SEGMENTATION OF ROAD EDGES FROM A VEHICLE-MOUNTED IMAGING RADAR

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ABSTRACT

This work treats a problem arising in intelligent vehicles and highway systems (IVHS), specifically in automatic road following, collision avoidance, and maneuver control. A millimeter-wave radar is placed on the front of the vehicle and an image of the radar backscatter of the terrain is acquired in polar coordinates. This image contains both road and offroad scatter components which must be segmented from each other. The radar sensor acquires the scene in polar coordinates while constraints on the road edges are naturally formulated in the cartesian cartographic domain. One novelty of our approach is to apply the road constraints in polar coordinates which allows us to estimate the road edge parameters directly from recorded data. Onother novelty is to construct an estimator which minimizes a cost criterion which is largely insensitive to the off-road scatter components. Numerical results are given for real data (L band radar images of a Southeast Michigan roadway) which indicate the accuracy and robustness of our approach.

1. INTRODUCTION

The recovery of road edges from front-mounted radar and optical sensors has been of recent interest in intelligent highway vehicle systems (IVHS) and presents several challenges. In particular edge detection must have high reliability and be robust to many changing conditions such as: weather, visibility, clutter and noise, road surface variability, presence of cross streets, exits, crosswalks, etc. Furthermore, any road edge segmentation algorithm must operate in real time. This requires a fairly simple model, a segmentation criterion which is easily to optimize, and fast convergence.

In this paper we present an algorithm which has the above properties. It is based on estimation of a pair of tightly coupled parabolic curves defined in the cartographic domain and which are transposed in the polar domain of the raw image acquisition. Unlike the method of [5], which applies a loosely constrained hyperbolic road model in cartesian image coordinates, the method proposed here performs curve estimation directly in the polar domain. Furthermore, robustness to off-road clutter is achieved by using a non-linear least squares estimation criterion which involves only pixels between the estimated road edges, i.e. estimated road pixels, for which faithful prior information is available. This simplification allows us to perform a simple numerical optimization without resorting to time consuming Monte Carlo optimization methods. An issue which is currently under study is the choice of search space for the parameter optimization. We will illustrate the proposed technique with real images acquired in Michigan from an L band radar mounted on a military transport vehicle on a country road.

2. THE MAIN ASSUMPTIONS

The segmentation procedure will be developed in this paper under the following assumptions.

(a) A road has a regular profile which can be modeled using a smooth function. Moreover, the road width is constant inside the field of view.

(b) Data $\boldsymbol{z}$ determine a field of view with azimuth between -31° and 32° and a range of up to 128 meters.

(c) The road has constant curvature within the field of view. Given the fact that the field of view is restricted, this assumption is realistic.

(d) The road constitutes a truly homogeneous region. No assumption can be formulated for the offroad zones.

(e) The angular resolution of the radar data decreases linearly with distance.

3. PARABOLIC ROAD EDGE MODEL

A birds-eye view of the road, which we call the cartographic domain, is represented on a cartesian lattice with axes (x, y). The left and right edges of the road, $\mathcal{X}_l$ and $\mathcal{X}_r$ respectively, are defined as functions of the vertical axis y. Following the example of [5, 2], these are modeled using parabolic curves:

$$\mathcal{X}_l(y) = ay^2 + by + c_l \quad \text{and} \quad \mathcal{X}_r(y) = ay^2 + by + c_r \quad (1)$$

where a, b and c_l, c_r are the parameters that we shall seek. Furthermore, the width of the road is denoted d and, since the

vehicle is between the left edge and the right edge, we have:

$$c_r > 0 \quad \text{and} \quad c_l = c_r - d < 0 \tag{2}$$

The recorded image $\boldsymbol{z}$ is acquired on a rectangular lattice (φ, ρ) whose axes,

$$\varphi \in [-31\pi/180, 32\pi/180] \text{ and } \rho \in [1/2, 128] \tag{3}$$

are the transform in polar coordinates of the cartesian coordinates,

$$x = \rho \sin(\varphi) \text{ and } y = \rho \cos(\varphi). \tag{4}$$

However, the model for the road edges is naturally defined in the (x, y) domain. An issue is to transform data $\boldsymbol{z}$ into the (x, y) domain: such an approach has been used in [5] which has the advantage of simplifying calculations. The drawbacks are that interpolating data into the cartographic domain generates errors which can be amplified during the edge detection stage and makes it difficult to exploit the advantages of assumption (e).

Instead, we tranpose the cartographic edge model (1) into the polar domain. Then, the range of edges $\mathcal{R}_l$, $\mathcal{R}_r$ is expressed as a function of the angular position φ using

$$\mathcal{R}_l \sin(\varphi) = a\mathcal{R}_l^2 \cos^2(\varphi) + b\mathcal{R}_l \cos(\varphi) + c_l \tag{5}$$

$$\mathcal{R}_r \sin(\varphi) = a\mathcal{R}_r^2 \cos^2(\varphi) + b\mathcal{R}_r \cos(\varphi) + c_r \tag{6}$$

The analysis of the roots of these equations permits to determine the road edges $\mathcal{R}_l(\varphi)$ and $\mathcal{R}_r(\varphi)$:

$$\text{if} \quad ac_{l,r} \leq 0 \tag{7}$$

$$\mathcal{R}_{l,r}(\varphi) = \frac{-\mathcal{B}(\varphi) + \text{sign}(a)\sqrt{\mathcal{B}^2(\varphi) + 4|ac_{l,r}|\cos^2(\varphi)}}{2a\cos^2(\varphi)}$$

$$\text{where} \quad \mathcal{B}(\varphi) = b\cos(\varphi) - \sin(\varphi)$$

$$\text{if} \quad ac_{l,r} > 0 \tag{8}$$

$$\mathcal{R}_{l,r}(\varphi) = \frac{-\mathcal{B}(\varphi) \pm \sqrt{\mathcal{B}^2(\varphi) - 4ac_{l,r}\cos^2(\varphi)}}{2a\cos^2(\varphi)} \quad \text{defined for}$$

$$a < 0 \text{ and } \varphi \leq \arg\tan(-2\sqrt{ac_{l,r}} + b)$$

$$a > 0 \text{ and } \varphi \geq \arg\tan(2\sqrt{ac_{l,r}} + b)$$

In these equations, subscript l, r means that the relevant parameter concerns both the left and the right edges. When $ac_{l,r} > 0$, edges $\mathcal{R}_l$ and $\mathcal{R}_r$ are multi-valued functions of φ: they take two different values for the same φ which correspond to the cases $+$ and $-$ in (8).

4. SEGMENTATION METHOD

The radar image $\boldsymbol{z}$ is given over a discrete lattice

$$\{(\varphi_m, \rho_n),\ -M/2 + 1 \leq m \leq M/2,\ 1 \leq n \leq N\} \tag{9}$$

(where without loss in generality M is supposed even). After discretization of $\mathcal{R}_l$ and $\mathcal{R}_r$ along the ρ-axis, (7-8) is inverted and the road edges are expressed in the polar domain as a function of the range, $\Phi_l(\rho_n)$ and $\Phi_r(\rho_n)$ for $1 \leq n \leq N$. The pixels belonging to the road are

$$\mathcal{S} := \{(n, m) \text{ such that } \Phi_l(\rho_n) \leq \frac{m\,\pi}{180} \leq \Phi_r(\rho_n)\} \tag{10}$$

Road $\mathcal{S} = \mathcal{S}(a, b, c_r, d)$ is a function of the model parameters. Estimating its edges amounts to determine the optimal $\mathcal{S}(\widehat{a}, \widehat{b}, \widehat{c}_r, \widehat{d})$. The left and right offroad regions are denoted by $\mathcal{T}_L$ and $\mathcal{T}_R$. The likelihood function of the radar image can be described using a log-normal distribution [1, 5]:

$$P(\boldsymbol{z}|\mathcal{S}) = \prod_{C \in \{\mathcal{S}, \mathcal{T}_L, \mathcal{T}_R\}} \prod_{(n,m) \in C} \frac{\exp[-\frac{(\log z_{n,m} - \mu_C)^2}{2\sigma_C^2}]}{z_{n,m}\sqrt{2\pi\sigma_C^2}} \tag{11}$$

where μ_C and σ_C^2 are the mean and the variance, respectively, of region C. This model supposes homogeneous side regions $\mathcal{T}_L$ and $\mathcal{T}_R$. Note that the evaluation of any criterion based directly on (11) involves all the pixels of $\boldsymbol{z}$. Henceforth, we use log-data $y_{n,m} = \log z_{n,m}$.

Let us observe that if we are given the road width $\widehat{d}$, a simple criterion, involving *only the pixels of the road*, can be constructed:

$$\mathcal{J}(a, b, c_r\ ;\ \widehat{d}, \boldsymbol{y}) = \frac{\sum_{(n,m) \in \mathcal{S}(.)} (y_{n,m} - \widehat{\mu}_{\mathcal{S}(.)})^2}{\#\{\mathcal{S}(a, b, c_r, \widehat{d})\}} \tag{12}$$

$$\widehat{\mu}_{\mathcal{S}(.)} = \frac{\sum_{(n,m) \in \mathcal{S}(.)} y_{n,m}}{\#\{\mathcal{S}(.)\}}$$

where $\#$ stands for cardinality, $\mathcal{S}(.)$ for $\mathcal{S}(a, b, c_r, \widehat{d})$, and $\widehat{\mu}_{\mathcal{S}(.)}$ is the empirical mean over the road. Parameters a, b, c_r are estimated as

$$(\widehat{a}, \widehat{b}, \widehat{c}_r) = \arg\min_{a,b,c_r} \mathcal{J}(a, b, c_r\ ;\ \widehat{d}, \boldsymbol{y}) \tag{13}$$

The proposed method (12-13) is to search over the recorded image for *the patch of constant width and curvature which has the minimum variance, i.e. which is the most homogeneous.* This constitutes a meaningful objective which leaves the side regions $\mathcal{T}_L$ and $\mathcal{T}_R$ out of consideration, and thus improves robustness to off-road variability (d).

Furthermore, (e) implies that the road width can reliably be estimated only from data corresponding to a few tens of meters in front of the vehicle. In our method, the road width d is estimated from a section of the road of about $\ell = 30$ meters. Over such a short section, the road edges can be assumed linear, which corresponds to $a = 0$ in (1). Then,

$$\widehat{d} = \arg\min_{b,c_r,d} \mathcal{G}(a = 0,\ b, c_r, d) \tag{14}$$

$$\mathcal{G}(a, b, c_r, d) = -\log P\{[\boldsymbol{z}]_0^\ell \mid \mathcal{S}(a, b, c_r, d)\} \tag{15}$$

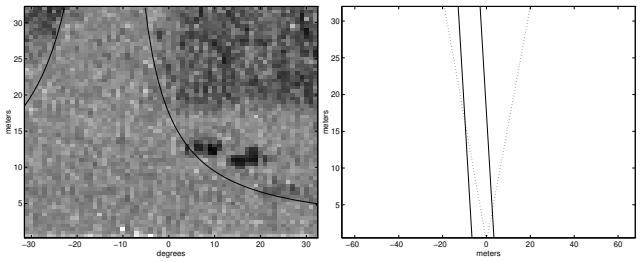

Figure 1: Coarse estimate of road width from the front parcel of the road, using a linear model for the road edges.

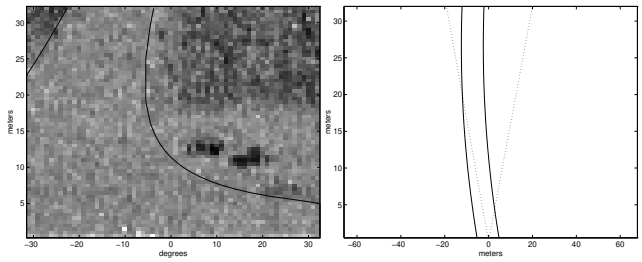

Figure 2: Refined estimate of road width from the same parcel of the road using a parabolic model for the road edges.

where $[\boldsymbol{z}]_0^\ell$ means that a ℓ-length section of the image is considered. In such a situation, only parameters (b, c_r, d) are to be estimated while these are related by a simpler expression:

$$\widetilde{\mathcal{R}}_{l,r}(\varphi) = \frac{c_{l,r}}{\sin(\varphi) - b\cos(\varphi)} \tag{16}$$

where $\widetilde{\mathcal{R}}_{l,r}$ are the road edges relevant to this linear local model. Moreover, the latter equation admits an analytical inversion which yields

$$\widetilde{\Phi}_{l,r}(\rho) = 2\arctan\frac{-\rho + \sqrt{(1+b^2)\rho^2 - c_{l,r}^2}}{b\rho - c_{l,r}} \tag{17}$$

which yields a unique result belonging to the admissible set (3).

A finer tuning of $\widehat{d}$ can be obtained by allowing $a \neq 0$ in (14) and performing a local minimization of $\mathcal{G}$ in the vicinity of the already obtained width estimate.

In spite of its quadratic form, criterion $\mathcal{J}$ is multimodal with respect to parameters a, b, c_r. Criterion $\mathcal{G}$ is multimodal as well. In a similar context, EM minimization of the relevant criteria has been proposed in [4]. However, our edge model depends on a very restricted number of parameters, while both criteria $\mathcal{J}$ and $\mathcal{G}$ concern only on a small parcel of the image: these two facts permit an optimization by "exhaustive" search.

5. EXPERIMENTAL RESULTS

In this section road segmentations are shown in the domain of the data, *i.e.* in the polar domain (φ, ρ). Next to these segmentations is plotted the corresponding birds-eye view of the road in the (x, y)-domain, and the radar sensor field of view is plotted with a dotted line and has a V-shaped boundary.

Fig. 1 shows a coarse estimate of road width using the linear model for the road edges, as given in (14-15). This yields an estimate $\widehat{d} = 10$ m.

A refined parabolic estimate of road width is obtained by finding the least squares fit for arbitrary $a \neq 0$ in (15). The result is given in Fig. 2 confirming that $\widehat{d} = 10$ m.

The segmentation presented in Fig. 3 corresponds to $\widehat{a} = 0.002$, $\widehat{b} = 0.1476$ and $\widehat{c}_r = 2.25$ and has been calculated using (12-13). The edges of the road are correctly retrieved and it is seen that the road has a gentle right turn.

The global minimization of criteria $\mathcal{J}$ and $\mathcal{G}$ has been performed using an "exhaustive" search over a discrete set of sampled parameters. The numerical efficiency of such a method critically depends on the number and the distribution of samples of the parameters used for the search. We reduce the number of samples by restricting the search domain. First we sample only over the *a priori* known feasibility domain. Second, we use a simple coarse estimation procedure to obtain a reduced region of interest (ROI) which contains the road with high likelihood.

The *a priori* feasibility domain contains all the parameters yielding a pair of road edges which remains in the field of view for at least $\mathcal{L}$ meters:

$$|av^2 + bv| \leq vF \quad \text{for} \quad 0 \leq v \leq \mathcal{L} \tag{18}$$

where $F = \frac{1}{2}(\tan\frac{32\pi}{180} + \tan\frac{-31\pi}{180})$. Note that $-vF$ and vF are the boundaries of the field of view. Parameter c_r is the distance between the radar (the car) and the right edge, which determiens its order of magnitude. The same holds for the road width d.

A tight region of interest (ROI) containing the road, at a low numerical cost is obtained by the following steps. First the image is partitioned into several sections (4, for instance) and in each of them the road edges are modeled by linear segments, similarly to (16-17), while ensuring the continuity of the road. More precisely, a term of the form (12) is defined over each segment, starting from the second segment (since the first one has been estimated by (14)). It is important to note that each road segment is described using one parameter b, since $a = 0$ whereas c_r are fixed by road continuity constraints. The criterion used to generate a piecewise linear segmentation of the road edges is given as the sum of these terms.

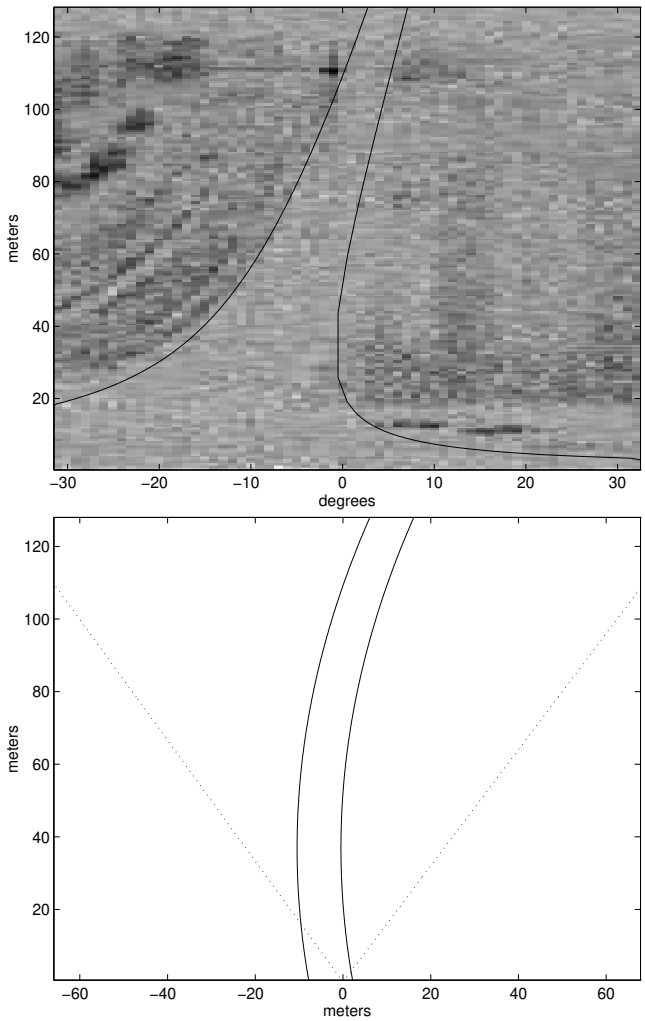

Figure 3: The final segmentation obtained using the proposed algorithm faithfully retrieves the locations of the road edges.

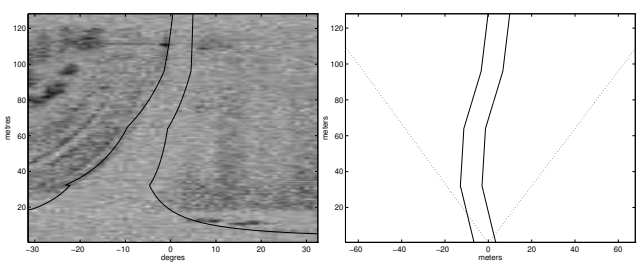

Figure 4: Coarse and rapid segmentation of the road using a locally linear model for its edges. It provides a faithful ROI for a further estimation of the location of the road.

Although this produces estimates of road edges which may exhibit sharp corners, it provides a faithful approximation of the road shape. Note that such a locally linear model is not very sensitive to the fineness of the sampling of the parameters. This coarse estimate is rapidly computed and it is used to restrict the search space for the parabolic model.

It can be seen from Fig. 4 that our ROI surrounds the above estimate: more closely just in front of the car and more loosely with the increase of the distance from the car. This ROI yields an additional set of strong but meaningful constraints for the parameters of the parabolic model.

6. CONCLUDING REMARKS

We have presented a novel technique for estimating road edges from polar scan images acquired from vehicle-mounted radar sensors. The technique proposed is accurate, robust and simple to implement. Some issues which remain to be studied are the following. Iterative methods may be useful for reducing the computation time. Further performance improvements may be obtained by: 1) modeling the on or off-road regions with a Gauss Markov field; 2) incorporating a more accurate model for the backscatter probability distribution; 3) adding another sensor modality, e.g. optical or multiple radar sensors; 4) using information from previous images in the sequence to track the road boundaries via local updating.

7. REFERENCES

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