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ON THE APPLICATION OF FACTS CONTROLLERS DERIVED FROM LOSSLESS MODELS TO LOSSY SYSTEMS

J. F. Gronquist
Dept. of Elec. and Comp. Engineering
University of Wisconsin-Madison
1415 Johnson Drive, Madison, Wisconsin, USA 53706
jim@uludag.ece.wisc.edu

I. A. Hiskens
Dept. of Elec. and Comp. Engineering
University of Newcastle
Callaghan, NSW, 2308,Australia
ianh@no-idea.newcastle.edu.au

Abstract: This paper studies the effects of applying controls for FACTS (Flexible AC Transmission System) devices derived from energy functions for lossless systems to systems with losses. The intent is to examine whether such controls can be effective, and under what circumstances they could produce a destabilizing effect.
Keywords: Electric Power Systems, FACTS Control, Energy Functions

1 Introduction

FACTS devices can be used to influence power flows, to support voltages, to increase stability margins, and to damp system oscillations. Due to their fast response times, it is imperative that they behave correctly. Otherwise they could quickly initiate system instability. Such instabilities can lead to power system outages, loss of generation, or unacceptable voltage fluctuations.

Controls for FACTS devices derived from energy functions for lossless power system models have been proposed in [2]. The advantages of these controllers are that their form is independent of the structure of the system (thus structural uncertainty is largely negated as an issue of concern), and they rely upon only local information (data measurable directly at the location of the controller). Also, as they are derived from a nonlinear system description, they should tend to have larger regions of validity than controllers derived from linearized models. The disadvantage is that the derivation relies upon an energy function analysis. For systems with transmission losses, or impedance load, such an energy function has not yet been found. In fact, it has even been argued that such a function may not exist [3]. It is therefore unclear just what sorts of effects may arise when such controllers are incorporated into lossy power system models. This paper studies the effects that can be expected when controllers for FACTS devices which were based upon lossless system models are applied to lossy systems.

2 Analytic Framework

The system dynamic behavior is described by:

$$\begin{aligned}\dot{\delta} &= \omega \\ \dot{\omega} &= H^{-1}F(\delta,u) \\ &= H^{-1}\left(\hat{F}(\delta)+\tilde{F}(\delta,u)\right)\end{aligned}$$

where:

$\delta,\omega \in \mathcal{R}^N$, δ is the generator rotor angle vector

$F \in \mathcal{R}^N$ is the vector of power balance equations

$u \in \mathcal{R}^p$ is a control parameter vector

$\tilde{F}(\delta,u)=0$ for $u=0$

H is the generator inertia matrix

If no infinite bus is assumed to be present in the system, the function $F(\delta,u)$ has translational symmetry [1], that is $F(\delta,u)=F(\delta+v,u)$ for all $v=C[1,1,...1]^t$, with C being some scalar. We can therefore define an equilibrium manifold for this system as the set of all pairs (δ^*,ω^*) such that $\hat{F}(\delta^*)=0$, $\omega_1^*=\omega_2^*=...=\omega_N^*=\omega_0$, where ω_0 is some constant. In the absence of control ($u=0\ \forall\ t$), the linearization of this system is:

$$\begin{bmatrix}\Delta\dot{\delta}\\ \Delta\dot{\omega}\end{bmatrix} = \begin{bmatrix}0 & I\\ H^{-1}A & 0\end{bmatrix}\begin{bmatrix}\Delta\delta\\ \Delta\omega\end{bmatrix}$$

where $A=\hat{F}_\delta(\delta^*)$. For lossless systems, A is symmetric. For systems with small losses, A is generically symmetrizable. It is shown in [1] that such a system is strongly stable iff $H^{-1}A$ has exactly one zero eigenvalue, and all the others are distinct and negative. If an infinite bus is present, the system is strongly stable iff $H^{-1}A$ has only distinct, negative eigenvalues.

A local Lyapunov function can be defined by:

$$\mathcal{V}=\frac{1}{2}\begin{bmatrix}\Delta\delta^t & \Delta\omega^t\end{bmatrix}\begin{bmatrix}P_{11} & 0\\ 0 & P_{22}\end{bmatrix}\begin{bmatrix}\Delta\delta\\ \Delta\omega\end{bmatrix} \tag{1}$$

where $P_{11}=-H^{\frac{1}{2}}W^tWH^{-\frac{1}{2}}A$, $P_{22}=H^{\frac{1}{2}}W^tWH^{\frac{1}{2}}$, and W is a matrix of left eigenvectors of $H^{-\frac{1}{2}}AH^{-\frac{1}{2}}$, that is $WH^{-\frac{1}{2}}AH^{-\frac{1}{2}}W^{-1}=-\Gamma^2$, with Γ diagonal, and $\Gamma\geq 0$ ($\Gamma>0$ for infinite bus case). It is easily verified that P_{11} is positive semi-definite, symmetric, and P_{22} is positive definite symmetric. For the lossless system, where $A=A^t$, W can be chosen such that $W^tW=I$. Equation (1) is then the local form of the Lyapunov function of [2].

The time derivative of $\mathcal{V}$ is given by:

$$\begin{aligned}\dot{\mathcal{V}} &= \begin{bmatrix}\Delta\dot{\delta}^t & \Delta\dot{\omega}^t\end{bmatrix}\begin{bmatrix}P_{11} & 0\\ 0 & P_{22}\end{bmatrix}\begin{bmatrix}\Delta\delta\\ \Delta\omega\end{bmatrix}\\ &= \left[\Delta\omega^t\right]\left[H^{\frac{1}{2}}W^tWH^{-\frac{1}{2}}\right]\left[H\Delta\dot{\omega}-A\Delta\delta\right]\end{aligned}$$

We can express $F(\delta,u)$ as a Taylor series expansion

$$F(\delta,u)=A\Delta\delta+\tilde{F}_u(\delta^*,0)\Delta u+F_2(\delta,\delta^*,u)$$

Where $F_2(\delta, \delta^*, u)$ captures the higher order terms. We have used the fact that $\tilde{F}(\delta, 0) = 0 \Rightarrow \tilde{F}_\delta(\delta^*, 0) = 0$. This yields that to linear approximation (dropping the 'Δ' notation for simplicity),

$$\dot{\mathcal{V}} = \left[\omega^t\right]\left[H^{\frac{1}{2}} W^t W H^{-\frac{1}{2}}\right]\left[\tilde{F}_u(\delta^*, 0)u\right] = \omega^t Q u \quad (2)$$

Therefore, to a linear approximation, $\dot{\mathcal{V}} = 0$ in the absence of control.

The control design objective is to force $\dot{\mathcal{V}} \leq 0$ by choice of the control u. To find this u we must examine just how the function $\tilde{F}(\delta, u)$ depends on u. For the reduced network model, F_k is given by:

$$\begin{aligned} F_k(\delta, u) &= P_k - \sum_{m=1, m\neq k}^{N} b_{km} E_k E_m \sin(\delta_k - \delta_m) \\ &\quad + g_{km} E_k \left(E_k - E_m \cos(\delta_k - \delta_m)\right) \end{aligned}$$

where g_{km}, b_{km} are functions of the control u.

The dependence of g_{km} and b_{km} on u is found by considering the transmission line to be composed of an inductor, a resistor, and a Thyristor Controller Series Capacitor (TCSC) connected in series. Let R be the line resistance, and X be fixed portion of the line reactance, where X possibly contains the contribution of fixed compensation and the TCSC set point. Let $\tilde{X}$ denote the modulation of the transmission line reactance due to the TCSC. Then the net impedance of the line is $R + j(X + \tilde{X})$. The net admittance of the line is:

$$\begin{aligned} g - jb &= \frac{1}{R + j(X + \tilde{X})} \\ &= (\hat{g} + \tilde{g}) - j(\hat{b} + \tilde{b}) \end{aligned}$$

where

$$\hat{g} = \frac{R}{R^2 + X^2} \qquad \hat{b} = \frac{X}{R^2 + X^2}$$

$$\tilde{g} = \left[\frac{-R}{R^2 + X^2}\right]\left[\frac{\tilde{X}(\tilde{X} + 2X)}{(\tilde{X} + X)^2 + R^2}\right]$$

$$\tilde{b} = \left[\frac{-X}{R^2 + X^2}\right]\left[\frac{\tilde{X}^2 + \left(\frac{X^2 - R^2}{X}\right)\tilde{X}}{(\tilde{X} + X)^2 + R^2}\right]$$

So for small $\tilde{X}$,

$$\tilde{g} \approx \left[\frac{-2RX}{(R^2 + X^2)^2}\right]\tilde{X} = -2\hat{g}\hat{b}\tilde{X}$$

$$\tilde{b} \approx \left[\frac{R^2 - X^2}{(R^2 + X^2)^2}\right]\tilde{X} = \left(\hat{g}^2 - \hat{b}^2\right)\tilde{X}$$

3 Dependence of the control on remote information

For each compensator j, $1 \leq j \leq p$, let u_j correspond to a compensator placed in the branch between buses k_j and m_j, $k_j < m_j$. Then column j of $\tilde{F}_u$ contains entries only in rows k_j and m_j. If the system is lossless, $Q = \tilde{F}_u$, $\frac{\partial \tilde{F}_{k_j}}{\partial u_j} = -\frac{\partial \tilde{F}_{m_j}}{\partial u_j}$, and

$$\dot{\mathcal{V}} = \omega^t \tilde{F}_u u = \sum_j \frac{\partial \tilde{F}_{k_j}}{\partial u_j}\left(\omega_{k_j} - \omega_{m_j}\right) u_j$$

implying u_j need only depend on ω_{k_j} and ω_{m_j} in order to ensure $\dot{\mathcal{V}} \leq 0$. In other words, a device in the (k_j, m_j) branch need only use local information for a control input in order to ensure system stability.

For a lossy system this will not, in general, be the case. Here $\hat{F}_\delta$ is not symmetric, so $W^t W \neq I$, implying

$$\dot{\mathcal{V}} = \omega^t Q u = \sum_i \sum_j \omega_i Q_{ij} u_j$$

In order to guarantee $\dot{\mathcal{V}} \leq 0$, u_j must rely on quantities other than those associated with buses k_j and m_j. To illustrate this, consider the system of figure 1. System data is given in table 1.

A controller is assumed in the $(1,3)$ branch, and the resistance of the $(1,2)$ line varied. As R_{12} is varied, the equilibrium δ is recalculated, yielding a new Q matrix in the equation $\dot{\mathcal{V}} = \omega^t Q u$. note: P is held fixed, so the infinite bus absorbs any mismatch due to losses. Here $Q = [Q_1 Q_2]^t$. The ratio Q_2/Q_1 as a function of the line $(1,2)$ R/X ratio is plotted in figure 2. We see that

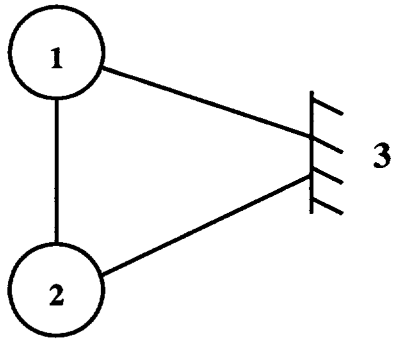

Fig. 1: *2 Generator, Infinite Bus System*

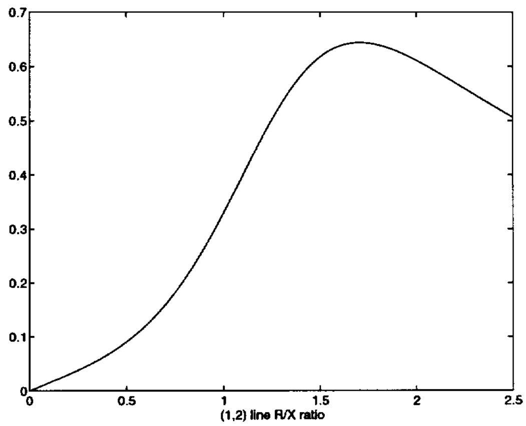

Fig. 2: *Ratio of Q_2 to Q_1 for control in $(1,3)$ branch*

this ratio achieves quite high values, indicating a substantial impact of ω_2 on the value of $\dot{\mathcal{V}}$. A controller designed based on the lossless system will depend only on ω_1, so large Q_2/Q_1 values indicate a large region of the (ω_1, ω_2) plane in which $\dot{\mathcal{V}}$ may be greater than 0, which is a very undesirable condition.

4 Impact of losses on system stability

The discussion of the previous section motivates the question of when do controllers based upon lossless system assumptions in fact stabilize lossy systems? To examine this, consider the same system used previously, and three different control designs.

1. Control design based entirely on the lossless system model. For $u = -GQ^t\omega$, Q is calculated from the lossless system, and G is chosen such that $\max_i \Re(\lambda_i)$, i.e. the maximum of the real parts of the closed loop eigenvalues, is minimized for the lossless system.

2. Calculate Q from the lossless system model, but choose G such the $\max_i \Re(\lambda_i)$ is minimized for the lossy system.

3. Calculate Q from the lossy system, and choose G such that $\max_i \Re(\lambda_i)$ is minimized for the lossy system.

Note that controllers 1 and 2 rely only on local measurements whilst controller 3 relies on remote measurement.

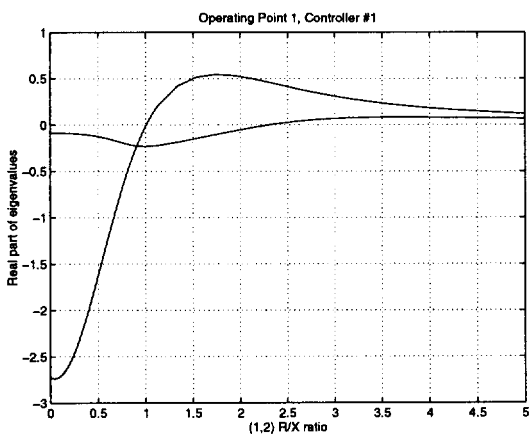

Fig. 3: *Controller 1, Real part of compensated system eigenvalues*

A controller is placed in the (1,2) branch, and the (1,2) branch line resistance varied. The three controllers are designed, with results plotted in figures (3) through (8).

The controller 1 system becomes unstable for an R/X ratio of approximately 1 (figure (3)). This is also the point where the controller 2 and controller 3 systems have $\max_i \Re(\lambda_i) \approx 0$.

It can be seen in figure 4 that the controller 2 systems have $\max_i \Re(\lambda_i) \approx 0$ over a range of R/X values.

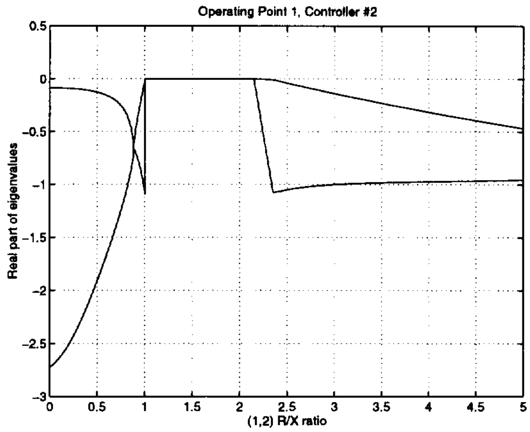

Fig. 4: *Controller 2, Real part of compensated system eigenvalues*

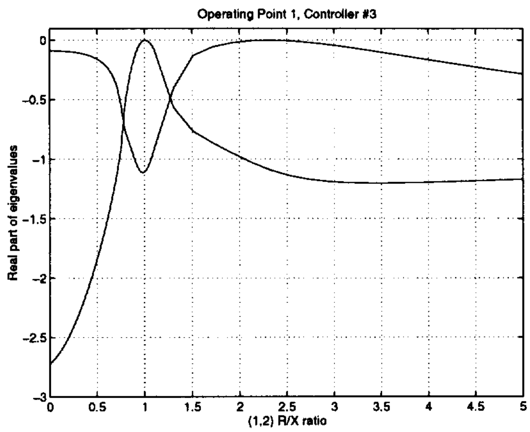

Fig. 5: *Controller 3, Real part of compensated system eigenvalues*

This behaviour can be explained with the help of root loci showing the variation of eigenvalues with controller gain.

Figure 8 shows root loci for the case where Q corresponds to the lossless system, i.e., controllers 1 and 2, for R/X ratios of 0.910 and 1.001. Consider the root loci for $R/X = 0.910$. The dotted lines show the general direction of eigenvalue movement as gain increases. The eigenvalues move toward the left as gain increases away from zero. (For zero gain the system is uncontrolled, and the eigenvalues lie on the imaginary axis.) Therefore a positive gain ensures that both eigenvalues are stable. Figure 6 confirms that.

Now consider the root loci for $R/X = 1.001$. The root locus for the eigenvalue near 5.9 rad/sec has shrunk to a point. Therefore, independent of gain, this eigenvalue lies on the imaginary axis. Because this point is independent of gain, it is independent of any input u, and so is also independent of Q. Hence all three controllers have a zero eigenvalue for this value of R/X, as indicated in Figures 3 to 5.

As the R/X ratio is increased away from 1.001, the root locus expands out from the point. However the orientation is reversed, i.e., as gain increases the eigenvalue moves to the right. Recall though that the lower

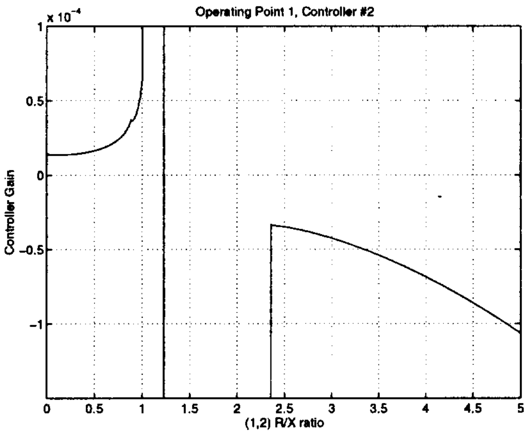

Fig. 6: *Controller 2 Gain*

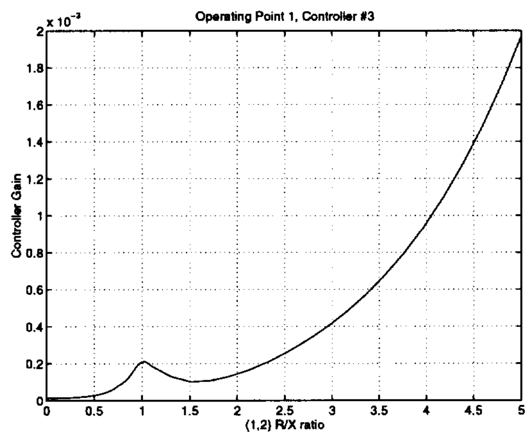

Fig. 7: *Controller 3 Gain*

eigenvalue moves to the left as gain increases. Therefore to minimize $\max_i \Re(\lambda_i)$, the gain must be zero.

Interestingly, as the R/X ratio increases further, the root locus of the slower eigenvalue shrinks whilst that of the faster eigenvalue expands. When $R/X \approx 2.4$, the shrinking root locus contracts to a point. Again the eigenvalue becomes independent of the value of gain. Figures 4 to 6 again show an eigenvalue with zero real part. With an increase in the R/X ratio beyond 2.4, the root locus expands out from the point again. But now its orientation is reversed, similar to the previous case. So for R/X ratios beyond 2.4, the root loci for both eigenvalues indicate that an increase in gain drives the eigenvalues to the right. Therefore minimizing $\max_i \Re(\lambda_i)$ is achieved with a negative gain. This is confirmed by Figure 7.

Controller 1 has a fixed positive gain. Therefore when $R/X < 1.001$, the real parts of both eigenvalues should be negative. This is shown in Figure 4. The reversal of one eigenvalue root locus when $1.001 < R/X < 2.4$ reflects as one eigenvalue with positive real part. The reversal of the second eigenvalue root locus for $R/X > 2.4$ leads to the other eigenvalue crossing to the right half plane.

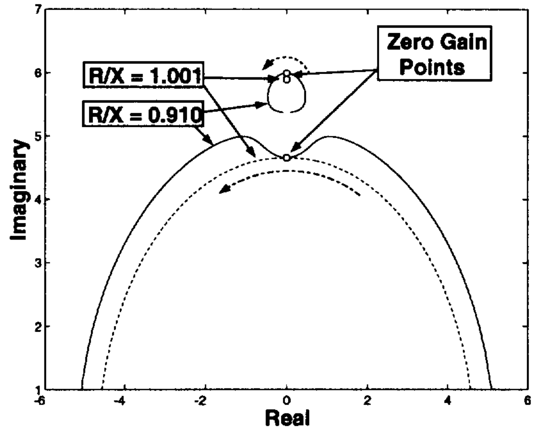

Fig. 8: *Eigenvalue trajectories*

Conclusions

Lyapunov techniques provide a useful way of motivating controls for FACTS devices. However some modelling assumptions underlying these techniques are rather restrictive. In particular, the assumption that systems are lossless is not true of real power systems.

Lyapunov-based control strategies for lossless systems require only local measurements. However losses cause cross-coupling effects, which introduce the need for remote measurements. If losses are small, the use of only local measurements is sufficient. However as losses increase, the remote measurements become more important, and in some cases crucial.

We have shown that FACTS controllers which are motivated by a lossless system Lyapunov function but then applied to a lossy system may encounter difficulties. However such difficulties tend to occur when R/X ratios are well above those which could normally be expected for transmission systems.

Simulations were also performed on larger systems, with results similar to those presented here. The small system used here was chosen to more clearly illustrate the ideas presented in this paper.

Table 1: Test System Parameters, Angles in degrees

$E_1 = 1.038$	$X_{12} = 19.61e-3$
$E_2 = 0.977$	$X_{13} = 11.96e-3$
$E_3 = 0.994$	$X_{23} = 7.96e-3$
$H_1 = 4.107 \quad H_2 = 3.991$	
$\delta_1 = 40$	$P_1 = 73.16$
$\delta_2 = 20$	$P_2 = 24.00$
$\delta_3 = 0$	$P_3 = -97.17$

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