On Wirelength Estimations for Row-Based Placement

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Wirelength Estimation

- Basic Formulation:

  What is expected WL of a net *N*?

- Fundamental to pre-routing performance analysis
- Studied in several contexts
  - floorplanning
  - row-based placement
  - hierarchical top-down layout
- Should match information, algorithmic context

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  - floorplanning and logic optimization
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  - placement feasibility analysis
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  - **high** accuracy ≈ 2%
  - **faster** than routing
- ***On-line*** (during placement)
  - e.g., top-down move-based placers
  - **intermediate** accuracy ≈ 2-10%
  - **very fast**

On-Line WL Estimation

Given k rectangles R_i each containing n_i cells of a net $\boldsymbol{N}$, what is ***expected*** WL of $\boldsymbol{N}$?

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- Rectangles R_i arise during top-down layout
- Terminals are localized in R_i by floorplanning, partitioning

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- Rectangles R_i arise during top-down layout
- Terminals are localized in R_i by floorplanning, partitioning
- Assumption: terminals are uniformly distributed within any given R_i

Previous Techniques

- WL ≈ rectilinear Steiner minimal tree (RSMT) cost
  - Beardwood et al. (1958): for n points in region R

$$cost(RSMT) \propto \sqrt{area(R)} \cdot \sqrt{n}$$

  - Chung/Graham (1979): for n points in bounding box R

$$\max cost(RSMT) \rightarrow \mathrm{HP}(R) \cdot \frac{(\sqrt{n}+1)}{2}$$

  - Discrepancy between two results

Previous Techniques

- WL ≈ rectilinear Steiner minimal tree (RSMT) cost
- WL ≈ bounding box half-perimeter
  - max/min terminal coordinates
  - efficiently updated (move-based placers)
  - center-to-center approximation
  - Cheng's (1994) scaling correction

Previous Techniques

- WL ≈ rectilinear Steiner minimal tree (RSMT) cost
- WL ≈ bounding box half-perimeter
- A priori/a posteriori WL estimations
  - Rent's rule
    - (Donath (1979), …, Davis et al. (1998))
  - neighborhood population technique
    - (Sechen (1988), Pedram/Preas (1989), Hamada et al. (1992))

Our Contributions

- Theoretical
  - Fast estimation of expected bounding box
  - WL dependency on # of terminals and bbox AR
  - First aspect ratio dependent WL estimators
- Practical
  - Substantially more accurate on-line WL estimators
  - Validation of new bbox and WL estimators using top-down partitioning-based placer
  - Early pruning of unpromising solution paths in top-down move-based placement

Outline

- Fast, accurate computation of expected bbox
  - inaccuracy of center-to-center approximation
  - exact and heuristic algorithms

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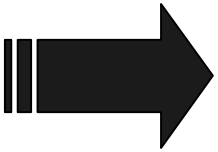

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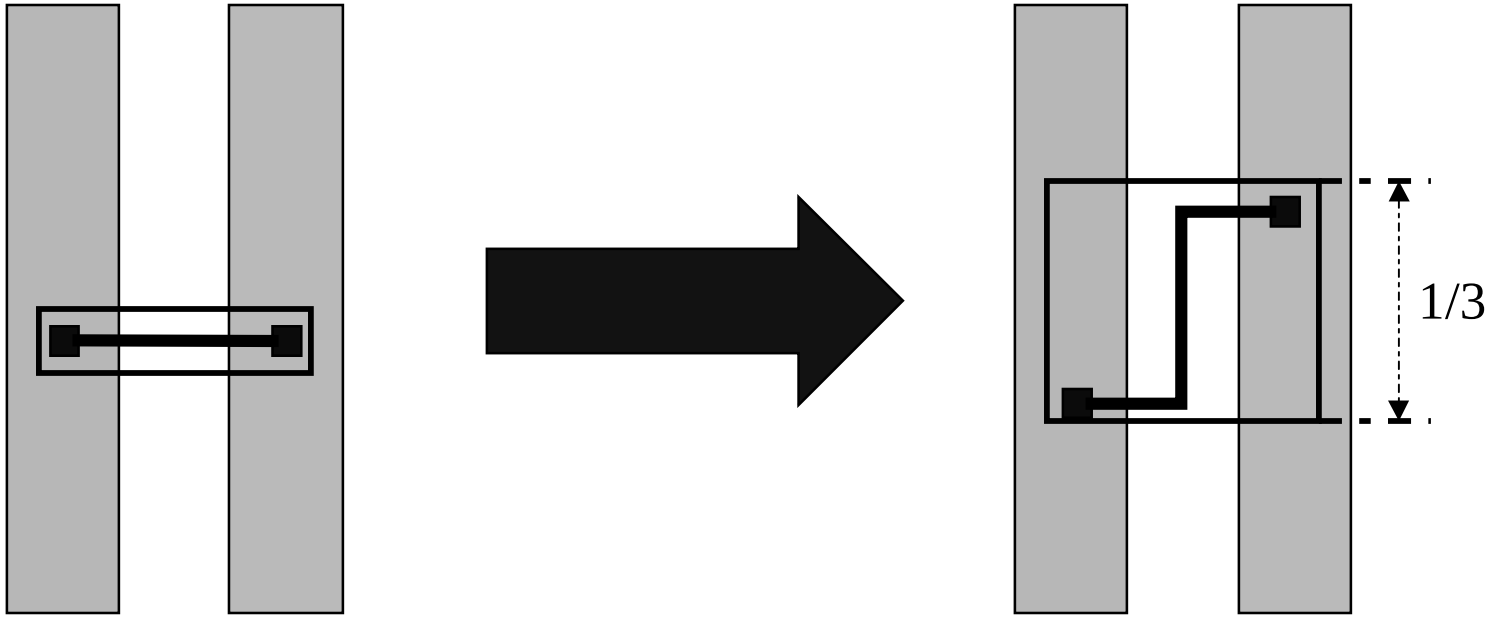
Inaccuracy of Center-to-Center Bbox Estimator



Expected bbox for 2-terminal net is ***underestimated*** by center-to-center method

Expected Bounding Box

Given N rectangles R_i each with n_i random points, find expected bbox of entire point set

- Treat horizontal and vertical dimensions separately
- Find expected minimum and maximum in each dimension
- Given n segments $[a_i, b_i]$ each with one random point, find expected minimum of all points
- Exact formula ($B = \min b_i$, $A = a_1 \geq ... \geq a_n$)

$$E = A + \int_A^B (1 - \frac{t - a_1}{b_1 - a_1}) dt - \sum_{i=2}^{n} \int_{a_i}^{B} \frac{t - a_i}{b_i - a_i} \prod_{j=1}^{i-1} (1 - \frac{t - a_j}{b_j - a_j}) dt$$

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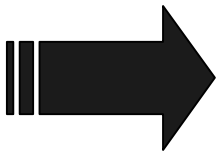

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Computing Expected BBox

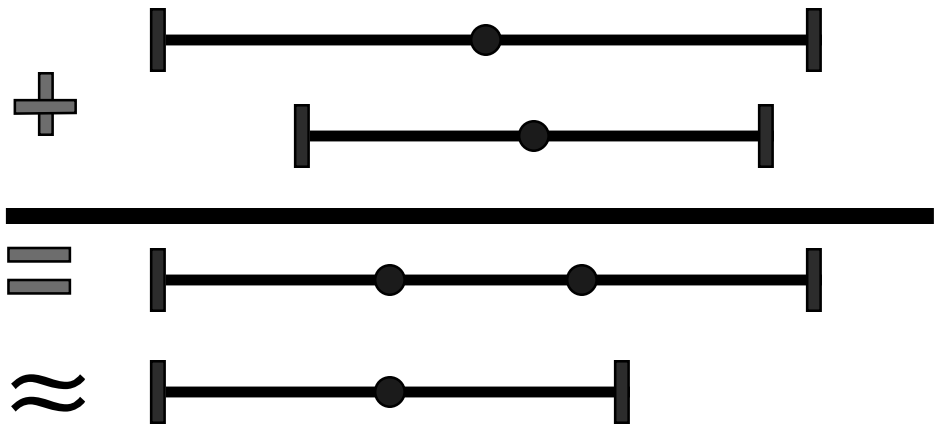
- Exact formula = $O(n^2)$ algorithm

$$E = A + \int_A^B (1 - \frac{t - a_1}{b_1 - a_1})dt - \sum_{i=2}^{n} \int_{a_i}^{B} \frac{t - a_i}{b_i - a_i} \prod_{j=1}^{i-1} (1 - \frac{t - a_j}{b_j - a_j})dt$$

  - multiply polynomials and symbolic integrate result, *n* times

Computing Expected BBox

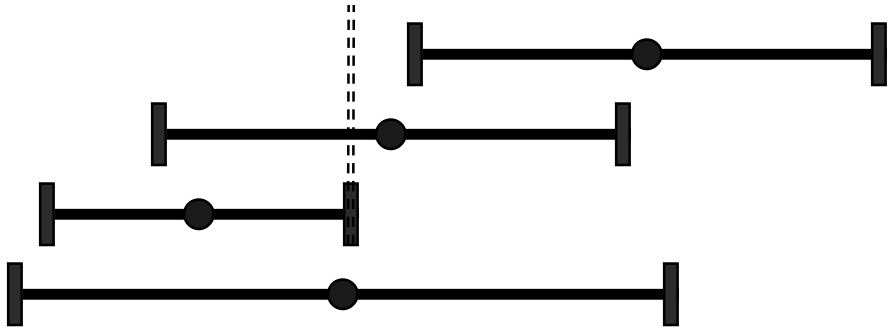
- Exact formula = $O(n^2)$ algorithm
- Very fast $O(n)$ heuristic:
  - replace exact distribution of two random points with linear distribution of one random point



  - Worst-case relative error < 100%, average < 1%

Computing Expected BBox

- Exact formula = $O(n^2)$ algorithm
- Very fast $O(n)$ heuristic
- More accurate $O(n \log n)$ heuristic:
  - sort by left endpoints $a_1 \geq \ldots \geq a_n$
  - omit all segments with $a_i > B = \min b_i$
  - run $O(n)$ fast heuristic on segments (in sorted order)



  - Worst-case relative error < 5.15%, average < .5%

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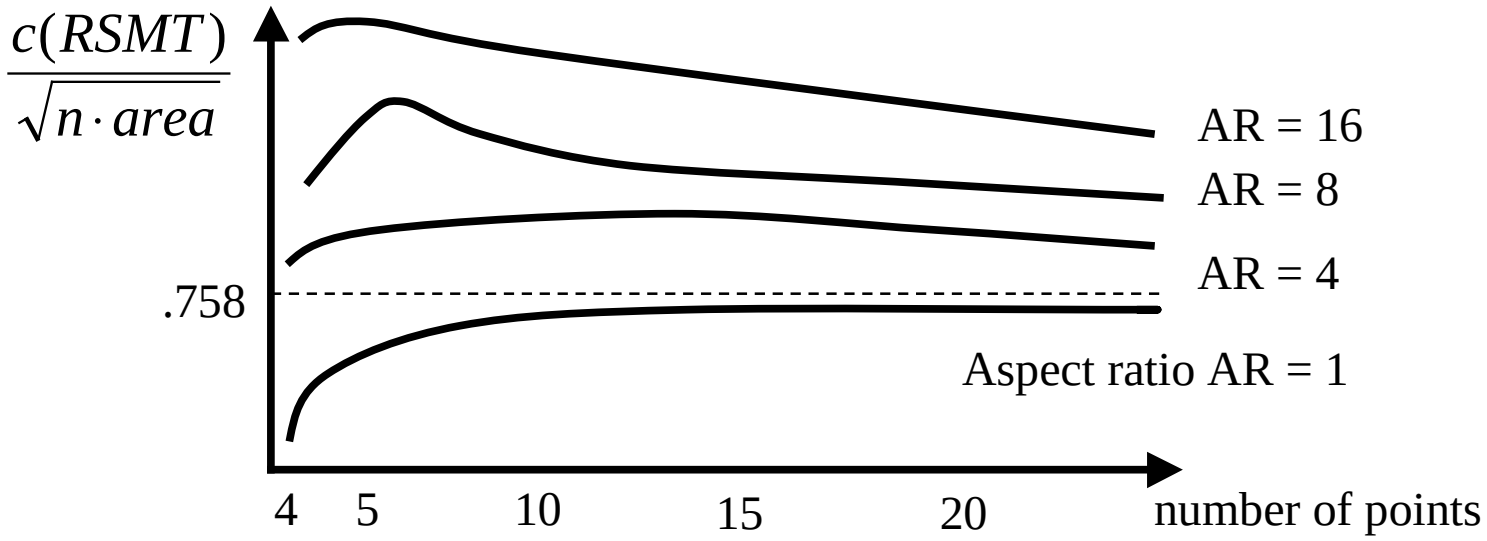
RSMT for n Points in Region

- For *n* random points chosen uniformly in rectangular region R
  - **Old result** (1958): $E[c(RSMT)] \propto \sqrt{area(R) \cdot n}$
  - **Theorem** For sufficiently large aspect ratio *w(R)/h(R)*

    $E[c(RSMT)] \propto HP(bbox)$
- Old result true for $n > N$, where $N = N(AR)$ depends on aspect ratio

RSMT Growth Rate



Growth rates of $c(RSMT)$ in rectangles
with different aspect ratios

How Well Bbox predicts RSMT Cost

- Maximum relative deviation (%) from $E[c(RSMT)]$
  - for 90% of all cases
  - n points within square region
  - n points with given square bounding box

# points	4	5	6	8	10	20	30
dev(region)	**40.6**	**34.8**	**31.0**	**25.1**	**20.6**	**13.0**	**10.0**
dev(bbox)	**11.2**	**12.5**	**14.5**	**14.1**	**13.5**	**10.3**	**8.6**

- **Conclusion:** On average, knowledge of bbox
  - halves maximum relative deviation
  - doubles confidence in expected WL

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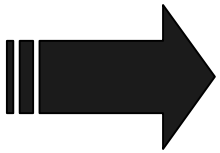

AR-Dependent Scaling

- 10000 random *n*-point samples with prescribed *AR*(*bbox*)

TABLE		NUMBER OF POINTS						
		4	5	6	8	10	20	30
AR	1*	1.08	1.15	1.22	1.34	1.45	1.89	2.23
	1	1.06	1.13	1.19	1.32	1.42	1.87	2.22
	2	1.05	1.11	1.16	1.27	1.36	1.78	2.10
	4	1.03	1.07	1.11	1.18	1.25	1.57	1.84
	10	1.01	1.03	1.05	1.12	1.12	1.29	1.45

- TABLE entries: average values of *cost*(*RSMT*)/HP(*bbox*)
- *E*[*cost*(*RSMT*)] = TABLE(*AR*(*bbox*), *n*) • HP(*bbox*)

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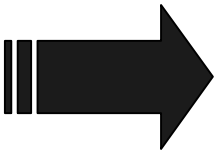

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Practical On-Line BBox Estimation

- Old and new bbox estimators in top-down partitioning
  - CBB: Standard center-to-center bbox
  - HBB: Our new heuristic for expected bbox
- Improving on the uniform distribution assumption
  - HBB0: HBB with 0-lengths for nets in one region
  - HBB6: HBB with dimensions of regions scaled by 1/6

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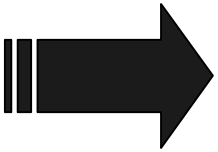

Practical On-Line WL Estimation

- Cheng's and new WL estimators in top-down partitioning:
  - Cheng: CBB scaled by Cheng coefficients
  - CBBtab: CBB scaled with TABLE coefficients
  - HBBtab: HBB scaled with TABLE coefficients
  - HBB0tab: HBB0 scaled with TABLE coefficients
  - HBB6tab: HBB6 scaled with TABLE coefficients

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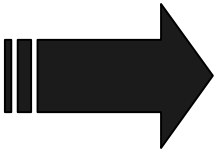

Results

Relative errors of estimated sums of HP(bbox) during top-down placement

Data for 5 test cases are normalized and averaged

	% OF LEVELS COMPLETED									
ESTIMATOR	10	20	30	40	50	60	70	80	90	100
CBB	.243	.185	.138	.110	.086	.059	.031	.011	.001	.000
HBB	2.36	1.59	.911	.511	.298	.169	.071	.029	.002	.000
HBB0	**.109**	**.071**	**.038**	.032	**.020**	**.016**	**.011**	.008	**.000**	**.000**
HBB6	.191	.110	.050	**.025**	.025	.022	.015	**.006**	.001	.000

Results

Relative errors of estimated sums of RSMT costs during top-down placement

Data for 5 test cases are normalized and averaged

	% OF LEVELS COMPLETED									
ESTIMATOR	10	20	30	40	50	60	70	80	90	100
Cheng	.208	.139	.085	.051	.038	.027	.039	.057	.065	**.065**
CBBtab	.259	.188	.132	.104	.080	.057	.032	.016	.010	**.010**
HBBtab	2.37	1.61	.920	.509	.293	.164	.066	.025	.010	.010
HBB0tab	.140	.097	.072	.047	**.026**	**.020**	**.011**	**.008**	**.010**	**.010**
HBB6tab	**.120**	**.069**	**.061**	**.045**	.046	.040	.032	.018	.010	.010

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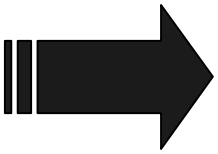

Conclusions and Future Work

- New WL estimation techniques for top-down floorplanning and placement
- Theoretical results:
  - Fast and accurate expected bbox estimation
  - AR(bbox) - influence on WL
- Improved accuracy of bbox and WL estimations for top-down placement context
- Experimental validation with industry test cases
- Recent work: incorporate new on-line estimators within move-based placement tools