Faster SAT and Smaller BDDs via Common Function Structure

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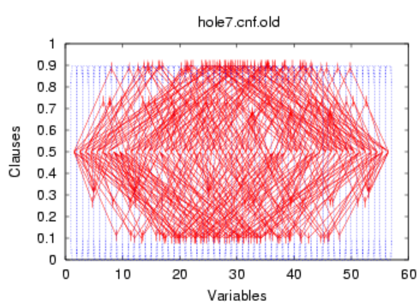
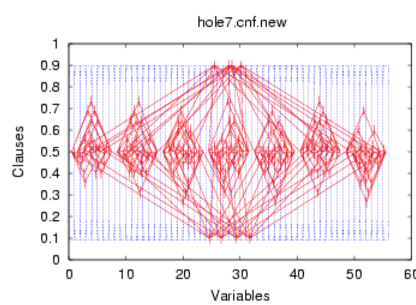
Motivation

Hole-7 Instance
(clauses in red)





Original Variable Order

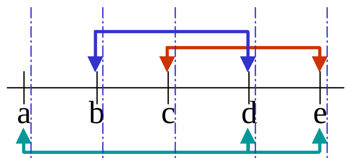
"New" Variable Order

Outline

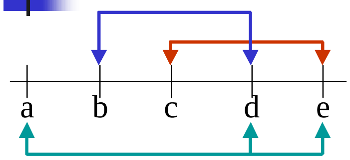
- Hypergraph Terminology
- Motivating Example
- Multilevel Partitioning
- MINCE Algorithm
- Experimental Results
- Conclusions

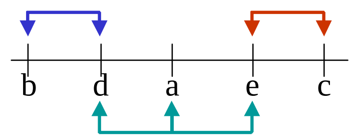
Linearly-Ordered Hypergraphs

- Given a hypergraph with V vertices and E hyperedges with a linear vertex order…
  - Span of hyperedge: difference between the greatest and smallest vertices connected by the same hyperedge
  - i-th cut: number of edges crossing vertex i+0.5
  - Cutwidth: maximum cut of all vertices i, i ∈ (0,..,n-1)
  - An objective of vertex ordering: identify a linear vertex order that minimizes the span and cutwidth of the instance



Bad vs. Good Vertex Orderings





Total Span = 8 Cutwidth = 3 Total Span = 4 Cutwidth = 1

How does vertex reordering help?

Converting CNF Formulas to Hypergraphs:

- Variables $\Rightarrow$ Vertices
- Clauses $\Rightarrow$ Hyperedges

$$f(a,b,c,d,e) = (a + d + e) \wedge (b + d) \wedge (c + e)$$

Related Work

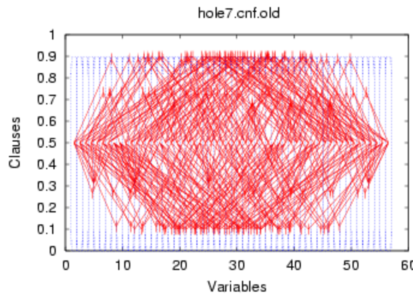
- Circuits with small cutwidth are theoretically "easy" for SAT [Prasad et al. 99]
- Sizes of BDDs are correlated with circuit cutwidth [Berman 91, McMillan 92]
- Extracted BDD variable orderings from linear spectral hypergraph placement [Wood et al. 98]

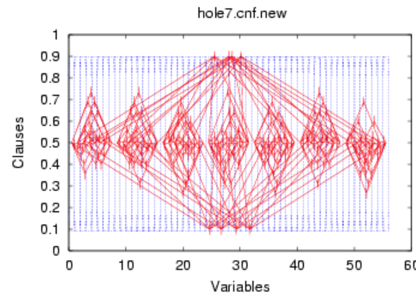
- This work considers average cutwidth instead of maximum cutwidth

Example

Hole-7 Instance

(clauses in red)

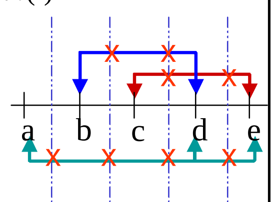




Original Variable Order | MINCE Variable Order

Observation: Crossing Minimization

$$TotalSpan = \sum_{e \in E} span(e) = \sum_{e \in E} \sum_{x \in e} 1 = \boxed{\#\,xings} = \sum_{c \in C} \sum_{x \in c} 1 = \sum_{i=0}^{V-1} cut(i)$$



$$AverageSpan = \frac{\sum_{e \in E} span(e)}{E}$$

$$AverageCut = \frac{\sum_{i=0}^{v-1} cut(i)}{V-1} = \frac{E}{V-1} \bullet \frac{\sum_{e \in E} span(e)}{E} = \frac{\overbrace{E}^{C}}{V} \bullet \overbrace{\frac{V}{V-1}}^{\approx 1} \bullet AverageSpan$$

$$AverageCut \approx \frac{C}{V} \bullet AverageSpan \qquad Min.\,AverageCut \leftrightarrow Min.\,AverageSpan$$

Known from VLSI placement:

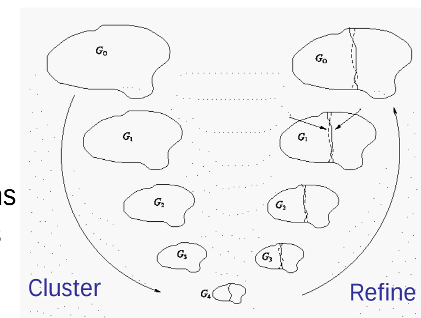
Recursive Min-cut Bisection $\Rightarrow$ Min. Total Net Length in LinPlacement

Linear Placement

- Net length objective (aka "bounding box")
  - For CNF instances, translates into $\sum$ clause span
- 30+ years of placement research
  - Recursive bisection a leading method
  - Applied to SAT in this work
- CAPO: Effecient hypergraph placement software
  - Caldwell, Kahng and Markov [DAC 00]
  - Based on Recursive Min-cut Bisection
  - Multilevel Fiduccia-Mattheyses (FM)
  - Open-source, free: http://vlsicad.cs.ucla.edu/software/PDtools
  - Runs in: $\Theta(N \log^2 N)$, N is size of input

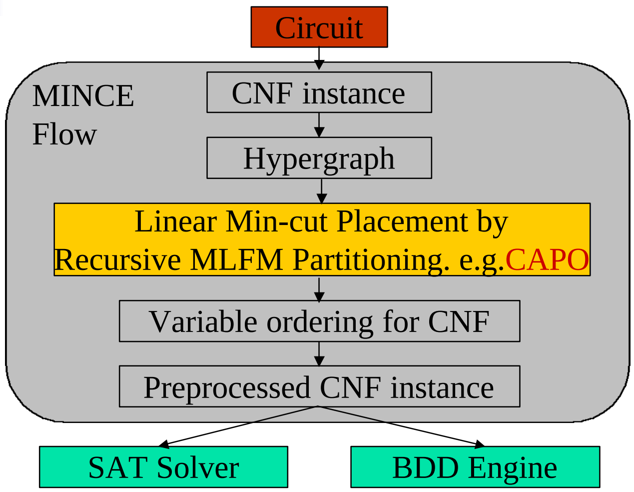
Min-Cut MLFM Partitioning

- MLPart: Efficient min-cut hypergraph partitioner
  - Caldwell, Kahng and Markov [ASPDAC 00]
  - Outperforms hMetis (Karypis et al. [DAC 97])
  - Runs in: $\Theta(N \log N)$
  - Called by CAPO
- Basic Idea:
  - Group original variables
  - Induce clustered hypergraphs
  - Partition clustered hypergraphs
  - Refine partitioned hypegraphs
  - Partition & refinement by Fiduccia-Mattheyses



*By G. Karypis, R. Aggarwal, V. Kumar and S. Shekhar

MINCE - Flow Diagram

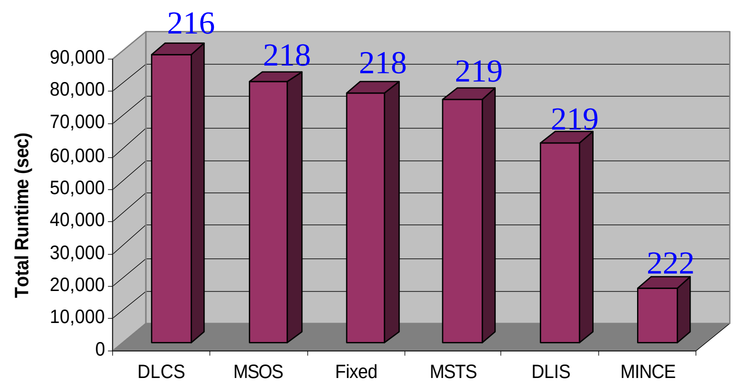


Experimental Setup

- SAT engine: GRASP SAT Solver
- BDD engine: CUDD Package
- Time-out limit: 10,000 seconds
- Memory limit: 500 Mb
- Platform: 333 MHz Pentium II with Linux
- Benchmarks: DIMACS, N-Queens, ISCAS89

SAT Results

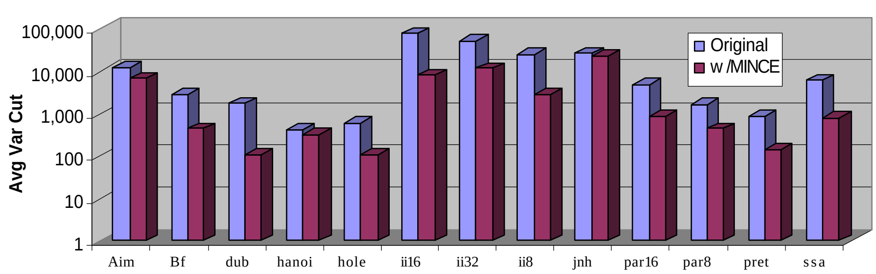
DIMACS Benchmarks*



*Except f, g, par32

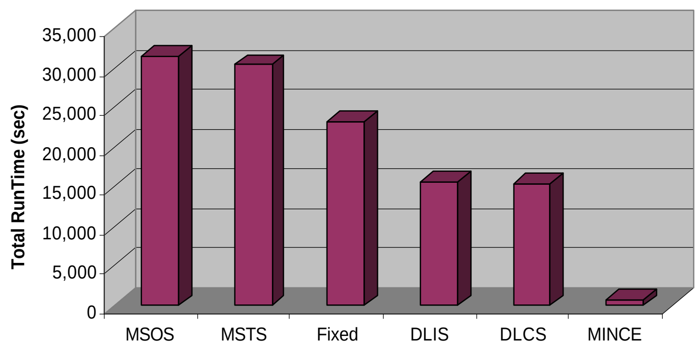
SAT Results

Selected DIMACS Instances



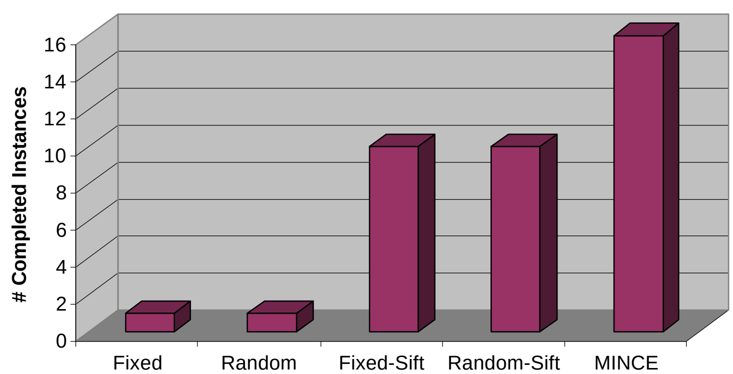
SAT Results

Selected NQueens Instances



BDD Results

ISCAS 89 Benchmarks



Best- vs. Worst-case Performance

- SAT/BDD
  - Worst-case: exp. Best-case: $\Theta(N)$
- Recursive min-cut bisection placement
  - Worst-case: $\Theta(N \log^2 N)$ Best-case: $\Theta(N \log^2 N)$
- Very easy problem instances
  - DLL/BDD run in near-linear time
  - Vertex ordering only slows DLL/BDD
  - MINCE is not helpful for easy instances

Conclusions

- MINCE is useful in capturing the structural properties of CNF instances
- MINCE ordering is very effective in reducing SAT runtime time and BDD runtime/memory requirements
- The ordering is easily generated in a preprocessing step
- No source code modification needed
- Tools are publicly available!

Future Work

- Dramatic speedup improvements possible
- Further improving the MINCE algorithm
- Accounting for polarities of literals in hypergraphs
- Applying the ordering to symbolic simulation
- Tracking empirical correlation between problem complexity and its cutwidth

- Check out MINCE @:
  http://andante.eecs.umich.edu/mince