Special Issue of IEEE Signal Processing Magazine on Mathematical Methods in Imaging Introduction

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Mathematical methods have had a long history in imaging. For example, as recently pointed out by Glasbey and Mardia [3], in the later part of the nineteenth century Sir Francis Galton[1] applied warping to align and average portrait photographs of known criminals in hope of discovering the archetypical "criminal face" [2]. More recent examples of mathematics in imaging are: analytical methods for tomographic image reconstruction, differential geometric methods for shape and surface representation, projective geometries for 3D computer vision, and group theoretic representations of image invariants. The articles in this special issue highlight the imaging applications of some emerging mathematical approaches including: fractals and self similarity; non-linear partial differential equations (PDE's), anisotropic diffusion; variational methods; pattern theory; stochastic point processes; and random graph theory.

The special issue opens up with three papers on applying PDE's to images. In the first paper Tschumperlé and Deriche present vector-valued diffusion PDE's for restoration and noise removal in color images. These multivalued PDE's are ingeniously constructed to produce a solution (image restoration) having desired local geometrical properties. In the second paper Pollak presents a non-linear diffusion approach to multiscale segmentation and noise suppression. Among other interesting results, the author shows that non-linear scale-space curve evolution is fast and optimal in the sense of maximizing a norm-penalized maximum likelihood (ML) criterion. Ben Hamza, Unal and Krim round out this group of papers by providing a unifying framework for PDE diffusion approaches covering both stochastic (Markovian) and deterministic (energy constrained) image models. Using insights gained from this unifying perspective they present an interesting hybrid gradient descent flow algorithm based on a combination of negentropy and total variation norms of the gradient field.

[1] F. Galton was a polymath geneticist, anthropologist, philosopher, and Africa explorer who established the field of biometry. Among his other exploits was founding the fields of eugenics, differential psychology, and fingerprint analysis, pioneering the field of meteorology, developing rank-order statistics and inventing the statistical methods of correlation and regression [1].

The special issue continues with two articles on stochastic image modeling. The article by Pesquet-Popescu and Lévy Véhel presents stochastic fractal and multifractal models for texture in images. These models are related to fractional Brownian motion (fBM) random field representations and the authors discuss extensions of these representations to useful non-isotropic models which can be applied to non-continuous digitized imagery. Srivastava presents an intriguing new class of mixture models for capturing observed heavytailed marginal densities of image gradients. These models are Bessel K forms which exploit the fact that natural images contain many separate objects that can be modeled as components following a Poisson point process.

The issue concludes with two papers on image analysis. Descombes and Zerubia present a marked point process framework as an alternative to Markov random fields to better capture the geometrical constraints and non-Gaussian noise attributes of natural images. The authors describe a novel birth and death process for expanding and pruning random graphs defining a path through the configuration space, e.g. a road network on a roadmap or the edges of buildings on a digital elevation map (DEM). We wrap up this special issue with a paper by Hero, Ma, Michel and Gorman who review the intriguing asymptotic theory of random graphs and use this theory to motivate minimal graph algorithms for pattern matching, image registration, and image retrieval. The key property underlying these algorithms is that good minimal graphs are "entropic:" their total weight converges to the Rényi entropy of the points connected by the graph.

References

[1] "Obituary of Sir Francis Galton D.C.L. D.Sc F.R.S.," *J. Royal Statistical Society*, pp. 314–320, Feb. 1911.

[2] F. Galton, "Composite portraits," *Journal of the Anthropological Institute of Great Britain and Ireland*, vol. 8, pp. 132–142, 1878.

[3] C. A. Glasbey and K. Mardia, "A penalised likelihood approach to image warping (with discussion)," *J. Royal Statistical Society, Ser. B*, vol. 63, pp. 465–514, 2001.