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An Enhanced MPC-Based Strategy for Non-Disruptive Load Shedding

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Abstract— **Model predictive control (MPC) methods have a well-earned reputation for providing on-line solutions to optimal feedback control problems, particularly for systems with control and parameter constraints. Previous work has shown the value of MPC in designing non-disruptive load-shedding strategies for power systems. The nonlinear, non-smooth dynamics of power systems make direct application of MPC difficult though. Therefore previous load-shedding applications of MPC have made use of an approximate discrete-time linear dynamic model that describes perturbations to the system's nominal behavior over a finite-time horizon. This approximate model is based on trajectory sensitivities. The article pursues several enhancements of such MPC-based load-shedding strategies. Specifically, at each MPC stage, we propose using a two-step optimization process to determine the optimal input sequence. This helps in combating the possibility of growing error in the discrete-time approximation if large input modifications are needed. We also consider the effects of varying voltage constraints over the MPC optimization horizon. The new two-step MPC strategies are used to design load-shedding controls that prevent voltage collapse in a ten-bus benchmark-system example.**

I. Introduction

Within the last several decades, the electric power infrastructure has become increasingly complicated: 1) new forms of generation (e.g., wind and solar generators) capabilities are being added with increasing frequency; 2) customer requirements are changing with newer forms of load (from electrical vehicles to data centers); 3) new measurement and cyber-techniques have been developed to obtain more extensive on-line data (e.g., real-time monitoring, contingency analysis); 4) market forces are becoming increasingly important in power-system operations. This trend will continue into the future, as further controllability is added through fast-acting demand responsiveness, energy storage and FACTS devices.

These changes and developments have in many cases significantly improved the power grid's performance. Nevertheless, they also introduce new challenges in developing control and protection strategies for a power grid subject to attacks from both natural and sentient adversaries (e.g., severe weather events, human/computer operation faults). When an abnormal event or attack occurs (and normal operating conditions are no longer in force), corrective actions may be required to return the system states to an acceptable range. These contingency events can have a relatively fast (usually on the order of tens of seconds to minutes) impact on the power grid, and the correction should be effective on that same time order. Due to the growing complexity of the power system, it is becoming increasingly difficult for human/computer operators to plan and respond efficiently within such a short time span. Hence, new feedback control strategies and automation tools are needed.

Several issues need to be addressed in order to develop contingency-management tools for modern power systems:

- Contingency-event modeling: a considered contingency can be any severe event that is beyond the abnormality envelope, e.g. unexpected failures of transmission lines or generators, dramatic changes in renewable energy sources, and cyber attacks to the computing systems, among many others. How to mathematically represent these contingency events and incorporate their impacts into the system-level dynamics is of importance. We note that the impact of contingencies also may vary as the operating point of the system changes, due to changes in variable renewable generation and storage.
- Prediction/estimation accuracy: In order to operate a control plan, one needs to obtain the current state of the system and dynamic trends in the near future. However, it may not be easy or possible to measure the full system state (even for the subregion of interest) at the current time. In reality, sensors or observers may not provide full coverage. Meanwhile, the measurement process may also be corrupted by noise, which may add difficulty to the estimation problem.
- Economic concern: From the viewpoint of both power suppliers and the system manager, an ideal corrective action will achieve the control task while imposing a minimum (economic) cost. Because of the *network* nature of the power system, economic solutions usually depend on both spatial and temporal properties of the grid (i.e., where and when to place control). Finding an economic solution therefore can be considered as an optimization problem subject to the system dynamics.
- Validation: It is also important to consider validation of a control plan, i.e. how to guarantee that the controller achieves system stability and addresses the abnormality. The validation task requires simulating or analyzing the controls, over the possible range of operating points and contingencies.

As a first effort in addressing the broad goal of contingency management, we revisit a classical load control problem here that has been introduced earlier in [1], [2]: response to unexpected, severe line-tripping events through load-shedding. Typically, failure of key transmission lines

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may cause a reduction in voltage levels oat both ends of the lines, which eventually can lead the system to voltage collapse. By shedding an appropriate amount of load, such voltage collapse can be avoided. However, traditional load control (shedding) schemes are unappealing because of the disruption they cause to consumers. On the other hand, there are generally some loads that can be curtailed with minimum consumer impact. From an economic perspective, the cost of different shedding strategies should be related to consumer disruption levels.

To develop a real-time load control strategy that achieves low consumer disruption, the earlier work of [1], [2] proposed a shedding strategy that is based on one-step *Model Predictive Control* (MPC) methods. This MPC-based shedding strategy provides an on-line shedding solution that can avoid voltage collapse while achieving low disruption over the design horizon. Because of the computational overhead needed for power-system simulation, numerical approximations are used within the MPC optimization to permit real-time design. More specifically, *trajectory sensitivity* analysis has been adopted in [1], [2] at each MPC stage, where the sensitivities with respect to shedding decisions along a nominal trajectory are calculated. Based upon these sensitivities, a linear mapping between shedding variations and state variations can be established. This linear mapping can be used to (approximately) determine the minimum load shedding requirements at each time-step that can restore voltages above a desired threshold. Simulation results also have shown the success of this one-step MPC load control strategy for a benchmark example with ten buses.

To enhance and extend the results from this earlier work, we first study how the look-ahead horizon for one-step MPC affects the shedding performance. We then focus on developing a two-step MPC-based load control method, i.e. where the MPC optimization is based on a look-ahead over two control actions. We also replicate the one-step MPC simulation for the ten-bus example considered in [1], [2], and provide some preliminary comparisons between the one-step and two-step methods.

The remainder of this article is organized as follows. Section II discusses the load shedding problem. Section III first details a two-step MPC optimization process which solves the load control problem, and then implements one- and two-step MPC on a ten-bus benchmark system. Some comparisons between the one-step two-step MPC designs are also presented.

II. Problem Formulation

This section focuses on the problem of preventing voltage collapse through load control (shedding), in the context of a benchmark example with ten buses. Specifically, we seek to design an optimal load shedding policy (i.e., one that requires minimal shedding), and propose an MPC-based solution. The ten-bus system that we study is shown in Figure 1. The particular disturbance considered here, that leads to voltage collapse, has been motivated and described in [1]. Briefly, the example involves tripping one of the five transmission

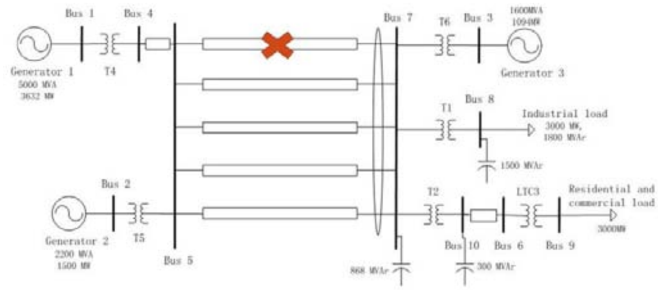

Fig. 1. The benchmark example with ten buses.

lines between Bus 5 and Bus 7. Upon tripping, the power flow on the remaining four transmission lines (which deliver power from the generation of the left-hand sub-network to the loads of the right-hand sub-network) increases. In fact, without load shedding, the loss of one feeder will cause power overloads on the remaining lines. The overloaded lines consume a large amount of reactive power, which causes voltages to drop at both ends, and ultimately leads to voltage collapse. Rapid load shedding can be used to prevent voltage collapse, since it reduces the requirement for power flow over the heavily loaded feeders.

Our goal is to shed a minimal amount of load to prevent voltage collapse. We consider load shedding in three consumer areas: an industrial load connected to Bus 8, and residential and commercial loads connected to Bus 9. The shedding amounts at time t, i.e. the percentage of the corresponding load that is removed, are denoted as $\lambda_1(t)$, $\lambda_2(t)$, and $\lambda_3(t)$, respectively. These shedding amounts are initially all zero (no shedding) and also are unchanging. After detection of the disturbance, we can change the value for these shedding parameters so as to restore the voltages to an allowable range. More specifically, we aim to minimize the overall amount of shedding during the post-disturbance period, such that the voltages at Bus 6 and Bus 8 ($V_6(t), V_8(t)$) are brought above and remain above a threshold (which is set at 0.98 pu).

We pursue an MPC solution to the load shedding problem. Specifically, let us consider an N-step MPC optimization process. In general, at each time-step, an N-step MPC process seeks to design a sequence of control inputs that optimizes an objective function subject to constraints over the next N time-steps, then implements the first control input and repeats the process at the next time-step. The complete optimization process at each time-step is also referred to as one *MPC stage* [3], [4]. In our case, the control inputs are the shedding parameters $\lambda_1(t), \lambda_2(t), \lambda_3(t)$. While the shedding parameters may be viewed as continuous-time signals, we find it convenient to design them over discrete time intervals, both to permit use of MPC and to simplify operational implementation.

We assume that the look-ahead time horizon for each MPC stage is T seconds (i.e. each time-step is T/N seconds), and that the MPC process starts at time 0. At each MPC stage k

(time $(k-1)T/N$), the goal becomes to minimize,

$$J_k = \sum_{i=1}^{N} \lambda_1(k+i-1) + \lambda_2(k+i-1) + \lambda_3(k+i-1), \quad (1)$$

subject to the power flow equations and the terminal voltage constraints,

$$V_6(k+N) \geq 0.98\text{pu}$$
$$V_8(k+N) \geq 0.98\text{pu}.$$

We will study whether successful load shedding can be achieved via the above N-step MPC process.

In [1], a one-step MPC-based load shedding strategy has been developed for the same benchmark system. Here, we focus on a two-step MPC-based strategy ($N = 2$) for this benchmark system, and discuss some potential advantages provided by the extra freedom in the MPC design.

III. RESULTS

So far, we have proposed a two-step MPC-based load shedding strategy to prevent voltage collapse. We will now discuss some details regarding computation of the two-step MPC solution, and then compare the performance of various two-step and one-step MPC solutions.

The key computation required for implementing MPC is the optimization of a cost through design of system inputs over a look-ahead horizon–in our case, the minimization of the cost J_k through design of appropriate load-shedding parameters. One salient advantage of MPC is that this optimization can often be achieved using standard numerical optimization tools, and so the optimal input sequence (in our case, minimal shedding amounts) can be obtained easily through numerical means. Standard optimization techniques can indeed be applied for the problem that we consider, but the optimization is complicated by the fact that the nonlinear system dynamics do not admit an analytical solution and instead must be computed by a rather expensive simulation. Thus, direct evaluation of the cost and constraints for a specified input sequence is computationally intensive, and hence an optimization algorithm that takes even a few iterations may be computationally unattractive for real-time implementation.

To overcome this complexity, we use a trajectory-sensitivity-based approximation (see [5], [6]) to simplify the cost/constraint evaluation and hence the MPC optimization. Specifically, we compute the sensitivity of the voltage-magnitude response at Buses 6 and 8 to the designable load parameters over the MPC horizon, and hence obtain the constrained voltages via a linear approximation. Based on this simplifying approximation (and the observation that the cost depends on the design parameters in simple ways), we see that the MPC optimization can be achieved easily using a linear programming algorithm. Since details of this procedure are given in [1], we omit them here. Briefly, the trajectory sensitivity analysis gives a linear mapping between changes in the load-shedding parameters at time-step k and voltage changes at time-step $k+1$ around a nominal trajectory,

$$\begin{bmatrix} \Delta V_6(k+1) \\ \Delta V_8(k+1) \end{bmatrix} = S_{v\lambda}(k+1,k) \begin{bmatrix} \Delta\lambda_1(k) \\ \Delta\lambda_2(k) \\ \Delta\lambda_3(k) \end{bmatrix}. \quad (2)$$

Moreover, the evolution of the sensitivity matrix $S(k+1,k)$ from k to $k+1$ is (approximately) captured by a linear time-varying differential equation. Therefore, at each time-step, the trajectory sensitivity analysis makes it easy to establish the dependence of voltage changes on load-shedding designs, around a nominal trajectory.

Next, we discuss differences in applying the one-step and two-step MPC strategies. For the one-step MPC case, because the optimization at each stage is over only one time-step, the sensitivity mapping is simply based on one nominal trajectory from time-step k to time-step $k+1$. That is, the voltage changes at time-step $k+1$ are approximated as a linear function of the initial shedding-rate changes at time k. However, for the two-step MPC case (from time-step k to time-step $k+2$), inputs (i.e., load-shedding changes) can be applied to the system at both time k and time $k+1$. Because there is still only one nominal trajectory over these two time-steps (between k and $k+2$), we need to consider the sensitivities differently from the one-step MPC case.

For the first time-step (between k and $k+1$), the sensitivity relationship is again simply given by (2). For the second time-step (from $k+1$ to $k+2$), because we still consider the same nominal trajectory as that for the first step, the changes of other dynamic states at time-step $k+1$, denoted $\Delta x(k+1)$, (which are caused by changes in the load shedding at time k, $[\Delta\lambda_1(k)\ \Delta\lambda_2(k)\ \Delta\lambda_3(k)]^T$) will also have an impact on the terminal voltages $V_6(k+2)$ and $V_8(k+2)$. That is,

$$\begin{bmatrix} \Delta V_6(k+2) \\ \Delta V_8(k+2) \end{bmatrix} = S_{v\lambda}(k+2,k+1) \begin{bmatrix} \Delta\lambda_1(k+1) \\ \Delta\lambda_2(k+1) \\ \Delta\lambda_3(k+1) \end{bmatrix} + S_{vx}(k+2,k+1)\Delta x(k+1).$$

Because,

$$\Delta x(k+1) = S_{x\lambda}(k+1,k) \begin{bmatrix} \Delta\lambda_1(k) \\ \Delta\lambda_2(k) \\ \Delta\lambda_3(k) \end{bmatrix},$$

we thus have,

$$\begin{bmatrix} \Delta V_6(k+2) \\ \Delta V_8(k+2) \end{bmatrix} = S_{v\lambda}(k+2,k+1) \begin{bmatrix} \Delta\lambda_1(k+1) \\ \Delta\lambda_2(k+1) \\ \Delta\lambda_3(k+1) \end{bmatrix} + S_{vx}(k+2,k+1)S_{x\lambda}(k+1,k) \begin{bmatrix} \Delta\lambda_1(k) \\ \Delta\lambda_2(k) \\ \Delta\lambda_3(k) \end{bmatrix}.$$

The voltage changes at time-step $k+2$ now depend on input changes at both time-step k and time-step $k+1$.

Simulation Results

Let us now present some simulation results and observations for the benchmark system shown in Figure 1. Figure 2 is the simulation of the voltage response at Buses 6

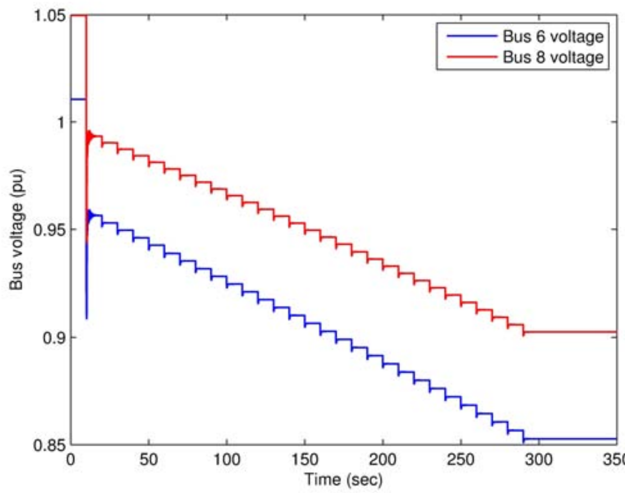

Fig. 2. The voltage response at Buses 6 and 8 without load shedding.

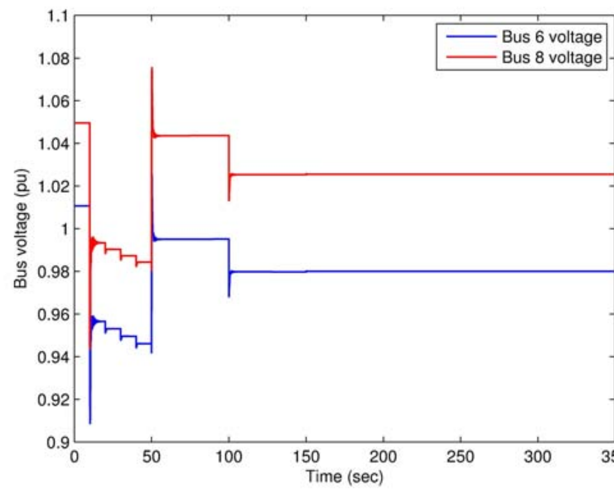

Fig. 3. The voltage response at Buses 6 and 8 with one-step MPC-based load shedding.

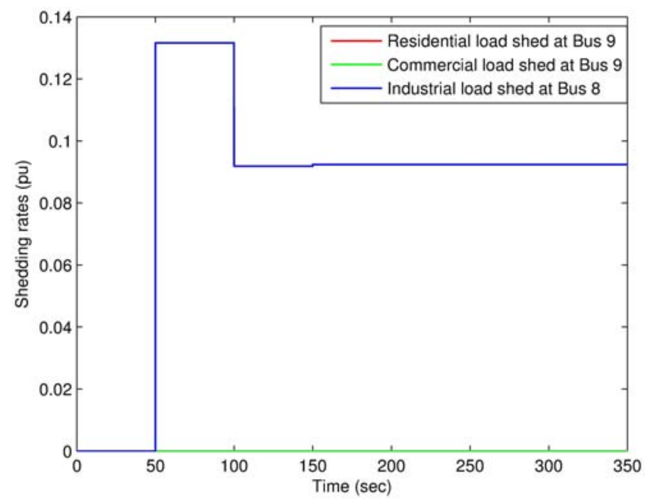

Fig. 4. The corresponding shedding requirements for one-step MPC.

and 8 without load shedding. We note that the voltages drop immediately when the line-tripping disturbance occurs at 10 seconds, and the tap-changing transformer starts to switch every 10 seconds (which drives the voltage-collapse phenomenon).

For the purpose of comparison, we also replicate the one-step MPC case here. Figure 3 shows the voltage response with the one-step MPC (where the one-step time-horizon T is chosen as 50 seconds), and Figure 4 presents the corresponding shedding amounts.

For the two-step MPC case, we use the same horizon T, so each step is 25 seconds. The following four different cases are considered:

Case 1: At each MPC stage k, we only apply the constraints $V_6(k+2), V_8(k+2) \geq 0.98$ pu, with no voltage constraint applied at time-step $k+1$. The minimal shedding policy for such a case is to shed zero load at time-step k (i.e. $\lambda_1(k) = \lambda_2(k) = \lambda_3(k) = 0$) and to shed about 13.2% of the industrial load (connected to Bus 8) at time-step $k+1$. In other words, because there is no voltage constraint at the middle of the time horizon, the cost is minimized by shedding load only for the second step. However, since MPC implements the inputs derived for time-step k (no shedding at all), the voltages continue to decrease, eventually leading to collapse. From this case, it follows that it is important to impose a further voltage constraint at time-step $k+1$ of the MPC optimization.

Case 2: At each MPC stage k, we enforce both $V_6(k+1), V_8(k+1) \geq 0.98$ pu and $V_6(k+2), V_8(k+2) \geq 0.98$ pu. Figures 5 and 6 show the voltage response and the corresponding shedding amounts for this case. When the first-step voltage constraint is the same as the terminal voltage constraint, the shedding policy is actually the same as the one-step MPC strategy with a 25 second time-step interval. In other words, because the voltages need to be brought above 0.98 pu for the first time-step, a certain amount of load has to be shed at time-step k. In this case, the two-step MPC strategy really does not show any advantage over one-step MPC. However, comparing to the one-step MPC strategy with a 50 second time-step, the total amount of load shed with this shorter time-step is smaller. One explanation is that the shorter duration for each time-step results in a smaller error in the trajectory-sensitivity-based approximation for the perturbed trajectory.

Case 3: For the first MPC stage $k=1$, set $V_6(k+1), V_8(k+1) \geq 0.97$ pu and $V_6(k+2), V_8(k+2) \geq 0.98$ pu, and for subsequent MPC stages $k \geq 2$, set $V_6(k+1), V_8(k+1) \geq 0.98$ pu and $V_6(k+2), V_8(k+2) \geq 0.98$ pu. In order to better leverage the design of the first time-step load shedding, we set a different voltage constraint in the middle of the horizon for the first MPC stage. Simulation results for the voltage response and shedding amounts are given in Figures 7 and 8. Notice that there is no over shedding for the first time-step, unlike the previous cases, which are shown in Figures 4

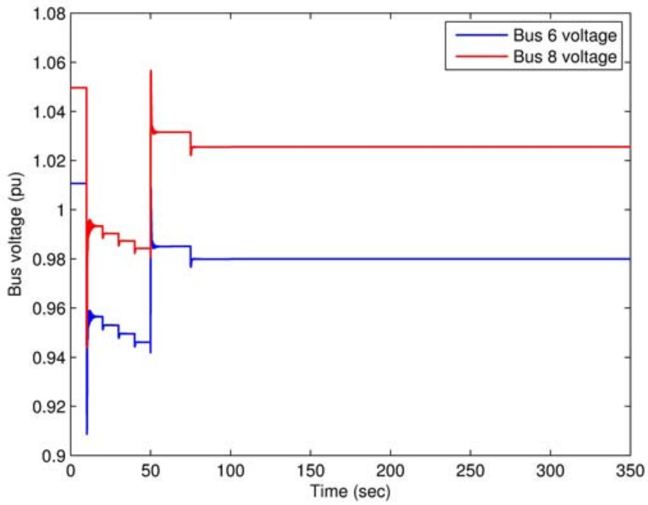

Fig. 5. The voltage response for the two-step MPC with a first-step voltage constraint $V \geq 0.98$ pu.

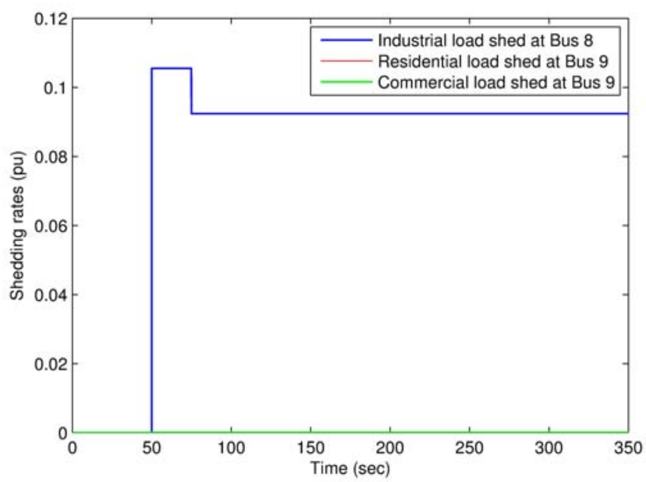

Fig. 6. The corresponding shedding requirements for the two-step MPC of Case 2.

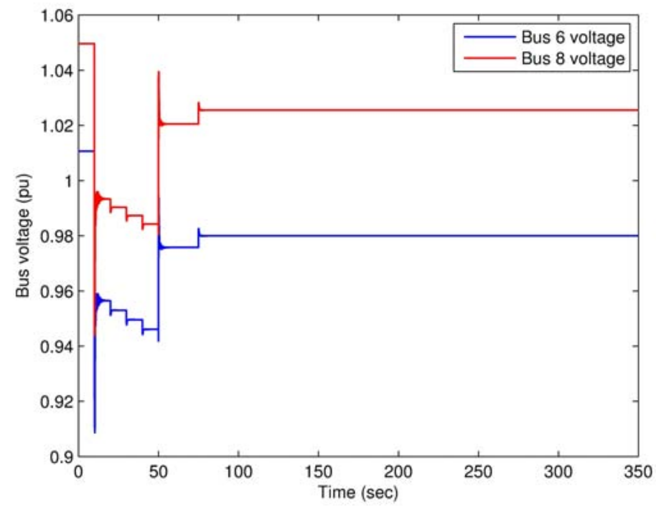

Fig. 7. The voltage response for the two-step MPC with a first-step voltage constraint $V \geq 0.97$ pu.

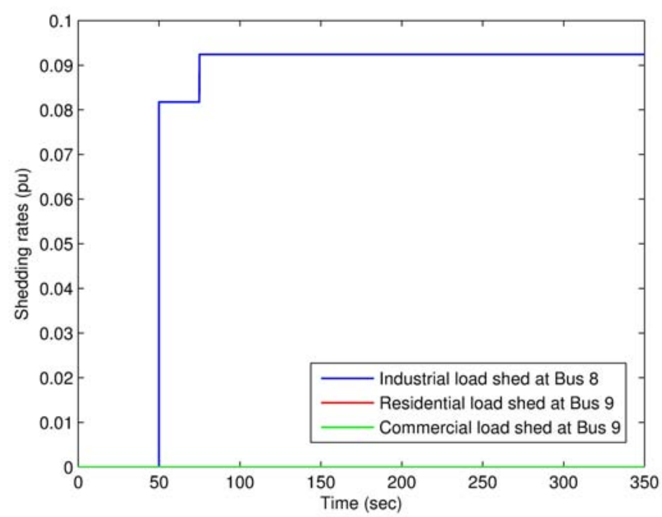

Fig. 8. The corresponding shedding requirements for the two-step MPC in Case 3.

and 6. Also, the overall cost is less. We note that the total load shedding cost (and final steady-state voltages) are comparable to those obtained in Case 2, but a much smaller shedding effort is employed at the first stage.

Case 4: Similar to Case 3, we again manipulate the voltage constraint in the middle for the first MPC stage. Here, we use a higher constraint level (0.975 pu) than that used for Case 3. Therefore, for the first MPC stage $k = 1$, we set $V_6(k+1), V_8(k+1) \geq 0.975$ pu and $V_6(k+2), V_8(k+2) \geq 0.98$ pu, and for subsequent MPC stages $k \geq 2$, we set $V_6(k+1), V_8(k+1) \geq 0.98$ pu and $V_6(k+2), V_8(k+2) \geq 0.98$ pu. The voltage response and the shedding amounts are shown in Figures 9 and 10. Again, there is no over shedding at the first time-step, and in this case the voltages remain effectively constant during the whole MPC process.

IV. Conclusion

A new two-step MPC strategy for voltage collapse prevention has been proposed. Compared to the earlier development of a one-step MPC-based load shedding strategy, this two-step MPC process utilizes a sensitivity chain analysis to link the terminal voltage changes to the shedding amounts over two time-steps. When there is no constraint on the voltages at the end of the first time-step, the optimal two-step MPC shedding policy only allocates shedding to the second time-step and hence fails to prevent voltage collapse. However, if appropriate voltage constraints are imposed at the end of the first time-step, the two-step MPC strategy can actually provide a control policy requiring less shedding then a one-step MPC strategy with the same time horizon. We note that these observations and conclusions are preliminary, as they are based on simulation results from a specific benchmark

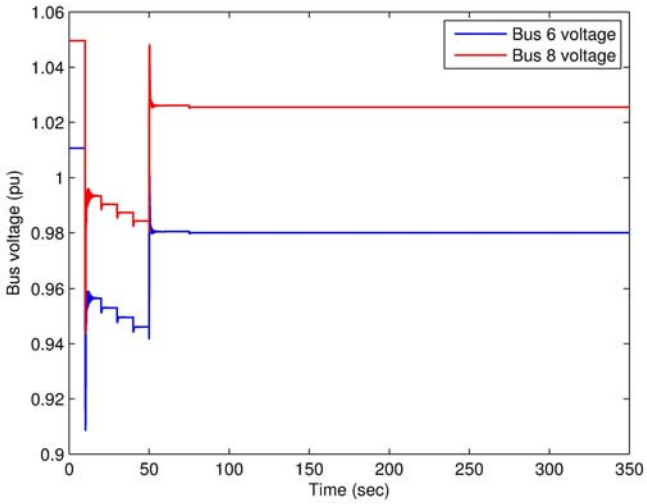

Fig. 9. The voltage response for the two-step MPC with a first-step voltage constraint ≥ 0.975 pu.

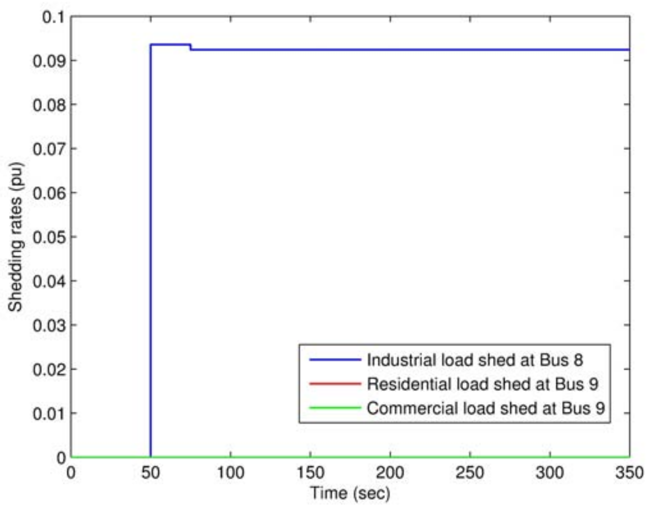

Fig. 10. The corresponding shedding requirements for the two-step MPC in Case 4.

system with particular voltage and shedding constraints. In particular, the terminal voltage constraints in our example may be too restrictive and shedding amounts at earlier time-steps may not have much weight in determining the voltage changes; different parameters may yield quite different designs. We also note that the two-step MPC design is based on a linearization around a single nominal trajectory, which may also introduce some error in the sensitivity calculation for the second time-step. Future work will undertake a detailed analysis of the properties of the proposed multi-step MPC scheme, and will explore more sophisticated approximations.

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