MIMO Capacity for Rician Fading Channels

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Outline

- Background
- Channel Model
- MIMO Capacity: without training
- MIMO Capacity: with training

Background: Capacity of MIMO Channels

What is the maximum possible rate for communication?

- Capacity of Rayleigh fading channels
  - Fading channel known at the receiver (Foschini, Telatar)
  - Fading unknown at the receiver (Marzetta and Hochwald)
  - Asymptotic expression for capacity (Zheng and Tse)
- Capacity of Rician fading channels
  - Fading known at the receiver (Farrokhi et. al.)
  - Isotropically random specular component (Godavarti et. al.)
  - Static specular component (Godavarti et. al.)

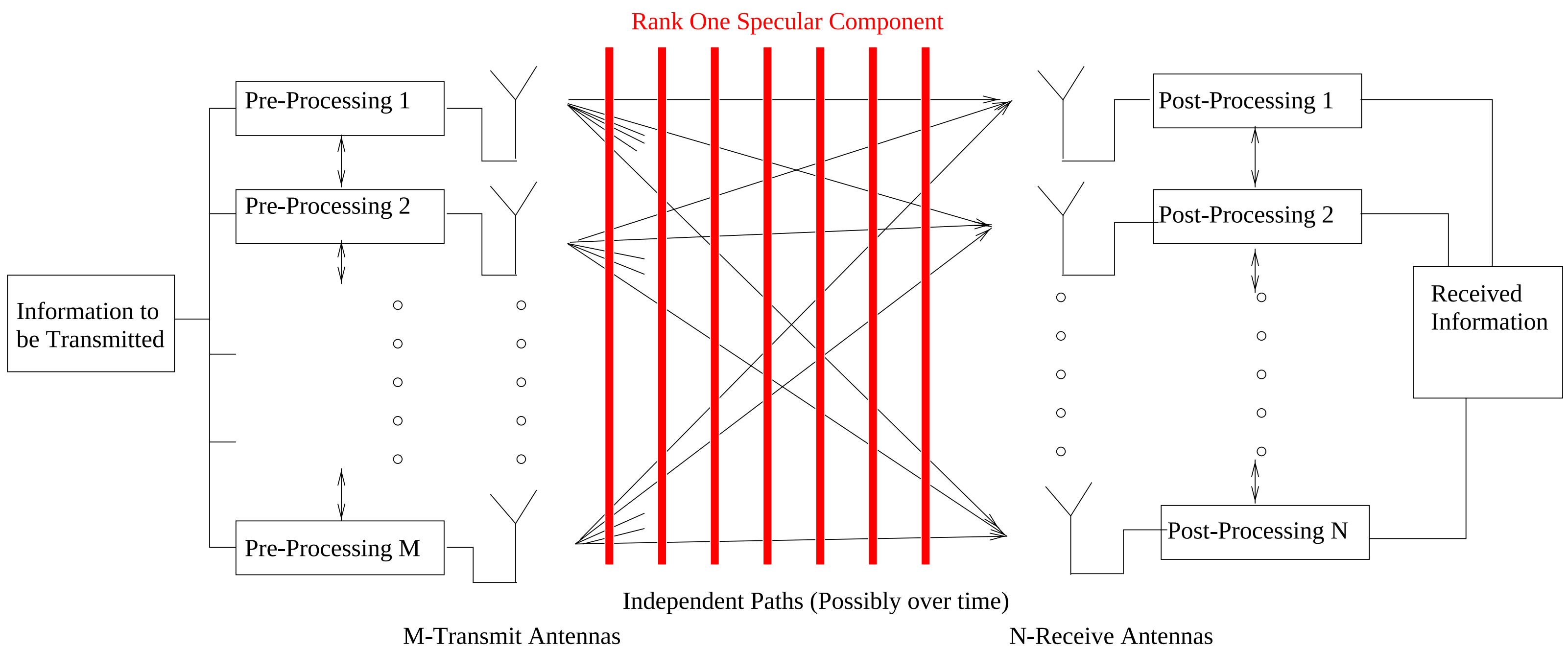

Figure 1. *Diagram of a multiple antenna communication system*

Channel Model

- Fading Model: Send $T \times M$ signal matrix S, Receive $T \times N$ signal matrix X

$$X = \sqrt{\frac{\rho}{M}} SH + W$$

  - H: $M \times N$ matrix of channel coefficients
    - Gaussian distributed for Rayleigh channel
    - deterministic for AWGN channel
  - W: $T \times N$ matrix of $\mathcal{CN}(0,1)$ random variables
  - M: Number of antennas at the transmitter
  - N: Number of antennas at the receiver
  - T: Symbol Coherence Interval
  - ρ: Average signal to noise Ratio at the receiver antennas

MIMO Rayleigh Capacity: avg power constraint: $\mathrm{tr}(E[SS^\dagger]) \le MT$

• T/R-informed capacity: $H = V\Lambda U^\dagger$ known to both T/R

$$\begin{aligned} C_1 &= E[C(H)], \quad (bits/sec/hz) \\ C(H) &= \max_{P_{S|H}} I(S,X|H) = T\ln\left|I_M + H^\dagger \Sigma_S H\right| \\ &= T \sum_{i=1}^{\min\{M,N\}} [\ln(\mu|\lambda_i|^2)]^+, \qquad \mu : \mathrm{tr}(\Sigma_S) = MT. \end{aligned}$$

After T/R spatial transformations (Beamforming)

$$S \to SV^\dagger, \qquad X \to XU$$

Capacity achieving source:

$$S \sim N\,(0,\, I_T \bigotimes \Sigma_S), \qquad \Sigma_S = \mathrm{diag}\left((\mu - 1/|\lambda_i|^2)^+\right)$$

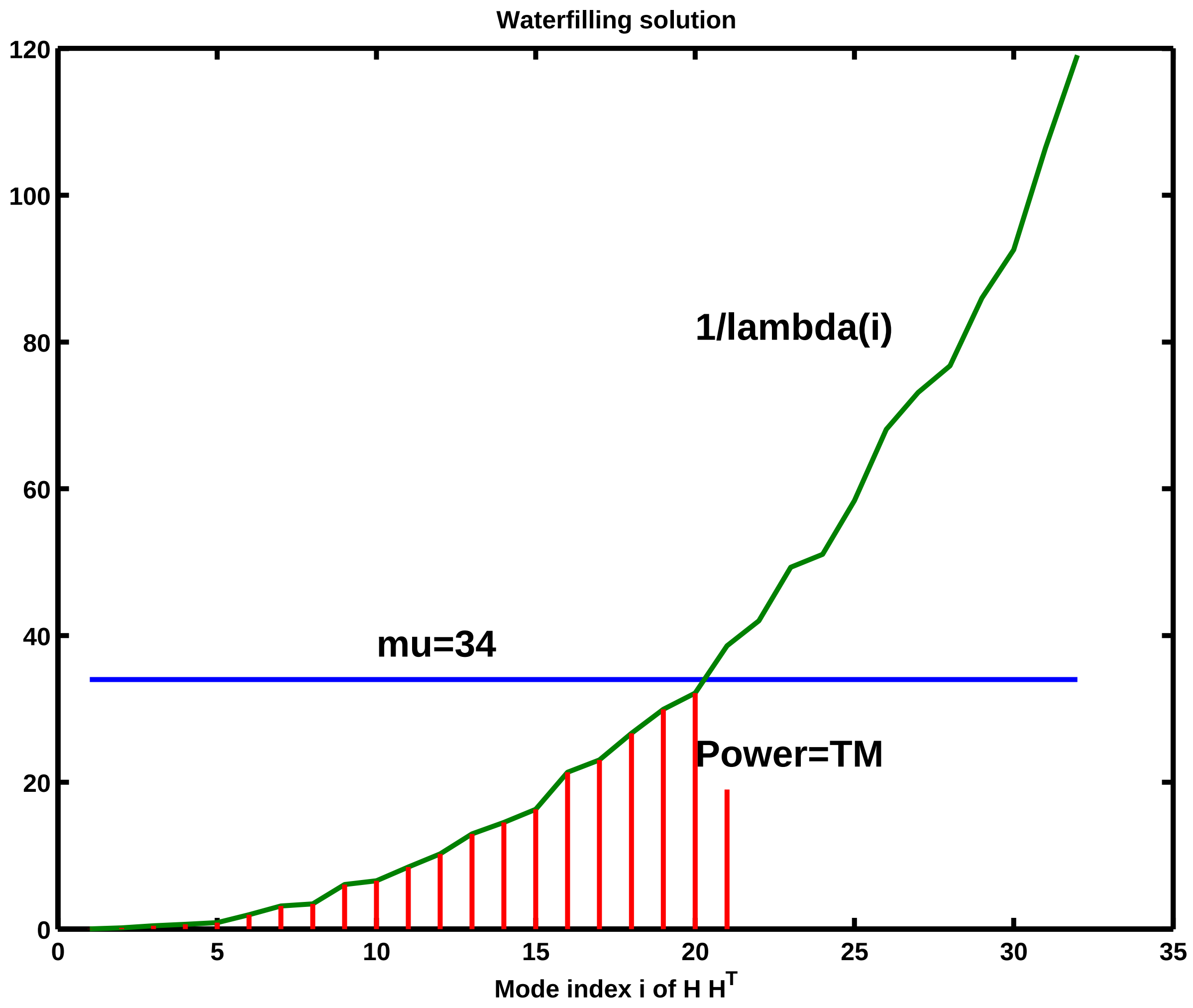

Figure 2. *Waterpouring solution for capacity achieving mode allocation*$(N = M = 32)$

MIMO Rayleigh Capacity: avg power constraint: $\mathrm{tr}(E[SS^\dagger]) \leq MT$

• R-informed capacity: H known to R only

$$C_2 = \max_{P_S} E[I(X,S|H)]$$

Capacity achieving source: i.i.d. Gaussian

$$S \sim N\left(0,\, I_T \bigotimes I_M\right)$$

Capacity achieving receiver: generalized beamformer $Y = XU$

• Uninformed capacity: H unknown to either T/R

$$C = \max_{P_S} E\left[\log P_{X|S}(X|S)/P_X(X)\right]$$

Capacity achieving source

$$S \sim V\Lambda$$

Rician Channel Model

- Combined Rayleigh and Specular Multipath Fading:

$$H = \sqrt{1-r}\, G + \sqrt{r}\, H_m$$

  - G_{mn} are i.i.d. $\mathcal{CN}(0,1)$
  - H_m deterministic matrix such that $\mathrm{tr}\{H_m H_m^{\dagger}\} = NM$
  - r fraction of channel energy devoted to specular component
  - H_m known to both the transmitter and receiver
  - G not known to the transmitter

- After spatial transformation (beamforming) at T/R: $H_m = [D, 0]$

R-informed Rician Capacity: Rank one H_m known to T/R

$$H_m = \sqrt{NM}\,\underline{e}_M \underline{e}_N^T = \begin{bmatrix} \sqrt{NM} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & 0 \end{bmatrix}$$

Capacity:

$$C_H = \max_{l,d} TE \log\det[I_N + \frac{\rho}{M} H^\dagger \Lambda^{(l,d)} H]$$

where

$$\Lambda^{(l,d)} = \begin{bmatrix} M-(M-1)d & l\underline{1}_{M-1} \\ l\underline{1}^\tau_{M-1} & dI_{M-1} \end{bmatrix}$$

- d is a positive real number such that $0 \le d \le M/(M-1)$
- l is a complex number such that $|l| \le \sqrt{(\frac{M}{M-1} - d)d}$

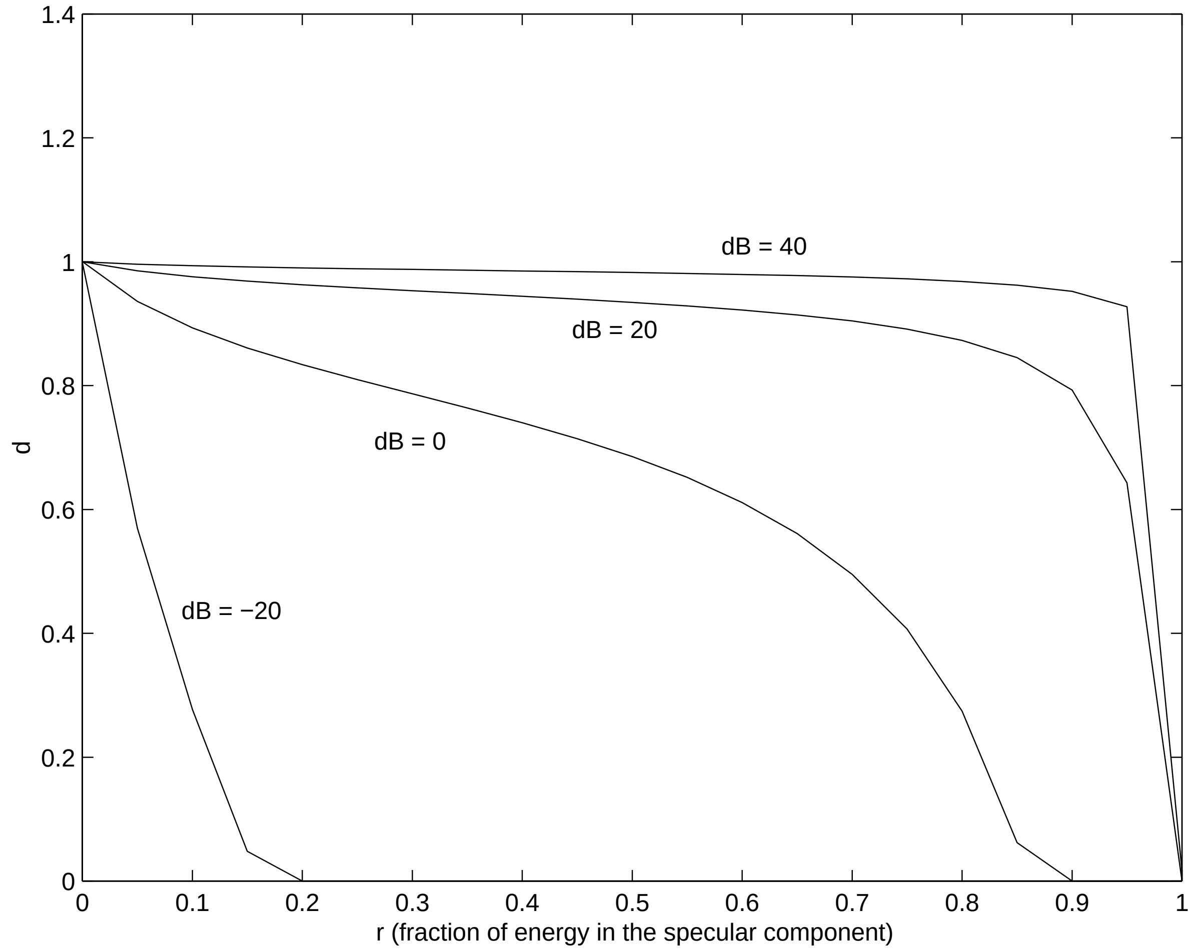

Figure 3. *Numerical optimization yeilds $l = 0$ and values of d shown as a function of r for different values of* ρ.

General Rank H_m

- Capacity achieving signal $S = \Phi V \Psi^\dagger$ where Φ is independent of V and Ψ
  - Φ: $T \times T$ isotropically random unitary matrix
  - V: $T \times M$ random diagonal matrix
  - Ψ: $M \times M$ random unitary matrix
- Low SNR Rician capacity

$$C_R \approx T\rho \left[r\lambda_{max}(H_m H_m^\dagger) + (1-r)N \right]$$

- High SNR Rician capacity for $T \geq 2M$ and $M < N$

$$C_R \approx M(T-M)\log\rho$$

achieved by $S = \sqrt{T}\Phi$ for $T \to \infty$.

Rayleigh Training-Based Communications

- T_t: number of channel uses devoted to training on G
- S_t: training signal
- κ: fraction of the energy devoted to communication
- $T_c = T - T_t$: number of channel uses devoted to communication
- S_c: communication signal
- Channel in the training phase

$$X_t = S_t(\sqrt{r}H_m + \sqrt{1-r}G) + W_t$$

  - X_t: $T_t \times N$
  - S_t: $T_t \times M$
  - Energy constraint on the training signal $E[\mathrm{tr}\{S_t S_t^\dagger\}] \leq (1-\kappa)TM$

- Generate MMSE estimate of G via recieved training signal
  $\hat{G} = \sqrt{1-r}(\sigma^2 I_M + (1-r)S_t^\dagger S_t)^{-1} S_t^\dagger [X - \sqrt{r} S_t H_m]$
- Channel in the communication phase

$$X_c = S_c(\sqrt{r} H_m + \sqrt{1-r}\hat{G}) + \tilde{W}_c$$

- Lower bound on normalized training capacity ($E[\mathrm{tr}\{S_c S_c^\dagger\}] \leq \kappa T M$)

$$C_T \geq (T - T_t) E \log\det\left(I_M + \frac{\rho_{eff}}{M} H_1 H_1^\dagger\right)$$

where

  - $H_1 = \sqrt{r_{new}} H_m + \sqrt{1 - r_{new}}\hat{G}$
  - $r_{new} = \frac{r}{r + (1-r)\sigma^2_{\hat{G}}}$
  - $\rho_{eff} = \frac{\kappa T \rho [r + (1-r)\sigma^2_{\hat{G}}]}{T_c + (1-r)\kappa T \rho \sigma^2_{\tilde{G}^2}}$
  - $\tilde{G} = G - \hat{G}$

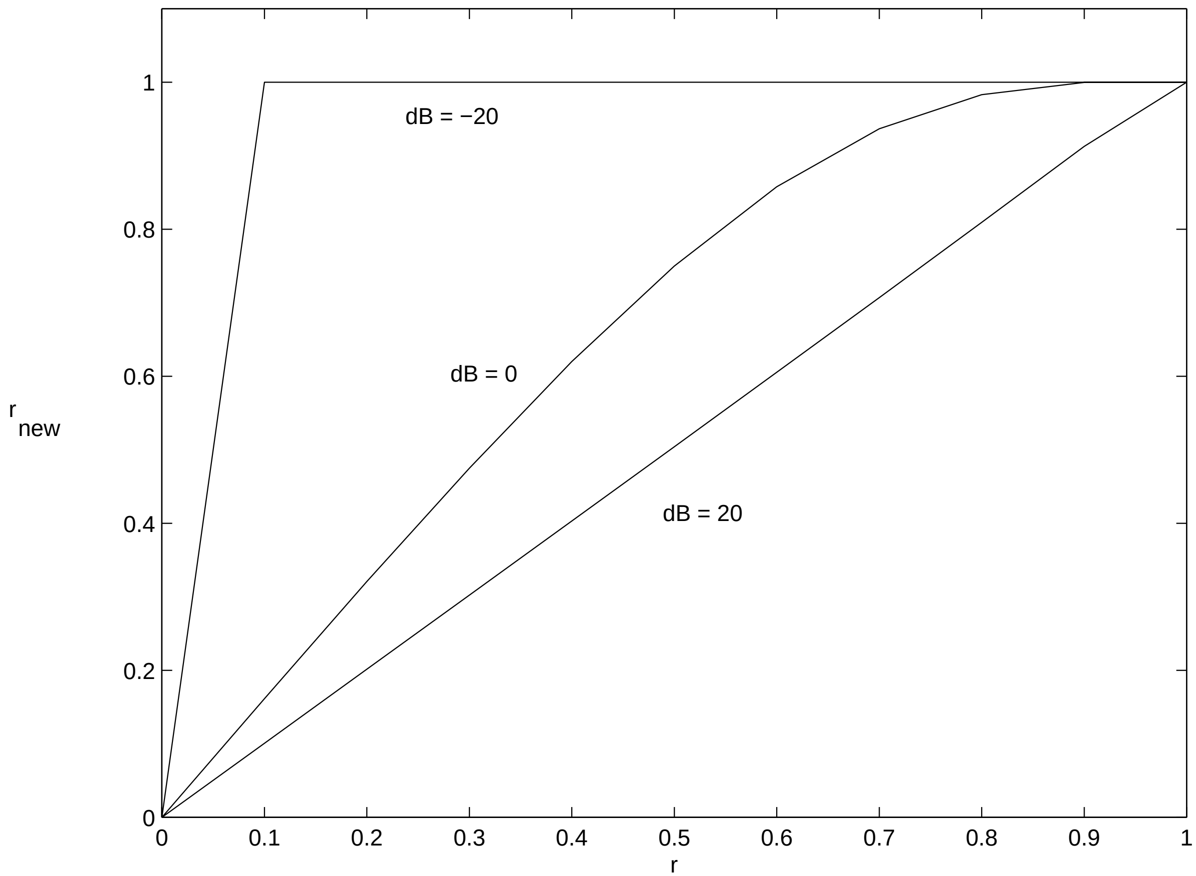

Figure 4. *Plot of r_{new} as a function of parameter r for $M = N = 5$, $T = 40$ and $H_m = I_M$*

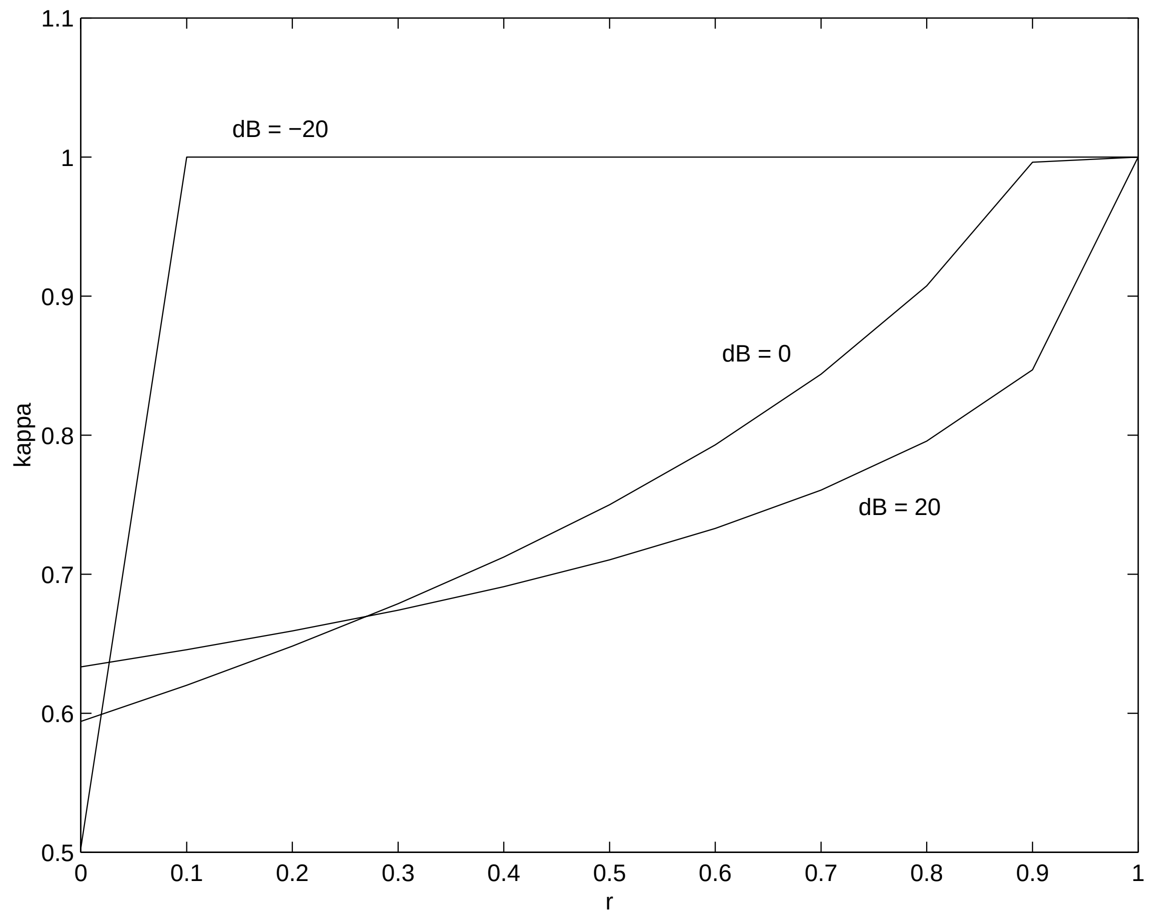

Figure 5. *Plot of optimal energy allocation* κ *as a function of Rician parameter* r *for* $M = N = 5$, $T = 40$ *and* $H_m = I_M$

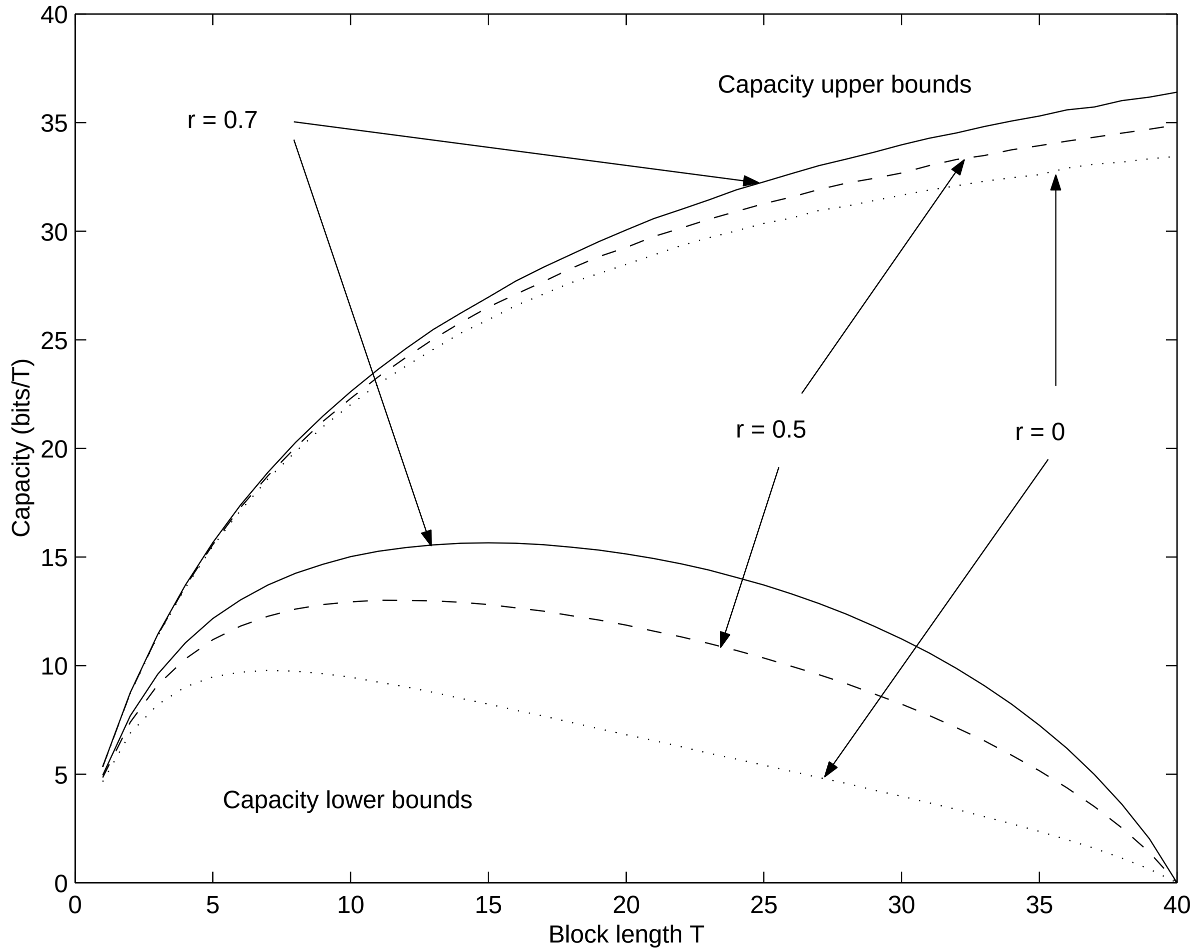

Figure 6. *Capacity upper bound* $E[C_R(H)]$ *and lower bound* C_T *as function of* M *for* $T = 40$, $N = 40$, $\rho = 1$, $H_m = [I_M, 0]$.

Conclusions

- Mixed specular and diffuse fading require new signaling strategies
- At low SNR ρ specular beamforming is optimal and $C_R = C_1$.
- At high SNR ρ combined beamforming and unitary signaling is optimal
- For high SNR and large coherence interval T Rayleigh optimal signaling achieves capacity
- Exploration of optimal power allocation and optimal transmit diversity for training via capacity bounds
- Codes that attain these capacities?