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Efficient Coordination of Electric Vehicle Charging using a Progressive Second Price Auction

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***Abstract*— An auction-based game is formulated for coordinating the charging of a population of electric vehicles (EVs) over a finite horizon. The proposed auction requires individual EVs to submit bid profiles that have dimension equal to two times the number of time-steps in the horizon. They compete for energy allocation at each time-step. Use of the progressive second price (PSP) auction mechanism ensures that incentive compatibility holds for the auction game. However, due to cross-elasticity between the charging time-steps, the marginal valuation of an individual EV at a particular time is determined by both the demand at that time and the total demand over the entire horizon. This difficulty is addressed by partitioning the allowable set of bid profiles according to the total desired energy over the entire horizon. It is shown that the efficient bid profile over the charging horizon is a Nash equilibrium of the underlying auction game. A dynamic update mechanism for the auction game is designed. A numerical example demonstrates that the auction system converges to the efficient Nash equilibrium.**

I. Introduction

It is anticipated that the penetration of electric vehicles (EVs) will substantially increase over the next few years [1]. If such growth does eventuate, it will become necessary to account for EV charging patterns in grid operation. Centralized coordination faces numerous challenges, from computational complexity to the loss of EV decision-making autonomy. Many distributed coordination methods have been proposed to address those difficulties. This paper studies EV charging coordination over multiple time intervals, as formulated in [2], [3], under an incentive compatibility mechanism [4], [5]. In particular, the paper utilizes a progressive second price (PSP) mechanism, designed by Lazar and Semret [6], [7] and initially applied in the allocation of network resources.

In a single divisible resource allocation problem under the PSP auction mechanism, each player only reports a two-dimensional bid profile. This bid profile is composed of a maximum amount of demand and an associated buying price, and is used to replace the player's complete (private) utility function. Under the PSP mechanism, the money transfer (or payment) of a player measures the externality that they impose on the system through their participation. As analyzed in [6], [7], the PSP auction mechanism is a VCG-style auction [8]–[10]. Therefore incentive compatibility holds, ensuring that all players submit truth-telling bid profiles, and indivisible items are allocated efficiently. Under this mechanism, as verified in [7], [11] in the context of single-unit network resource allocations, the efficient bid profile is a Nash equilibrium.

This paper studies EV charging coordination over a multi-period time horizon, where players (EVs) must consider tradeoffs between energy costs that vary over the charging horizon, the benefit derived from the total acquired energy, and battery degradation. Individual EVs are inter-temporal cross-elastic loads, as defined in [12]. As a result, the underlying auction has the form of an auction-based allocation of a collection of divisible resources, with electric energy at each time-step of the horizon being a separate divisible resource. The dimension of each EV's bid profile is double the number of divisible resources to be shared (equivalently double the number of time intervals in the charging horizon). Such auctions have received limited attention in the literature. A key contribution of the paper is to show that the efficient bid profile over the charging horizon is a Nash equilibrium of the underlying auction game. However, due to the inter-temporal cross-elasticity of a bid profile over the multi-step time horizon, it is infeasible to directly verify the Nash equilibrium property of the efficient bid profile using analysis that is applicable for a single-resource auction game [7], [11]. An alternative approach is proposed in this paper.

The paper is organized as follows. Section II establishes the problem structure by formulating a class of EV charging coordination problems over a multiple time-step horizon. Distributed charging problems under the PSP auction mechanism are introduced in Section III. Section IV shows that the efficient (truth-telling) bid profile is a Nash equilibrium of the underlying PSP auction game. A dynamic process is designed in Section V to implement the PSP auction. Section VI provides an example that illustrates the convergence of this process to the efficient Nash equilibrium. Section VII concludes the paper.

II. Coordination of Electric Vehicle Charging

This paper focuses on charging coordination of a population of EVs, $\mathcal{N} \triangleq \{1, \cdots, N\}$, over a finite charging horizon, $\mathcal{T} \triangleq \{0, \cdots, T-1\}$. For each EV, $n \in \mathcal{N}$, the

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energy delivered[1] over the t-th time period is denoted x_{nt}, and the battery state of charge (SoC) evolves according to

$$s_{n,t+1} = s_{nt} + \frac{1}{\Theta_n} x_{nt} \qquad (1)$$

where Θ_n is the battery capacity and s_{nt} is the normalized SoC for the n-th EV at time t. An *admissible charging strategy*, $\boldsymbol{x}_n \equiv (x_{nt}, t \in \mathcal{T})$, satisfies the constraints:

$$x_{nt} \begin{cases} \geq 0, & \text{when } t \in \mathcal{T}_n \\ = 0, & \text{otherwise} \end{cases}, \quad \text{with} \sum_{t=0}^{T-1} x_{nt} \leq \Gamma_n, \qquad (2)$$

where $\mathcal{T}_n \subset \mathcal{T}$ denotes the charging interval of the n-th EV, $\Gamma_n = \Theta_n(s_n^{max} - s_{n0})$ gives the maximum energy that it can receive, and $0 \leq s_{n0} \leq s_n^{max} \leq 1$ gives the (normalized) minimum and maximum SoC, respectively. The values for $\mathcal{T}_n$, Γ_n and s_{n0} follow from the driving style and vehicle battery capacity, see for example [13]. The set of all possible admissible charging strategies is denoted by $\mathcal{X}_n$. Also, define the collection of admissible charging strategies for all EVs by $\boldsymbol{x} \equiv (\boldsymbol{x}_n; n \in \mathcal{N})$, with its corresponding set being $\mathcal{X}$.

The utility function of the n-th EV, for the charging strategy $\boldsymbol{x}_n$, is given by:

$$w_n(\boldsymbol{x}_n) = -\sum_{t=0}^{T-1} f_n(x_{nt}) - \delta_n \Big(\sum_{t=0}^{T-1} x_{nt} - \Gamma_n \Big)^2, \qquad (3)$$

where $\delta_n > 0$ is a fixed parameter, and $f_n(\cdot)$ denotes the battery degradation cost of the n-th EV. This cost is governed by the charging rate x_{nt}, and provides a measure of the cost associated with the decrease in the battery energy capacity due to battery resistance growth [14]. The second term in (3) captures the cost of not fully charging the EV, with δ_n weighting the relative importance of delivering the maximum energy over the charging interval, see [15]. Therefore, $w_n(\boldsymbol{x}_n)$ establishes the tradeoff between the battery degradation cost and the benefit derived from delivering the full charge.

Subject to a collection of admissible charging strategies $\boldsymbol{x}$, the system cost is given by:

$$J_s(\boldsymbol{x}) = \sum_{t=0}^{T-1} c(D_t + \sum_{n=1}^{N} x_{nt}) - \sum_{n=1}^{N} w_n(\boldsymbol{x}_n), \qquad (4)$$

where $c(\cdot)$ denotes the generation cost, D_t the aggregate inelastic background demand at time t, and $D_t + \sum_{n=1}^{N} x_{nt}$ is the total demand at time t.

It is desirable to determine the collection of efficient (socially optimal) charging strategies $\boldsymbol{x}^{**}$ that minimizes the system cost (4). This centralized EV charging coordination problem can be formulated as the following optimization problem.

Optimization Problem 1:

$$\min_{\boldsymbol{x} \in \mathcal{X}} \; J_s(\boldsymbol{x}) \qquad (5)$$

such that $\boldsymbol{x}$ satisfies constraints (2) for all $n \in \mathcal{N}$. ■

The efficient charging strategy $\boldsymbol{x}^{**}$ of Optimization Problem 1 can be characterized by its associated KKT conditions. Firstly, the Lagrangian can be written:

$$L(\boldsymbol{x}, \boldsymbol{\lambda}) = J_s(\boldsymbol{x}) + \sum_{n=1}^{N} \lambda_n \Big(\sum_{t=0}^{T-1} x_{nt} - \Gamma_n \Big),$$

where λ_n is the Lagrangian multiplier associated with the constraint $\sum_{t=0}^{T-1} x_{nt} \leq \Gamma_n$ from (2). The KKT conditions for Optimization Problem 1 are therefore given by:

$$\frac{\partial}{\partial x_{nt}} L(\boldsymbol{x}, \boldsymbol{\lambda}) \geq 0, \quad x_{nt} \geq 0, \qquad \frac{\partial}{\partial x_{nt}} L(\boldsymbol{x}, \boldsymbol{\lambda}) x_{nt} = 0 \qquad (6a)$$

$$\sum_{t=0}^{T-1} x_{nt} - \Gamma_n \leq 0, \quad \lambda_n \geq 0, \quad \lambda_n \Big(\sum_{t=0}^{T-1} x_{nt} - \Gamma_n \Big) = 0, \qquad (6b)$$

for all $t \in \mathcal{T}$ and $n \in \mathcal{N}$, where:

$$\frac{\partial}{\partial x_{nt}} L(\boldsymbol{x}, \boldsymbol{\lambda}) = c'(D_t + \sum_{n=1}^{N} x_{nt}) - \frac{\partial}{\partial x_{nt}} w_n(\boldsymbol{x}_n) + \lambda_n. \qquad (6c)$$

Assumptions: The following conditions will be assumed throughout the remainder of the paper.

(A1) $c(y)$ is monotonically increasing, strictly convex and differentiable on y;

(A2) $f_n(x)$, for all $n \in \mathcal{N}$, is monotonically increasing, strictly convex and differentiable on x.

Remarks: The generation cost $c(\cdot)$ is widely assumed to be a convex function of the total generation, see for example [16]–[18]. The battery degradation cost $f_n(\cdot)$ is governed by the chemical processes inherent in charging. It is shown in Fig. 7 of [14] that the growth of battery resistance, hence the fade of battery energy capacity, is generally increasing and convex with respect to charging rate. This provides some justification for (A2) since f_n measures the cost related to the fade of battery capacity with respect to charging rate.

Lemma 2.1: The collection of efficient charging strategies $\boldsymbol{x}^{**}$ for Optimization Problem 1 is unique.

Proof. Under Assumptions (A1,A2), the cost function $J_s(\boldsymbol{x})$ is strictly convex and differentiable. Also, the constraints (2) determine a convex domain. Therefore Optimization Problem 1 is a strictly convex optimization problem. Thus there exists a unique solution. ■

This *centralized* charging coordination strategy can only be effectively implemented when the system has complete information and can directly schedule the behavior of all EVs. In practice, however, individuals are often unwilling to share their private information with others. Thus the paper focuses on the development of a distributed control method that is based on the *progressive second price (PSP) auction mechanism*.

[1]It is assumed that the energy is delivered at a constant rate (power) over each time interval, and that the time intervals are of unit length. Therefore the charging rate is also given by x_{nt}.

III. Distributed EV Charging Coordination under a PSP Auction Mechanism

A. Bid profiles of individual EVs

Each EV $n \in \mathcal{N}$ submits a $2T$-dimensional bid profile, $\boldsymbol{b}_n \equiv (b_{nt}, t \in \mathcal{T})$, where $b_{nt} = (\beta_{nt}, d_{nt})$, with β_{nt} specifing the price that the n-th EV is willing to pay for energy at time t, and

$$d_{nt} \begin{cases} \geq 0, & \text{when } t \in \mathcal{T}_n \\ = 0, & \text{otherwise} \end{cases}, \text{ and } \sum_{t=0}^{T-1} d_{nt} \leq \Gamma_n, \tag{7}$$

establishing the maximum electrical energy that is desired at that time. The corresponding feasible allocation $\boldsymbol{x}_n \equiv (x_{nt}, t \in \mathcal{T})$ with respect to $\boldsymbol{b}_n$ must satisfy:

$$0 \leq x_{nt} \leq d_{nt}, \tag{8}$$

for all $t \in \mathcal{T}$. Let $\mathcal{B}_n$ denote the allowable set of bids for the n-th EV, so that $\boldsymbol{b}_n \in \mathcal{B}_n$.

Each EV's revealed utility function is defined as:

$$\widehat{w}_n(\boldsymbol{x}_n(\boldsymbol{b}_n); \boldsymbol{b}_n) \triangleq \sum_{t=0}^{T-1} \beta_{nt} \min(x_{nt}, d_{nt}) = \sum_{t=0}^{T-1} \beta_{nt} x_{nt} \tag{9}$$

where the last equality holds by (8). The revealed system cost with respect to a collection of bid profiles $\boldsymbol{b} \equiv (\boldsymbol{b}_n, n \in \mathcal{N})$ is then given by:

$$J(\boldsymbol{x}(\boldsymbol{b}); \boldsymbol{b}) = \sum_{t=0}^{T-1} c(D_t + \sum_{n=1}^{N} x_{nt}) - \sum_{n=1}^{N} \widehat{w}_n(\boldsymbol{x}_n(\boldsymbol{b}_n); \boldsymbol{b}_n). \tag{10}$$

Auction-based EV charging allocation can be written as the following optimization problem.

Optimization Problem 2:

$$J^*(\boldsymbol{b}) = \min_{\boldsymbol{0} \leq \boldsymbol{x} \leq \boldsymbol{d}} J(\boldsymbol{x}(\boldsymbol{b}); \boldsymbol{b}), \tag{11}$$

where the bid profile $\boldsymbol{b}$ satisfies the constraint (7) and $\boldsymbol{x}$ satisfies (8). The objective of the auctioneer is to assign an optimal allocation $\boldsymbol{x}^*(\boldsymbol{b})$ with respect to bid profiles $\boldsymbol{b}$ to minimize the revealed system cost given by J. ■

Lemma 3.1: Suppose $\boldsymbol{x}^*(\boldsymbol{b}) \equiv (x_t^*, t \in \mathcal{T})$ is the optimal allocation subject to bid profile $\boldsymbol{b}$. Then

$$\boldsymbol{x}_t^*(\boldsymbol{b}) \equiv \boldsymbol{x}_t^*(\boldsymbol{b}_t), \qquad \text{for all } t \in \mathcal{T}, \tag{12}$$

i.e., the optimal allocation at time t is completely determined by the bid profile at that time.

The proof follows directly from (9) and (10).

The optimal charging allocation $\boldsymbol{x}^*$ of Optimization Problem 2 can be characterized by the associated KKT conditions. Firstly, the Lagrangian can be written:

$$L^a(\boldsymbol{x}, \boldsymbol{\sigma}; \boldsymbol{b}) = J(\boldsymbol{x}(\boldsymbol{b}); \boldsymbol{b}) + \sum_{n=1}^{N} \sum_{t=0}^{T-1} \sigma_{nt}(x_{nt} - d_{nt}),$$

where σ_{nt} is the Lagrangian multiplier associated with the constraint (8). The KKT conditions for Optimization Problem 2 are given by:

$$\frac{\partial}{\partial x_{nt}} L^a(\boldsymbol{x}, \boldsymbol{\sigma}; \boldsymbol{b}) \geq 0, \ x_{nt} \geq 0, \ \frac{\partial}{\partial x_{nt}} L^a(\boldsymbol{x}, \boldsymbol{\sigma}; \boldsymbol{b}) x_{nt} = 0 \tag{13a}$$

$$x_{nt} - d_{nt} \leq 0, \ \sigma_{nt} \geq 0, \qquad (x_{nt} - d_{nt})\sigma_{nt} = 0, \tag{13b}$$

for all $t \in \mathcal{T}$ and $n \in \mathcal{N}$, where

$$\frac{\partial}{\partial x_{nt}} L^a(\boldsymbol{x}, \boldsymbol{\sigma}; \boldsymbol{b}) = c'(D_t + \sum_{k=1}^{N} x_{kt}) - \beta_{nt} + \sigma_{nt}. \tag{13c}$$

It is now possible to establish a connection between the optimal charging strategies given by the two optimization problems (5) and (11).

Lemma 3.2: Consider a collection of bid profiles,

$$b_{nt}^* = (\beta_{nt}^*, d_{nt}^*) = \left(\frac{\partial}{\partial x_{nt}} w_n(\boldsymbol{x}_n^{**}), x_{nt}^{**} \right), \tag{14}$$

for all $n \in \mathcal{N}$ and $t \in \mathcal{T}$. Then, under Assumptions (A1,A2), $\boldsymbol{x}^*(\boldsymbol{b}^*) = \boldsymbol{x}^{**}$, i.e., the optimal charging allocation $\boldsymbol{x}^*$ of Optimization Problem 2 with respect to $\boldsymbol{b}^*$ is *efficient*. Also,

$$\beta_{nt}^* \begin{cases} = c'(D_t + \sum_{k=1}^{N} d_{kt}^*), & \text{if } x_{nt}^* > 0 \\ \leq c'(D_t + \sum_{k=1}^{N} d_{kt}^*), & \text{if } x_{nt}^* = 0 \end{cases}, \tag{15}$$

for all $n \in \mathcal{N}$ and $t \in \mathcal{T}$, i.e., the EVs with an allocation larger than zero share the same marginal price as the generation, which is larger than or equal to the price of EVs with zero allocation.

Lemma 3.2 is essentially the so-called *fundamental theorem of welfare economics* [19]. Verification uses the KKT conditions (6) and (13), with full details provided in [20].

Incentive compatibility holds under the PSP auction mechanism [6], [7]. Therefore, a bid profile with price satisfying $\beta_{nt} = \frac{\partial}{\partial d_{nt}} w_n(\boldsymbol{d}_n)$, for all $t \in \mathcal{T}$, as is the case in (14), is the best choice among all possible bid profiles. It follows from (3) that the truth-telling bid profile of the n-th EV is given by:

$$\beta_{nt} = -f_n'(d_{nt}) + 2\delta_n \Big(\Gamma_n - \sum_{t=0}^{T-1} d_{nt} \Big). \tag{16}$$

This implies that an EV's marginal valuation at each time-step is determined by both its electrical energy request d_{nt} at that time and its total energy request $\sum_t d_{nt}$ over the entire multi-period charging horizon.

B. Calculation of EV payment and payoff

The payment incurred by each EV will be specified with respect to the allocation law defined by Optimization Problem 2. Each EV's payment is exactly the externality imposed on the system through its participation in the auction. For the n-th EV, this is given by the system-wide utility when it does not join the auction process, minus the system-wide utility (but excluding its own contribution) when it joins the auction.

To express this payment, it is convenient to introduce a slight abuse of notation by writing the collection of bid

$$\tau_n(\boldsymbol{b}) = \sum_{t=0}^{T-1} \underbrace{\left\{ -c(D_t + \sum_{m\neq n} x_{mt}^{*,-n}) + c(D_t + \sum_{m\neq n} x_{mt}^{*} + x_{nt}^{*}) + \sum_{m\neq n} \beta_{mt}\left(x_{mt}^{*,-n} - x_{mt}^{*}\right) \right\}}_{\triangleq\, \tau_{nt}(\boldsymbol{b}_t)} \tag{18}$$

profiles as $\boldsymbol{b} \equiv (\boldsymbol{b}_n, \boldsymbol{b}_{-n})$, where $\boldsymbol{b}_{-n} \equiv (\boldsymbol{b}_k, k \in \mathcal{N} \setminus \{n\})$. The payment of the n-th EV, for a collection of bid profiles $\boldsymbol{b}$, is then given by:

$$\tau_n(\boldsymbol{b}) = -J^*(\boldsymbol{0}_n, \boldsymbol{b}_{-n}) - \left(-J^*(\boldsymbol{b}) - \sum_{t=0}^{T-1} \beta_{nt} x_{nt}^*(\boldsymbol{b}) \right) \tag{17}$$

where $(\boldsymbol{0}_n, \boldsymbol{b}_{-n})$ denotes the bid profile without the n-th EV's participation, i.e., with the bid d_{nt} replaced by $d_{nt} = 0$ for all $t \in \mathcal{T}$, and $\boldsymbol{x}^*(\boldsymbol{b})$ is the optimal charging allocation given by (11), with respect to $\boldsymbol{b}$. Thus by (10), (17) and Lemma 3.1, the payment of the n-th EV can be expressed as (18), where $x_{mt}^{*,-n}$ denotes the optimal charging allocation of EVs $m \neq n$ given by Optimization Problem 2 with respect to $(\boldsymbol{0}_n, \boldsymbol{b}_{-n})_t$. (Recall from Lemma 3.1 that the allocations at time t are unrelated to other times.)

The *payoff function* of the n-th EV is given by the difference between the EV's utility and its payment:

$$u_n(\boldsymbol{b}) = w_n(\boldsymbol{x}_n^*(\boldsymbol{b})) - \tau_n(\boldsymbol{b}). \tag{19}$$

This payoff function provides the basis for defining a Nash equilibrium for the PSP auction game.

Definition 3.1: A collection of bid profiles $\boldsymbol{b}^0$ is a *Nash equilibrium* for Optimization Problem 2 if:

$$u_n(\boldsymbol{b}_n^0, \boldsymbol{b}_{-n}^0) \geq u_n(\boldsymbol{b}_n, \boldsymbol{b}_{-n}^0),$$

for all $\boldsymbol{b}_n \in \mathcal{B}_n$ and for all $n \in \mathcal{N}$. That is, no EV can benefit by unilaterally deviating from its bid profile $\boldsymbol{b}_n^0$. ■

IV. Efficiency of the Charging Coordination PSP Auction Game

Suppose $\boldsymbol{b}^*$ is the bid profile specified in Lemma 3.2, such that the corresponding optimal charging allocation is efficient. It will be shown in this section that $\boldsymbol{b}^*$ is a Nash equilibrium for the underlying auction game. By Definition 3.1, this implies:

$$u_n(\boldsymbol{b}_n^*, \boldsymbol{b}_{-n}^*) \geq u_n(\boldsymbol{b}_n, \boldsymbol{b}_{-n}^*), \quad \text{for all } \boldsymbol{b}_n \in \mathcal{B}_n. \tag{20}$$

Due to the cross-elasticity arising from the second term in (16), it is infeasible to directly verify that the efficient bid profile is best for every individual EV. Accordingly, the approach developed in [11] for auction games of a single divisible resource is not applicable. In order to overcome this difficulty, an alternative approach is required. This involves partitioning the set of bid profiles $\mathcal{B}_n$ into a collection of subsets:

$$\mathcal{B}_n(A) \triangleq \left\{ \boldsymbol{b}_n \in \mathcal{B}_n; \quad \text{s.t. } \sum_{t=0}^{T-1} d_{nt} = A \right\}, \tag{21}$$

each of which is composed of those bid profiles that are admissible for the n-th EV and that possess a common total for the desired charge energy over the charging horizon $\mathcal{T}$. The set of bid profiles is then given by:

$$\mathcal{B}_n = \bigcup_{A \in [0, \Gamma_n]} \mathcal{B}_n(A), \tag{22}$$

noting that $\mathcal{B}_n(\widehat{A}) \bigcap \mathcal{B}_n(\widetilde{A}) = \emptyset$ whenever $\widehat{A} \neq \widetilde{A}$. For all bid profiles in a subset $\mathcal{B}_n(A)$, it follows from (16) that the marginal valuation price β_{nt} at any time t includes a variable part $-f_n'(d_{nt})$ that is dependent upon the request d_{nt} at that time, and a fixed part $2\delta_n(\Gamma_n - A)$ that is identical for all bid profiles in $\mathcal{B}_n(A)$.

By Definition 3.1 and the specification of $\mathcal{B}_n(A)$, it is sufficient to show that $\boldsymbol{b}^*$ is a Nash equilibrium, if for every fixed $A \in [0, \Gamma_n]$:

$$u_n(\boldsymbol{b}_n^*, \boldsymbol{b}_{-n}^*) \geq u_n(\widehat{\boldsymbol{b}}_n, \boldsymbol{b}_{-n}^*), \quad \text{for all } \widehat{\boldsymbol{b}}_n \in \mathcal{B}_n(A), \tag{23}$$

and for all $n \in \mathcal{N}$. This is easier to verify than (20), since the difficulty associated with cross-elasticity is avoided when considering each specific subset $\mathcal{B}_n(A)$ with fixed $A \in [0, \Gamma_n]$. To do so requires the following lemma.

Lemma 4.1: Suppose $\beta_{nt}(d_{nt}; A)$ is the bidding price given by (16) for the n-th EV at time t, but with the summation $\sum_t d_{nt}$ in the second term replaced by A. Then $\beta_{nt}(d_{nt}; A)$ satisfies the properties:

$$\beta_{nt}(d_{nt}^1; A) > \beta_{nt}(d_{nt}^2; A) > 0, \text{ when } d_{nt}^1 < d_{nt}^2, \text{ for all } A, \tag{24a}$$

$$\beta_{nt}(d_{nt}; A_1) > \beta_{nt}(d_{nt}; A_2), \quad \text{when } A_1 < A_2, \text{ for all } d_{nt}. \tag{24b}$$

In other words, $\beta_{nt}(d_{nt}, A)$ decreases with increasing d_{nt} and A.

Proof. By (16), $\beta_{nt}(d_{nt}; A) = 2\delta_n(\Gamma_n - A) - f_n'(d_{nt})$. It is straightforward to verify (24) under Assumption (A2). ■

The desired result (23) will be established by considering the two cases $A \geq \sum_{t=0}^{T-1} d_{nt}^*$ and $0 \leq A < \sum_{t=0}^{T-1} d_{nt}^*$ separately.

A. Verification of (23) when $A \geq \sum_{t=0}^{T-1} d_{nt}^*$

For notational simplicity, let $\boldsymbol{x}^*$ and $\widehat{\boldsymbol{x}}$ denote the optimal allocations with respect to $\boldsymbol{b}^*$ and $(\widehat{\boldsymbol{b}}_n, \boldsymbol{b}_{-n}^*)$ respectively.

Lemma 4.2: If $A \geq \sum_{t=0}^{T-1} d_{nt}^*$ then $\widehat{x}_{mt} = d_{mt}^*$, for all $m \in \mathcal{N} \setminus \{n\}$, i.e., each of the EVs $m \in \mathcal{N} \setminus \{n\}$ is fully allocated.

The proof relies on Lemmas 3.2 and 4.1, and KKT conditions (13). Full details are given in [20].

The main result for this section can now be established.

Theorem 4.1: Under Assumptions (A1,A2), (23) holds when $A \geq \sum_{t=0}^{T-1} d^*_{nt}$.

Proof. Considering the n-th EV, the first step is to compute the difference between the payoff given by the optimal strategy $u_n(\boldsymbol{b}^*)$ and that obtained from an alternative strategy $u_n(\widehat{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n})$. Using (19), these payoffs are given by,

$$u_n(\boldsymbol{b}^*) = w_n(\boldsymbol{d}^*_n) - \tau_n(\boldsymbol{b}^*) \tag{25a}$$

$$u_n(\widehat{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n}) = w_n(\widehat{\boldsymbol{x}}_n) - \tau_n(\widehat{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n}). \tag{25b}$$

Hence, the difference is:

$$\begin{aligned}\Delta u_n &\triangleq u_n(\boldsymbol{b}^*) - u_n(\widehat{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n}) \\ &= w_n(\boldsymbol{d}^*_n) - w_n(\widehat{\boldsymbol{x}}_n) \\ &\quad + \left(J^*(\widehat{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n}) + \sum_{t=0}^{T-1} \widehat{\beta}_{nt}\widehat{x}_{nt} - J^*(\boldsymbol{b}^*) - \sum_{t=0}^{T-1} \beta^*_{nt} d^*_{nt}\right)\end{aligned}$$

where the equality follows from (17). Straightforward analysis using Lemma 4.2 gives:

$$\begin{aligned}\Delta u_n = w_n(\boldsymbol{d}^*_n) - w_n(\widehat{\boldsymbol{x}}_n) + \sum_{t=0}^{T-1} \Big\{ c(D_t + \sum_{m\neq n} d^*_{mt} + \widehat{x}_{nt}) \\ - c(D_t + \sum_{m\neq n} d^*_{mt} + d^*_{nt}) \Big\}.\end{aligned} \tag{26}$$

Also, using Lemma 4.2,

$$\begin{aligned}J_s(\boldsymbol{x}^*) - J_s(\widehat{\boldsymbol{x}}) &= J_s(\boldsymbol{d}^*) - J_s(\widehat{\boldsymbol{x}}_n, \boldsymbol{d}^*_{-n}) \\ &= \sum_{t=0}^{T-1} \Big\{ c(D_t + \sum_{m\neq n} d^*_{mt} + d^*_{nt}) - c(D_t + \sum_{m\neq n} d^*_{mt} + \widehat{x}_{nt}) \Big\} \\ &\qquad - w_n(\boldsymbol{d}^*_n) + w_n(\widehat{\boldsymbol{x}}_n) \\ &= -\Delta u_n.\end{aligned}$$

Since $\boldsymbol{x}^*$ is the efficient allocation, $J_s(\boldsymbol{x}^*) \leq J_s(\widehat{\boldsymbol{x}})$, so

$$\Delta u_n = J_s(\widehat{\boldsymbol{x}}) - J_s(\boldsymbol{x}^*) \geq 0.$$

This implies that the n-th EV cannot benefit by unilaterally changing its bid profile $\boldsymbol{b}^*_n$ to any other bid profile $\widehat{\boldsymbol{b}}_n \in \mathcal{B}_n(A)$ with $A \geq \sum_{t=0}^{T-1} d^*_{nt}$. ∎

B. Verification of (23) when $0 \leq A < \sum_{t=0}^{T-1} d^*_{nt}$

In considering $0 \leq A < \sum_{t=0}^{T-1} d^*_{nt}$, it is convenient to first establish the n-th EV's optimal bid profile in the set $\mathcal{B}_n(A)$ and under the constraint,

$$0 \leq \widehat{d}_{nt} \begin{cases} < d^*_{nt}, & \text{when } d^*_{nt} > 0, \\ = 0, & \text{otherwise,} \end{cases} \quad \text{for all } t \in \mathcal{T}. \tag{27}$$

It will then be shown that the resulting bid profile remains optimal when the constraint (27) is released.

Lemma 4.3: Consider a bid profile $\widehat{\boldsymbol{b}}^*_n \equiv \widehat{\boldsymbol{b}}^*_n(A) \equiv \big((\widehat{\beta}^*_{nt}, \widehat{d}^*_{nt}), t \in \mathcal{T}\big)$, with $A \in [0, \sum_{t=0}^{T-1} d^*_{nt})$, such that

$$\widehat{\boldsymbol{b}}^*_n = \underset{\substack{\widehat{\boldsymbol{b}}_n \in \mathcal{B}_n(A) \\ \text{Constraint (27)}}}{\operatorname{argmax}} \; u_n(\widehat{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n}). \tag{28}$$

Let $\widehat{\boldsymbol{x}}^* \equiv (\widehat{x}^*_{kt}, k \in \mathcal{N}, t \in \mathcal{T})$ denote the optimal allocations with respect to $(\widehat{\boldsymbol{b}}^*_n, \boldsymbol{b}^*_{-n})$. Then,

$$\widehat{\boldsymbol{x}}^*_n = \widehat{\boldsymbol{d}}^*_n, \qquad \widehat{\boldsymbol{x}}^*_m = \boldsymbol{d}^*_m \text{ for all } m \in \mathcal{N} \setminus \{n\}, \tag{29}$$

i.e., all EVs are fully allocated. Furthermore, define the function,

$$g_{nt}(d) \triangleq c(D_t + \sum_{m\neq n} d^*_{mt} + d) + f_n(d). \tag{30}$$

Then, under Assumptions (A1,A2), $\widehat{\boldsymbol{b}}^*_n$ satisfies the property:

$$g'_{nt}(\widehat{d}^*_{nt}) \begin{cases} = \mu, & \text{when } \widehat{d}^*_{nt} > 0 \\ \geq \mu, & \text{when } \widehat{d}^*_{nt} = 0 \end{cases}, \quad \text{for all } t \in \mathcal{T}, \tag{31}$$

where μ is a constant. ∎

Proof. The proof uses Lemmas 3.2 and 4.1, together with the KKT conditions (13), to establish (29). The argument also indicates that $x^{*,-n}_{mt} = x^*_{mt}$ for all $m \in \mathcal{N} \setminus \{n\}$, and so (18) becomes,

$$\tau_n(\widehat{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n}) = \sum_{t=0}^{T-1} \left\{ c(D_t + \sum_{m\neq n} d^*_{mt} + \widehat{d}_{nt}) - c(D_t + \sum_{m\neq n} d^*_{mt}) \right\}. \tag{32}$$

The KKT conditions for (28) then give (31). Full details are provided in [20].

It will now be shown that the bid profile $\widehat{\boldsymbol{b}}^*_n$ established in Lemma 4.3 remains optimal when the constraint (27) is released.

Theorem 4.2: Suppose that $\widehat{\boldsymbol{b}}^*_n$ is the optimal bid profile from Lemma 4.3. Then, under Assumptions (A1,A2) and with $A \in [0, \sum_{t=0}^{T-1} d^*_{nt})$, $\widehat{\boldsymbol{b}}^*_n$ satisfies:

$$\widehat{\boldsymbol{b}}^*_n = \underset{\widehat{\boldsymbol{b}}_n \in \mathcal{B}_n(A)}{\operatorname{argmax}} \; u_n(\widehat{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n}). \tag{33}$$

Proof. Consider a bid profile $\widehat{\boldsymbol{b}}_n \equiv \big((\widehat{\beta}_{nt}, \widehat{d}_{nt}), t \in \mathcal{T}\big) \in \mathcal{B}_n(A)$ such that $\widehat{d}_{nt} \geq d^*_{nt}$ for some $t \in \mathcal{T}$. Then the desired result follows if it can be proven that $\widehat{\boldsymbol{b}}_n$ cannot be the optimal bid profile in the subset $\mathcal{B}_n(A)$.

The proof relies on the following:

(i) $\beta_{nt}(\widehat{d}_{nt}; A) > \beta_{nt}(\widehat{d}_{nt}; \sum_{t=0}^{T-1} d^*_{nt})$ because $A < \sum_{t=0}^{T-1} d^*_{nt}$, by (24b).

(ii) $c'(D_t + \sum_{m\neq n} d^*_{mt} + d_{nt})$ increases with d_{nt} under Assumption (A1).

(iii) (β^*_{nt}, d^*_{nt}) is the point at which $\beta_{nt}(d_{nt}; \sum_{t=0}^{T-1} d^*_{nt})$ and $c'(D_t + \sum_{m\neq n} d^*_{mt} + d_{nt})$ coincide, by Lemma 3.2.

Using (i)–(iii) together with (24a) gives,

$$0 < d^*_{nt} < \widehat{d}^1_{nt}, \qquad \beta^*_{nt} < \widehat{\beta}^1_{nt},$$

where $(\widehat{\beta}^1_{nt}, \widehat{d}^1_{nt})$ denotes the point of intersection between $\beta_{nt}(d_{nt}; A)$ and $c'(D_t + \sum_{m\neq n} d^*_{mt} + d_{nt})$.

Let $(\widehat{\beta}^2_{nt}, \widehat{d}^2_{nt})$ denote the bid profile with respect to A such that $\widehat{\beta}^2_{nt} = \beta^*_{nt}$, in other words $\widehat{\beta}^2_{nt} = \beta^*_{nt} = \beta_{nt}(\widehat{d}^2_{nt}, A)$. It follows from (24a) that $\widehat{d}^1_{nt} < \widehat{d}^2_{nt}$. Based on the ordering $0 < d^*_{nt} < \widehat{d}^1_{nt} < \widehat{d}^2_{nt}$, the set $[0, \infty)$ can be partitioned into four disjoint regions,

$$\mathcal{R}_0 \triangleq [0, d^*_{nt}), \quad \mathcal{R}_1 \triangleq [d^*_{nt}, \widehat{d}^1_{nt}),$$
$$\mathcal{R}_2 \triangleq [\widehat{d}^1_{nt}, \widehat{d}^2_{nt}), \; \mathcal{R}_3 \triangleq [\widehat{d}^2_{nt}, +\infty).$$

Suppose there exists a $\widehat{\boldsymbol{b}}_n \in \mathcal{B}_n(A)$ such that at time t_2, $\widehat{d}_{nt_2} \in \mathcal{R}_1 \cup \mathcal{R}_2 \cup \mathcal{R}_3$. Then, because $\sum_{t=0}^{T-1} \widehat{d}_{nt} = \sum_{t=0}^{T-1} \widehat{d}^*_{nt} = A < \sum_{t=0}^{T-1} d^*_{nt}$, and $\widehat{d}^*_{nt} < d^*_{nt}$ due to (27), there must exist another time t_1 such that $\widehat{d}_{nt_1} < \widehat{d}^*_{nt_1}$.

Consider two bid profiles $\widehat{\boldsymbol{b}}_n, \widetilde{\boldsymbol{b}}_n \in \mathcal{B}_n(A)$ such that,

$$\widehat{d}_{nt_1} < \widetilde{d}_{nt_1} < \widehat{d}^*_{nt_1} \tag{34a}$$

$$d^*_{nt_2} < \widetilde{d}_{nt_2} < \widehat{d}_{nt_2} \tag{34b}$$

$$\widehat{d}_{nt_1} + \widehat{d}_{nt_2} = \widetilde{d}_{nt_1} + \widetilde{d}_{nt_2} \tag{34c}$$

$$\widehat{d}_{nt} = \widetilde{d}_{nt}, \quad \text{for all } t \neq t_1, t_2. \tag{34d}$$

Then, by considering the cases where $\widehat{d}_{nt_2}, \widetilde{d}_{nt_2}$ lie in regions $\mathcal{R}_1$, $\mathcal{R}_2$ and $\mathcal{R}_3$ respectively, it is shown in [20] that,

$$u_n(\widehat{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n}) < u_n(\widetilde{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n}). \tag{35}$$

Hence, $\widehat{\boldsymbol{b}}_n$ cannot be the optimal bid profile with respect to $\boldsymbol{b}^*_{-n}$, which is the desired result.

Moreover, because Assumptions (A1,A2) ensure differentiability of $c(\cdot)$ and $f_n(.)$ for all n, the payoff function of every EV is continuous in the bid profiles. Therefore, it follows from (35) that the optimal bid profile for the n-th EV must lie in region $\mathcal{R}_0$ for all $t \in \mathcal{T}$. ∎

Theorem 4.3: Under Assumptions (A1,A2), (23) holds when $0 \leq A < \sum_{t=0}^{T-1} d^*_{nt}$.

Proof. By Theorem 4.2, $\widehat{\boldsymbol{b}}^*_n$ is the optimal bid with respect to $\boldsymbol{b}^*_{-n}$ in $\mathcal{B}_n(A)$, when $A \in [0, \sum_{t=0}^{T-1} d^*_{nt})$. Therefore, the best possible payoff for the n-th EV is,

$$u_n(\widehat{\boldsymbol{b}}^*_n, \boldsymbol{b}^*_{-n}) = w_n(\widehat{\boldsymbol{d}}^*_n) - \tau_n(\widehat{\boldsymbol{b}}^*_n, \boldsymbol{b}^*_{-n}). \tag{36}$$

Using (29) and recalling that $\boldsymbol{x}^* = \boldsymbol{d}^*$ gives,

$$\begin{aligned} J_s(\boldsymbol{x}^*) - J_s(\widehat{\boldsymbol{x}}^*) &= J_s(\boldsymbol{d}^*) - J_s(\widehat{\boldsymbol{d}}^*_n, \boldsymbol{d}^*_{-n}) \\ &= \sum_{t=0}^{T-1} \Big\{ c(D_t + \sum_{m\neq n} d^*_{mt} + d^*_{nt}) - c(D_t + \sum_{m \neq n} d^*_{mt} + \widehat{d}^*_{nt}) \Big\} \\ &\qquad - w_n(\boldsymbol{d}^*_n) + w_n(\widehat{\boldsymbol{d}}^*_n) \\ &= -\Big(u_n(\boldsymbol{b}^*) - u_n(\widehat{\boldsymbol{b}}^*_n, \boldsymbol{b}^*_{-n}) \Big) \equiv -\Delta u_n \end{aligned}$$

where the final equality makes use of (25a) and (36). By Lemma 3.2, $\boldsymbol{x}^* \equiv \boldsymbol{x}^*(\boldsymbol{b}^*)$ is the efficient allocation solution, so $J_s(\boldsymbol{x}^*) \leq J_s(\widehat{\boldsymbol{x}}^*)$. Therefore,

$$\Delta u_n = J_s(\widehat{\boldsymbol{x}}^*) - J_s(\boldsymbol{x}^*) \geq 0,$$

which implies that the n-th EV cannot benefit by unilaterally changing its bid profile $\boldsymbol{b}^*_n$ to any other bid profile $\widehat{\boldsymbol{b}}^*_n \in \mathcal{B}_n(A)$ with $A \in [0, \sum_{t=0}^{T-1} d^*_{nt})$. ∎

C. Existence of efficient Nash equilibrium

Using the results from Sections IV-A and IV-B, it can now be shown that the efficient bid profile $\boldsymbol{b}^*$ specified in (14) is a Nash equilibrium of the EV charging coordination game under the PSP auction mechanism.

Corollary 4.1: Under Assumptions (A1,A2), the efficient bid profile $\boldsymbol{b}^* \equiv (\boldsymbol{b}^*_n; n \in \mathcal{N})$ specified in (14) satisfies the property:

$$u_n(\boldsymbol{b}^*_n, \boldsymbol{b}^*_{-n}) \geq u_n(\boldsymbol{b}_n, \boldsymbol{b}^*_{-n}), \qquad \text{for all } \boldsymbol{b}_n \in \mathcal{B}_n. \tag{37}$$

Proof. It was shown in Theorems 4.1 and 4.3, under Assumptions (A1,A2), that

$$u_n(\boldsymbol{b}^*_n, \boldsymbol{b}^*_{-n}) \geq u_n(\boldsymbol{b}_n, \boldsymbol{b}^*_{-n}), \quad \text{for all } \boldsymbol{b}_n \in \mathcal{B}_n(A), \tag{38}$$

holds when $A \geq \sum_{t=0}^{T-1} d^*_{nt}$ and when $0 \leq A < \sum_{t=0}^{T-1} d^*_{nt}$, respectively. The desired result (37) holds since $\mathcal{B}_n = \bigcup_{A\in[0,\Gamma_n]} \mathcal{B}_n(A)$. ∎

V. A Dynamic PSP Auction Process

In order to implement the PSP-auction based distributed charging process, each EV must be able to determine its best bid profile, given the collection of bid profiles for the other EVs. It will be shown that this process can be formulated as a dynamic programming problem. This update mechanism can then be embedded in an algorithmic description of the underlying auction game.

Recall that $\boldsymbol{b}^*_n(\boldsymbol{b}_{-n})$ denotes the best bid profile of the n-th EV with respect to the collection of bid profiles $\boldsymbol{b}_{-n}$ of all the other EVs,

$$\boldsymbol{b}^*_n(\boldsymbol{b}_{-n}) = \underset{\boldsymbol{b}_n \in \mathcal{B}_n}{\text{argmax}}\, u_n(\boldsymbol{b}_n, \boldsymbol{b}_{-n}), \tag{39}$$

with $u_n(\boldsymbol{b}_n, \boldsymbol{b}_{-n})$ being the individual payoff of the n-th EV, as established in (19). However, due to the cross-temporal coupling arising from the summation term $\sum_{t=0}^{T-1} d_{nt}$ in truth-telling bid profiles, as identified in (16), it is impractical to directly implement the best response $\boldsymbol{b}^*_n(\boldsymbol{b}_{-n})$ which is incentive compatible. This can be addressed by determining the best response when bid profiles are constrained to possess a common total desired demand $A = \sum_{t=0}^{T-1} d_{nt}$, and then optimizing over $A \in [0, \Gamma_n]$. The resulting optimization is given by,

$$u_n(\boldsymbol{b}^*_n, \boldsymbol{b}_{-n}) = \max_{A\in[0,\Gamma_n]} \max_{\boldsymbol{b}_n \in \mathcal{B}_n(A)} u_n(\boldsymbol{b}_n, \boldsymbol{b}_{-n}). \tag{40}$$

The remainder of this section describes a dynamic programming approach to solving the inner optimal bidding problem that arises for each fixed total demand request $A \in [0, \Gamma_n]$.

The dynamics associated with the physical charging process (1) can be rewritten,

$$s_{n,t+1}(\boldsymbol{b}_n, \boldsymbol{b}_{-n}) = s_{nt}(\boldsymbol{b}_n, \boldsymbol{b}_{-n}) + \frac{1}{\Theta_n} x_{nt}(\boldsymbol{b}_n, \boldsymbol{b}_{-n}), \quad t \in \mathcal{T} \tag{41}$$

where $x_{nt}(\boldsymbol{b}_n, \boldsymbol{b}_{-n})$ denotes the allocated charging rate of the n-th EV at time t, with respect to the collection of bid profiles $(\boldsymbol{b}_n, \boldsymbol{b}_{-n})$.

Lemma 5.1: Consider a bid profile $\boldsymbol{b}_n \in \mathcal{B}_n(A)$, for any fixed $A \in [0, \Gamma_n]$. Then the payoff function of the n-th EV has the summation form:

$$u_n(\boldsymbol{b}_n, \boldsymbol{b}_{-n}) = \sum_{t=0}^{T-1} a_n(\boldsymbol{b}_t) - \delta_n \Theta_n^2 \left(s_n^{max} - s_{nT}\right)^2, \tag{42}$$

where $a_n(\boldsymbol{b}_t) \equiv -f_n(x_{nt}(\boldsymbol{b}_t)) - \tau_{nt}(\boldsymbol{b}_t)$, and $\tau_{nt}(\boldsymbol{b}_t)$ is defined in (18).

Proof. From (19),

$$\begin{aligned} u_n(\boldsymbol{b}) &= w_n(\boldsymbol{x}_n^*(\boldsymbol{b}_n)) - \tau_n(\boldsymbol{b}) \\ &= \sum_{t=0}^{T-1} \Big\{ - f_n(x_{nt}(\boldsymbol{b}_t)) - \tau_{nt}(\boldsymbol{b}_t) \Big\} \\ &\qquad - \delta_n \Big(\sum_{t=0}^{T-1} x_{nt}(\boldsymbol{b}_t) - \Gamma_n \Big)^2 \end{aligned}$$

where equality holds by (3), Lemma 3.1, and (18). Then (42) follows directly from (41). ■

Let $\mathcal{T}_t \equiv \{t, ..., T-1\}$, and define the value function $v_n(t, s_{nt}) \equiv v_n(t, s_{nt}; A, \boldsymbol{b}_{-n})$, for all $t \in \mathcal{T}$ as,

$$v_n(t, s_{nt}) \triangleq \max_{\boldsymbol{b}_n(\mathcal{T}_t) \in \mathcal{B}_n(\mathcal{T}_t, s_{nt}; A)} \left\{ \sum_{s=t}^{T-1} a_n(\boldsymbol{b}_s) - \delta_n \Theta_n^2 \left(s_n^{max} - s_{nT}\right)^2 \right\} \tag{43}$$

where $a_n(\boldsymbol{b}_s)$ is defined in Lemma 5.1, and

$$\mathcal{B}_n(\mathcal{T}_t, s_{nt}; A) \triangleq \begin{cases} \Big\{ \big((\beta_{ns}, d_{ns}), s \in \mathcal{T}_t\big) \text{ s.t. } \beta_{ns} = -f_n'(d_{ns}) + 2\delta_n \left(\Gamma_n - A\right), \\ \qquad \text{and } \sum_{s=t}^{T-1} d_{ns} \le \min\left\{A, \Theta_n(s_n^{max} - s_{nt})\right\} \Big\}, & \text{when } t > 0 \\ \mathcal{B}_n(A), & \text{when } t = 0 \end{cases} \tag{44}$$

with $\mathcal{B}_n(A)$ defined in (21). The terminal value function is defined as,

$$v_n(T, s_{nT}) \triangleq -\delta_n \Theta_n^2 \left(s_n^{max} - s_{nT}\right)^2. \tag{45}$$

Note that the set of bid profiles of the n-th EV over the interval $\mathcal{T}_t$ specified in (44) is defined in such a way that the total bidding demand over the whole interval $\mathcal{T}$ is guaranteed to equal A.

Based on the value function definition,

$$v_n(0, s_{n0}; A, \boldsymbol{b}_{-n}) = \max_{\boldsymbol{b}_n \in \mathcal{B}_n(A)} u_n(\boldsymbol{b}_n, \boldsymbol{b}_{-n}) \tag{46}$$

and therefore,

$$u_n(\boldsymbol{b}_n^*, \boldsymbol{b}_{-n}) = \max_{A \in [0, \Gamma_n]} v_n(0, s_{n0}; A, \boldsymbol{b}_{-n}). \tag{47}$$

Let $\boldsymbol{b}_n^*(\mathcal{T}_t) \equiv \Big((\beta_{ns}^*, d_{ns}^*), s \in \mathcal{T}_t\Big) \equiv \boldsymbol{b}_n^*(\mathcal{T}_t, s_{nt}; A, \boldsymbol{b}_{-n})$ denote the best bid profile of the n-th EV solving the optimal problem (43) over the interval $\mathcal{T}_t$ with respect to A and $\boldsymbol{b}_{-n}$, and let $\Pi_n^*(t, s_{nt}; A, \boldsymbol{b}_{-n})$ denote the total bidding demand over the interval $\mathcal{T}_t$ of the n-th EV subject to the best bid profile $\boldsymbol{b}_n^*(\mathcal{T}_t, s_{nt}; A, \boldsymbol{b}_{-n})$,

$$\Pi_n^*(t, s_{nt}; A, \boldsymbol{b}_{-n}) \triangleq \sum_{s=t}^{T-1} d_{ns}^*(\mathcal{T}_t, s_{nt}; A, \boldsymbol{b}_{-n}). \tag{48}$$

Define $\mathcal{B}_n(t, s_{nt}) \equiv \mathcal{B}_n(t, s_{nt}; A, \boldsymbol{b}_{-n})$, for any $t \in \mathcal{T}$, as the set of bid profiles at time t, such that

$$\mathcal{B}_n(t, s_{nt}; A, \boldsymbol{b}_{-n}) \triangleq \begin{cases} \Big\{ (\beta_{nt}, d_{nt}) \text{ s.t. } \beta_{nt} = -f_n'(d_{nt}) + 2\delta_n \left(\Gamma_n - A\right), \\ \quad \text{and } d_{nt} + \Pi_n^*(t+1, s_{n,t+1}; A, \boldsymbol{b}_{-n}) \\ \qquad \le \min\left\{A, \Theta_n(s_n^{max} - s_{nt})\right\} \Big\}, & \text{when } t > 0 \\ \Big\{ (\beta_{nt}, d_{nt}) \text{ s.t. } \beta_{nt} = -f_n'(d_{nt}) + 2\delta_n \left(\Gamma_n - A\right), \\ \quad \text{and } d_{nt} + \Pi_n^*(t+1, s_{n,t+1}; A, \boldsymbol{b}_{-n}) = A \Big\}, & \text{when } t = 0 \end{cases} \tag{49}$$

where $s_{n,t+1}$ is given by (41). As with the set of bid profiles of the n-th EV over the interval $\mathcal{T}_t$ specified in (44), the set of bid profiles of the n-th EV at each time $t \in \mathcal{T}$ specified in (49) is defined such that the total bidding demand over the whole interval $\mathcal{T}$ is guaranteed to equal A.

Theorem 5.1: The value function $v_n(t, s_{nt}; A, \boldsymbol{b}_{-n})$ of the n-th EV, with respect to a fixed $A \in [0, \Gamma_n]$ and a collection of bid profiles of the other EVs $\boldsymbol{b}_{-n}$, can be implemented by solving the Bellman equation,

$$v_n(t, s_{nt}; A, \boldsymbol{b}_{-n}) = \max_{b_{nt} \in \mathcal{B}_n(t, s_{nt}; A, \boldsymbol{b}_{-n})} \Big\{ a_n(\boldsymbol{b}_t) + v_n(t+1, s_{n,t+1}; A, \boldsymbol{b}_{-n}) \Big\}, \tag{50}$$

for $t \in \mathcal{T}$, where $\mathcal{B}_n(t, s_{nt}; A, \boldsymbol{b}_{-n})$ is defined in (49), $a_n(\boldsymbol{b}_t)$ is specified in Lemma 5.1, and $v_n(T, s_{nT})$ is given by (45).

It is straightforward to verify Theorem 5.1 by applying the *optimality principle* for the underlying optimization problem defined in (46) for the n-th EV with respect to A and $\boldsymbol{b}_{-n}$.

Finally, the n-th EV's overall best bid profile, with respect to bid profiles of the other EVs $\boldsymbol{b}_{-n}$, is given by (47).

By applying Algorithm 5.1, each EV updates its own best response with respect to the bid profiles of all the other

Algorithm 5.1: (Nash equilibrium implementation)

- Provide an initial collection of bid profiles $\boldsymbol{b}^{(0)}$.
- Set the iterative step $k = 0$.
- Set the iteration termination criterion $\epsilon > \epsilon_0$ for some $\epsilon_0 > 0$.
- While $\epsilon > \epsilon_0$
  - For $n : 1 = N$
    Determine the best response for the n-th EV, $\boldsymbol{b}_n^{(k+1)}$, with respect to $\boldsymbol{b}_1^{(k+1)}, \cdots, \boldsymbol{b}_{n-1}^{(k+1)}, \boldsymbol{b}_{n+1}^{(k)}, \cdots, \boldsymbol{b}_N^{(k)}$, by maximizing the payoff function,
    $$\boldsymbol{b}_n^{(k+1)} = \operatorname*{argmax}_{\boldsymbol{b}_n \in \mathcal{B}_n} u_n(\boldsymbol{b}_n; \boldsymbol{b}_1^{(k+1)}, \cdots, \boldsymbol{b}_{n-1}^{(k+1)}, \boldsymbol{b}_{n+1}^{(k)}, \cdots, \boldsymbol{b}_N^{(k)}),$$
    which can be achieved by applying the dynamic programming update.
  - Update $\epsilon := ||\boldsymbol{b}^{(k+1)} - \boldsymbol{b}^{(k)}||$.
  - Update $k := k + 1$.

End of Algorithm

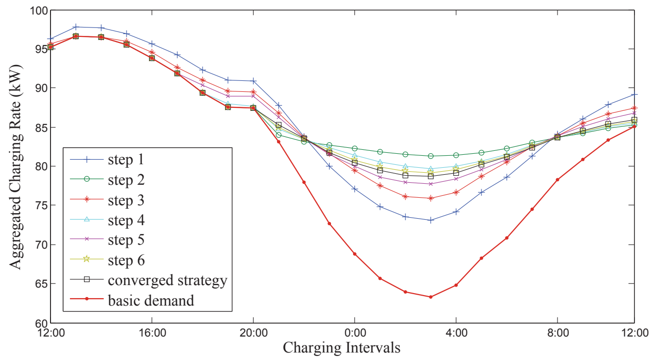

Fig. 1. Convergent updates of Algorithm 5.1.

EVs. This process continues until the updates become insignificant. Upon convergence, on the basis of Definition 3.1, the bid profile implemented by Algorithm 5.1 is a Nash equilibrium.

VI. Numerical Illustration

To illustrate the auction-based coordination process, a numerical example considers EV charging over a common time horizon $T = 24$, from 12:00 on one day to 12:00 the next day, with a time-step of $\Delta T = 1$ h. The background demand D_t is shown in Fig. 1. For the purpose of demonstration, a small population of 5 vehicles is considered. Each EV has a common battery capacity of 30 kWh and a common maximum SoC value $s_n^{max} = 0.9$. Heterogeneity is introduced by letting the initial SoC values s_{n0} for the five EVs take the values $\boldsymbol{s}_0 = [0.1 \ \ 0.15 \ \ 0.23 \ \ 0.14 \ \ 0.08]^\intercal$.

The generation cost is given by $c(\boldsymbol{x}_t, D_t) = 0.005(\sum_{n\in\mathcal{N}} x_{nt} + D_t)^{1.7}$ and the battery degradation cost by $f_n(x_{nt}) = 0.002x_{nt}^2$. Both these functions are strictly convex. The weighting factor for the quadratic charging deviation cost of each EV is set to $\delta_n = 10$ for all $n \in \mathcal{N}$.

The efficient EV charging trajectory, given by Optimization Problem 1, is shown in aggregation ($\sum_{n=1}^N x_{nt}^{**}$, $t \in \mathcal{T}$) by the black line with squares in Fig. 1. In contrast, the distributed approach to charging coordination, described by Algorithm 5.1, gave the update evolution shown by the other curves in Fig. 1. At each iteration, all EVs determine their optimal bid profile by solving the dynamic programming problem formalized by Theorem 5.1. Fig. 1 shows the aggregate allocation ($\sum_{n=1}^N x_{nt}^*(\boldsymbol{b}^{(k)})$, $t \in \mathcal{T}$) obtained by the auctioneer solving Optimization Problem 2 with respect to the bid profile $\boldsymbol{b}^{(k)}$ at the k-th iteration. It is clear that the auction game converges to the efficient charging solution.

VII. Conclusions

The paper considers the coordination of EV charging over a finite horizon. A distributed approach, based on the progressive second price (PSP) auction mechanism, has been developed. It was proven that the efficient (centrally optimal) coordination solution is a Nash equilibrium of the PSP auction game. A dynamic update mechanism for the underlying auction game was established. A numerical example which implemented this update mechanism illustrated convergence of the auction system to the efficient Nash equilibrium.

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