current, and to the right for temperatures less than about 0.9 $T_c$, where there is also hysteresis.
We derive the RF Coupling Loss from:

\[ L_c = \frac{P_{LO,B} - P_{LO,A}}{P_{DC,A} - P_{DC,B}} \]  

(22)

where \( P_{LO,A} \) and \( P_{LO,B} \) are defined at the coaxial connector input to the liquid helium dewar. The coupling loss thus includes the attenuation in the input coaxial cable, and the microstrip circuit to which the device is connected. The sum of these contributions has been estimated to about 3 dB (see Fig. 20).

The incremental step in LO-power used in our experiments was typically between a half and one dB. Apart from the circuit losses of about 3 dB, we find that the coupling loss depends on the bias, as shown in Fig. 14. The bias-dependent coupling loss can be interpreted as the reflection loss of the device impedance with respect to the 50 ohm microstrip circuit. If we assume that the device impedance is \( R_0 \), then we can calculate the reflection loss as shown in the full drawn curve in Fig 14. At about 3 mV bias, \( R_0 \), as defined above, is close to 50 ohm, and the reflection loss very small. The measured coupling loss shows a similar trend, but is about 1.5 dB higher at low bias voltages. The device resistance at 20 GHz thus is fairly close to \( R_0 \). If the frequency of the RF radiation were higher than the bandgap frequency in the entire strip, then we would expect the resistance to be equal to \( R_N \), and independent of the bias. This is likely to be the situation in THz versions of the HEB mixer, but is not the case in the 20 GHz prototype. The frequency and bias dependence of the device impedance thus requires further study, as we attempt to increase the frequency at which the HEB mixer operates.

![Diagram](image_url)

**Fig. 14.** Measured 20 GHz RF coupling loss (points) vs. bias voltage, and calculated reflection loss under the assumption that the bolometer microwave resistance is equal to its DC resistance. (sample #7)

**B) CONVERSION LOSS**

The lowest conversion loss of 7 investigated devices was around 1 dB at 20 MHz IF at an operating temperature of 2.1 K (sample #7). The IF of 20 MHz was chosen for the experiments since it is well below the cut-off frequency (Compare Fig. 17). In Fig. 15 one can see how the conversion loss depends on bias voltage (sample #7). The lowest conversion loss is found for the smallest bias, i.e. the lowest bolometer DC resistance. Comparing devices for equal bias, such as the points for samples 11.2 and 15.5 in Fig. 15, it is seen that sample #7 is not the best device, and it is reasonable to assume that samples 11.2 and 15.5 would attain lower conversion loss than sample #7, if measured at lower bias. Also of some importance for the mixer properties are \( R_N \), \( \Delta T_C \), \( T_0 \), and the number of strips. So far we have not carried out a complete investigation of the variation of conversion loss with these parameters. However in general terms we have found the following:

The best conversion gain is found for samples with the narrowest superconducting-normal transition \( \Delta T_C \), i.e. \( dR/dT \) is large.
The conversion gain improves for lower temperatures. Typically we observed one or two dB improvement per Kelvin for a mixer temperature of about 3K. More parallel strips yields a smoother IV-curve and a more stable output signal.

We have not observed any strong dependence on $R_N$.

In Fig. 15 the measured intrinsic conversion loss of sample #7, the most extensively investigated device, is compared with the theoretical conversion loss calculated from Eq. (1) and (8). In order to find the intrinsic conversion loss, we used the coupling loss, given by expression (22). The agreement is quite good despite the simplified model, which only takes into account uniform heating of the hot electrons, and for example does not consider the slight roughness of the IV-curve. The bias point and a set of DC IV-curves utilised for the RF coupling calculation are shown in Fig. 16. $P_{LO}$, $R_0$, $P_{DC}$, $I_0$, $I_{oo}$ and $(dV/dI)_DC$ were obtained from the DC IV-plot. The absorbed pump and signal powers, about -43 dBm and -72 dBm respectively, are typical values in our experiments.

Fig. 15. Experimental and calculated conversion loss vs. bias voltage for sample #7 shown together with the conversion loss of two other samples. The LO power is optimised for the lowest bias and kept constant.

Fig. 16. IV curves for different pump-power. (sample #7)
Note in Fig. 16 that for low bias and no LO, a region with negative resistance is clearly observed. The differential resistance along the isotherm shown in Fig. 16 for different LO powers follows the prediction of Eq. (10). A probable explanation for the occurrence of the negative resistance is the following. When the bias is lowered, the heating is lowered and the temperature of the Nb lattice is lowered as well. This should lead to an increase in the maximum current density, and we consequently will see a negative differential resistance. The time constant for the combined heating phenomenon will be determined by \( \tau_e - \tau_{ph} \) since \( \tau_{ph} \approx \tau_e - \tau_{ph} \).

C) IF BANDWIDTH

The conversion loss has dropped 3 dB at around 100 MHz IF, see Fig. 17. For larger bias there is a small increase in bandwidth, i.e. the mixer time constant will be slightly shorter. Assuming the relaxation time constant \( \tau_e = 1.78 \) ns, and calculating \( C_0 I_0^2 \) and \( R_0 \) from the DC IV-characteristic, we obtain theoretically from (Eq. 14) \( \tau_{mix} = 2.6, 2.14, \) and \( 1.83 \) ns, respectively, for the three bias voltages 1.1, 1.4 and 3.2 mV respectively, in excellent agreement with the experimental time constants. In Fig. 17 it can also be seen that there is an increased response at lower IF around 1-10 MHz. This is probably due to influence from slower processes, such as the relaxation of the normal domain size and back flow of phonons from the substrate.

![Conversion Gain vs. frequency and bias graph](image)

Fig. 17. Conversion Gain vs. frequency and bias, showing an increased mixing time constant for lower bias. (sample #7)

It should be possible to achieve a more uniform absorption of RF power in the strips by: (1) Application of a magnetic field, which reduces the superconducting energy-gap below \( f_{RF - h} \), allowing absorption also in the superconducting regions, (2) Use of higher LO and signal frequencies, comparable to or above the gap frequency of the superconductor at the actual temperature, (3) reduction of the superconducting energy-gap by increasing the lattice temperature to a value closer to \( T_c \). It is noteworthy that a higher-frequency HEB mixer is expected to have more ideal performance than the low frequency prototype we have investigated.

D) IF IMPEDANCE

The device IF-impedance has been measured between 1 and 500 MHz (see Fig. 18). There is a transition of the impedance in the range 10 to 100 MHz from 90 \( \Omega \), which is close to the DC differential resistance (108 \( \Omega \)) in the bias point, to 26 \( \Omega \), which is the DC resistance in the bias point. The impedance could not be measured for lower biases due to influence from the network analyser.
Fig. 18. The measured bolometer impedance for sample #7 (dots). The solid lines show the real and imaginary part of the circuit in Fig. 19. $R_0$ is 26 $\Omega$ and $(dV/df)_{DC} = 108$ $\Omega$. Optimum LO power is applied.

In Fig. 19 is shown the equivalent circuit of the device, which satisfies Eq. (16), as well as the measured data in Fig. 18. The time constant $\tau = R_2 C$ is 5.37 ns. Using the value for $C_0 I_0^2$ in the bias point and Eq. (17) we find a value for $\tau_c = 1.89$ ns, very close to the number obtained from the conversion loss measurement ($\tau_c = 1.78$ ns).

$$ Z = \frac{R_1}{1 + j\omega R_2 C} $$

Fig. 19. Equivalent circuit for the bolometer. The values for $R_1$, $R_2$ and $C$ apply to sample #7 and were derived from the measured bolometer impedance data in Fig. 18.

E) NOISE

The receiver and device output noise temperatures, respectively, of the pumped bolometer were measured using the measurement set-ups shown in Figs. 20 and 21.
Fig. 20. Set-up for mixer noise measurement.

Since no isolator is available for the IF-range of interest, a coolable load was fabricated from lumped resistors and a capacitor, with values as shown in Fig. 19. The impedance of this load was checked with a network analyser, and agreed well with the measured data for the bolometer (see Fig. 18). This load was used as a hot and cold load for calibration of the room temperature low noise amplifier over the frequency range 1-100 MHz (Fig. 22).

The DSB receiver noise temperature of one of the HEB mixers was measured with a solid state noise source. The resulting mixer receiver noise temperature was between 470 and 690 K. A DSB mixer noise temperature between 100 and 450 K can be derived from these values, given the conversion loss in the biaspoint of $7 \pm 1$ dB. The influence of the components between the noise source and the mixer had to be subtracted, introducing an uncertainty in the determination of the mixer receiver noise temperature, thus the large interval quoted. The output noise from the mixer at intermediate frequencies of 20-90 MHz was measured to be 30-50 K (Fig. 22). For this measurement, the mixer was not biased at the optimum DC bias point. However, the optimum LO power was supplied. In this measurement, the conversion loss was 7 dB, and we can predict a DSB mixer noise temperature between 80 and 100 K, based on the measured output noise temperature. The predicted DSB mixer noise temperature is consistent with the above measurements of the mixer receiver noise.

Fig. 22. Output noise temperature of mixer, and amplifier noise when connected to the mixer. $R_0 = 26 \, \Omega$, $T_0 = 2.1 \, K$, $P_{LO}$ is optimised for maximum conversion gain.
CONCLUSIONS

We have presented a simple phenomenological theoretical model for the bolometric mixer. This model allows us to derive conversion loss and bandwidths from knowledge of the pumped or unpumped IV-characteristic.

We do see very low conversion loss, which is related to negative resistance in the unpumped IV, and fits the theoretical model quite well.

The negative resistance is likely due to a bias dependent heating effect of the Nb lattice. When the bias is lowered, the temperature of the superconductor is lowered, and a larger current density possible. Hence the current can increase while the bias is decreased.

It is predicted that conversion gain can be achieved, if proper IF impedance load is chosen. The penalty for larger gain is smaller IF bandwidth.

The measured noise is quite low. We see noise temperatures out from the device typically around 50 K, and a double sideband noise temperature expected for an optimised mixer is calculated to about 100-150 K.

The impedance of the device at 20 GHz is bias dependent, which means that this frequency is lower than the bandgap frequency of the superconducting regions of the device.

The frequency dependence of the IF-impedance and conversion gain has been modelled and measured. The measurements are consistent with an electron energy relaxation time of 1.8-1.9 ns. This means that the maximum IF frequency for this particular device is about 100 MHz. It is pointed out that the IF-frequency dependence can be improved by a proper IF load circuit.

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REFERENCES


Appendix

Modelling IV-characteristics.

For an IV-curve, with or without LO power, the model suggests that in the bias point $V$ and $I$, we have

$$C_0 = \frac{dR_0}{dP} = \frac{1}{I_0^2} \left( \frac{dV_0}{dI_0} \frac{V_0}{I_0} \right)$$

(A1)

Assume that for a certain bias voltage $V_0$ and current $I_0$, the differential resistance $dV/dI=\infty$. Hence in this bias point we have $C_0=1/(I_0)^2$. Furthermore, $C_0$ is constant and the same for given ratio $V_0/I_0=R_0$, i.e., a given device temperature (device isothermal). Hence

$$C_0 = \frac{dR_0}{dP} = \frac{1}{I_0^2} \left( \frac{dV_0}{dI_0} \frac{V_0}{I_0} \right) = \frac{1}{I_0^2}$$

(A2)

For simplicity and as an illustration, let us assume that for no LO power I is constant, $I=I_0$, independent of bias voltage ($\nu=0$ in Fig. 5 in paragraph 5). Since $R_0$ represents an isothermal, we have

$$\frac{P_{LO} + P_{DC}}{P_{DC}} = \frac{P_{LO} + R_0 I_0^2}{R_0 I_0^2} = \frac{R_0 I_{\infty}^2}{R_0 I_0^2}$$

(A3)

Together with Eq. (A2), we now get

$$\frac{dV_0}{dI_0} = \frac{P_{LO}}{R_0 I_{\infty}^2} \left( \frac{1+(I_0/ I_{\infty})^2}{1-(I_0/ I_{\infty})^2} \right)^2$$

(A4)

We define the parameter $\nu=P_{LO}/I_0$, where $I_0=1$ in this diagram.

$$\frac{P_{LO}}{P_{DC}} = \frac{P_{LO}}{R_0 I_{\infty}^2} = \frac{\nu}{R_0 I_{\infty}^2} = \frac{\nu}{V_0}$$

(A5)

where $V_0$ corresponds to the voltage where $R_0$ intersects $I=I_{\infty}$. 