

**LOW POWER SIGNAL PROCESSING STRATEGIES  
DoD MURI Low Energy Electronics Design for Mobile  
Platforms**

**DOD-G-DAAH4-96-1-0377**

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**July 9, 2002**

## **ARO-MURI RESEARCH MILESTONES**

1. **Reduced wordlength strategies** for channel equalization and detection (Gupta)
  - **Fixed point LMS:** Transient and steady-state behavior (Hero&Gupta ISIT-98, SSAP-98, MILCOM-98, ICASSP-99, ICASSP-00, **TSP-01a**).
  - **Optimal compression for detection** (Gupta&Hero URSI-00, ISIT-01, SIAM-01, **TIT-01a**)
2. **Partial update strategies** for equalization and beamforming (Godavarti)
  - Stability analysis of S-LMS and P-LMS for cyclostationary signals (Godavarti&Hero ICASSP-99, ICASSP-01, **TSP-01b**).

- New stochastic P-LMS algorithm with better convergence  
(Godavarti&Hero SAM-00, ICASSP-01, **TSP-01c**).
3. **Space-time processing** and space-time coding (Godavarti)
- **Cutoff rates and optimal power allocation** for Rayleigh fading  
(Hero&Marzetta SAM-00, ISIT-00, **TIT-01b**)
  - **Channel Capacity, diversity and DOF** for Rician Channels  
(Godavarti, Hero, Marzetta ISIT-01, Godavarti&Hero ISIT-02,  
ICASSP-02, URSI-02, **TIT-01c, TIT-01d**)
4. **Secure space time modulation** (Hero **TIT-01e**)

## **INDUSTRY/GOV'T. INTERACTIONS**

### 1. Invited seminars on low power MURI topics (Hero)

Hughes Network Systems, Lucent Technologies, MIT Lincoln Laboratory, France Telecom

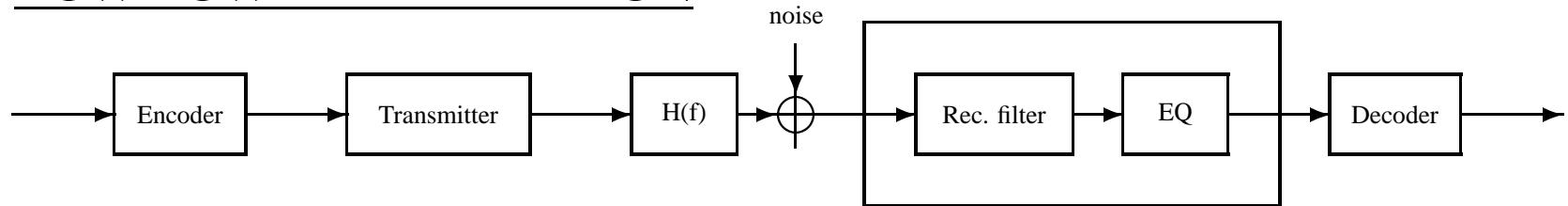
### 2. Industrial Internships

- Bell Labs, Lucent Technologies (Murray Hill) - Hero (1999)
- Bell Labs, Lucent Technologies (Murray Hill) - Godavarti (2000)
- Rockwell Inc. (LA) - Gupta (2000)

### 3. Government Panels and Study Groups (Hero)

DARPA (2000), SEDD-ARL (2002)

## LOW POWER ADAPTATION



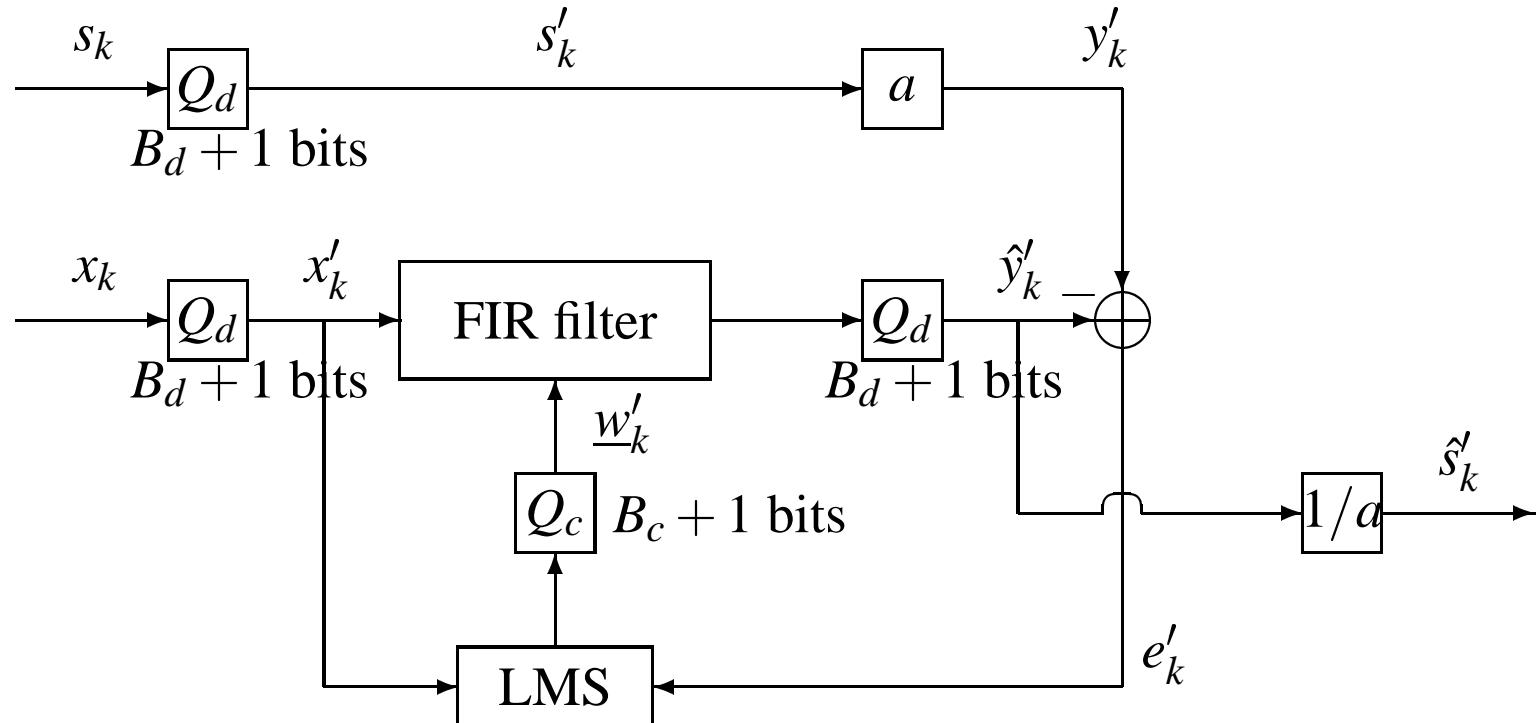
**Objective:** Reduce power consumption of channel equalizer with minimal loss in adaptation performance.

### Other applications:

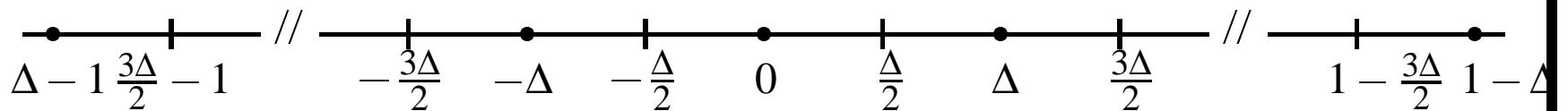
- System ID (channel identification)
- Adaptive beamforming
- Adaptive interference cancellation

**Innovation on previous work:** Power-optimal bit allocation.  
 (Caraicos&Liu 84, Alexander 87, Douglas&Meng 91,  
 Bershad&Bermudez 96)

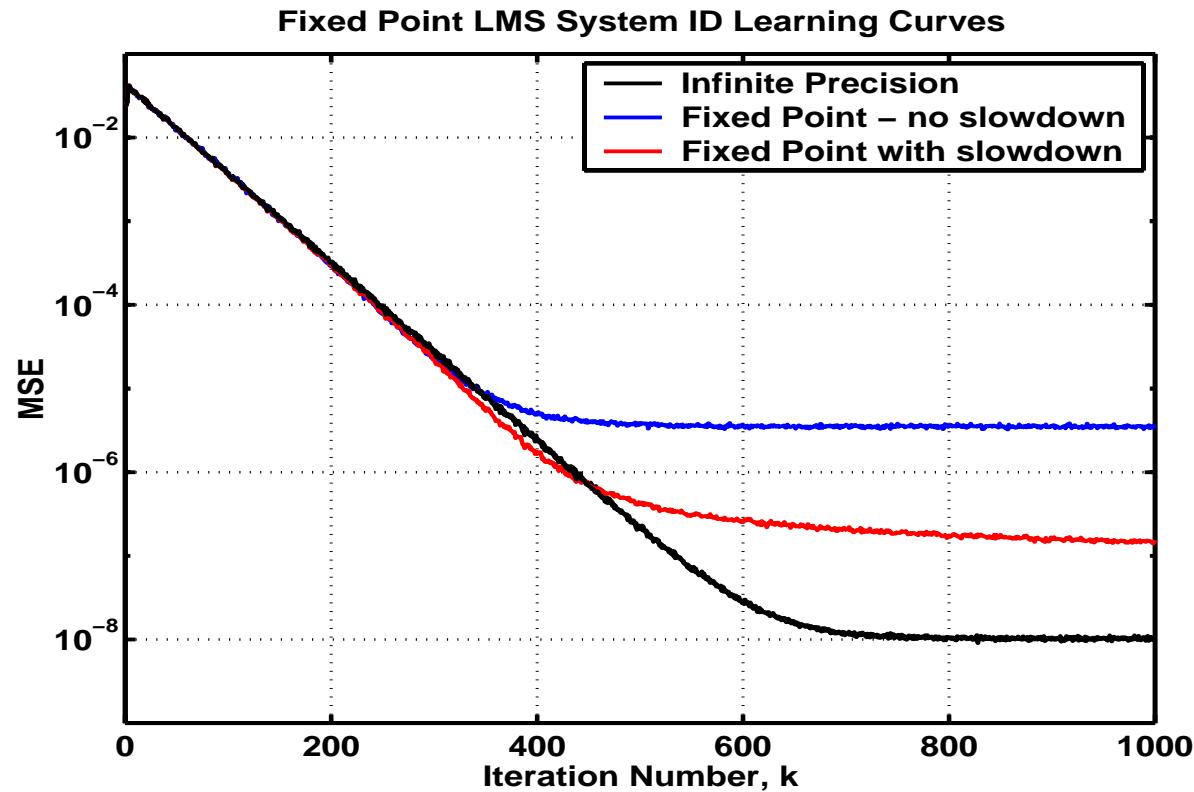
## Fixed point LMS adaptation



**Figure 1. Finite precision LMS algorithm.**



## Fixed Point LMS Adaptation



**Figure 2. Fixed point LMS learning curves.**

Note “slowdown” in red curve and residual MSE in blue curve.

## Power Optimization of Finite Resolution MSE

Finite resolution LMS power consumption:

$$P_T = 4p[(2B_d + B_c + 2)\eta_a + (3B_d + B_c)\eta_t]$$

$\eta_a, \eta_t$  = power per add, table lookup

Bit allocation factor  $\rho$ :

$$\rho = \frac{B_d}{B_d + B_c}$$

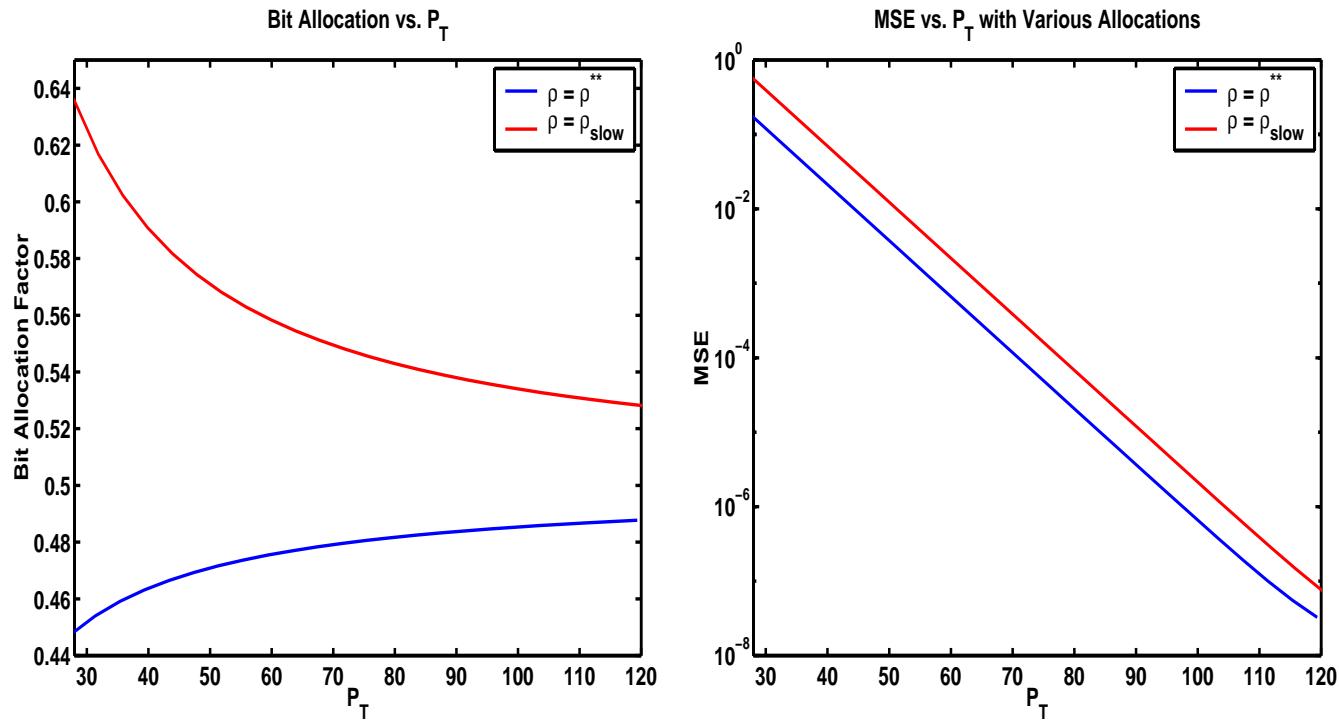
Can **optimize**  $\rho$  with respect to steady-state MSE with **power constraint**, assuming no slowdown.

$$\rho^P = \min\{\rho_{slow}, \rho^{**}\}$$

$\rho_{slow}$  = maximum  $\rho$  such that slowdown does not occur.

$\rho^{**}$  = optimal  $\rho$  with respect to MSE.

## Power Optimization: Example

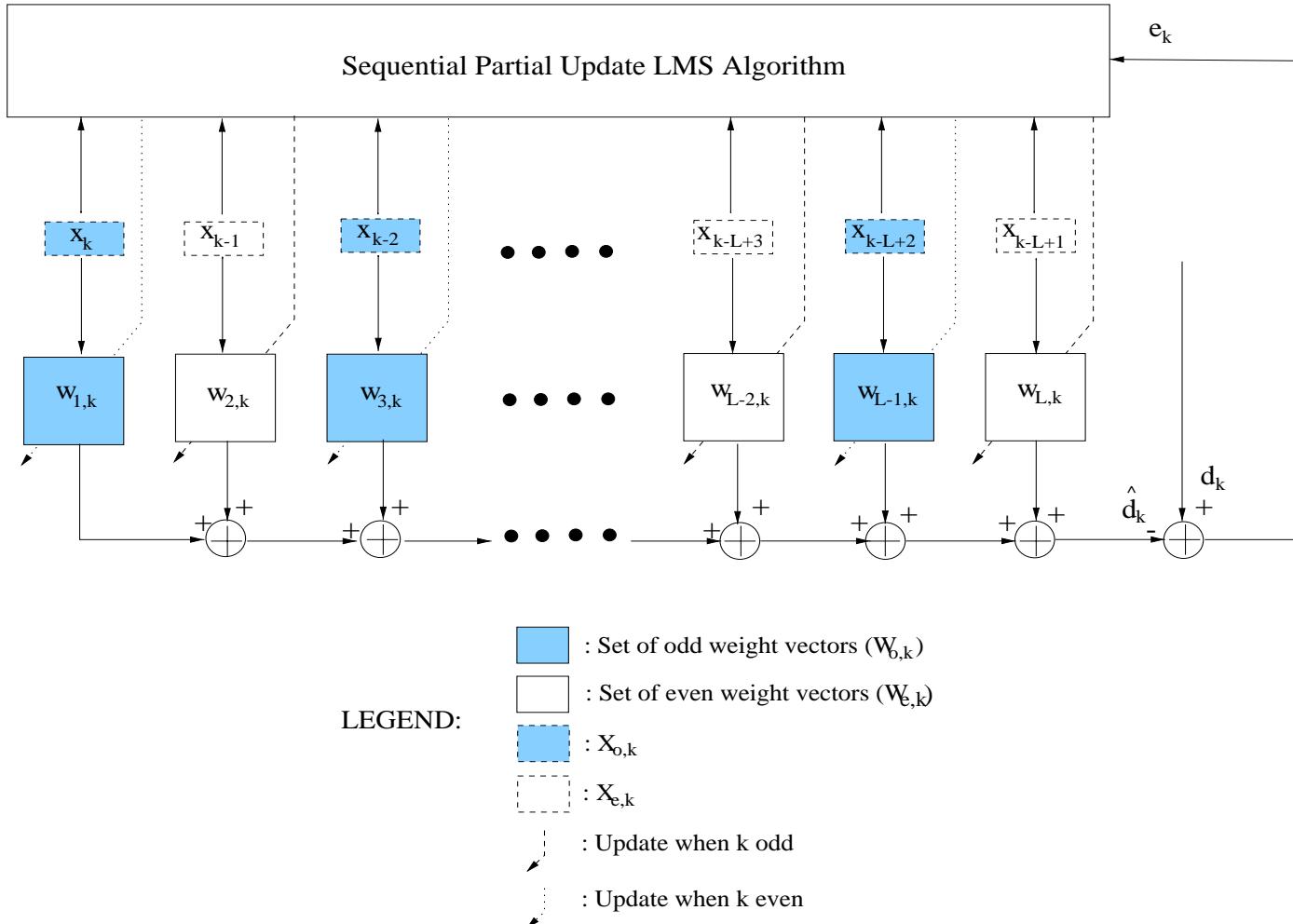


**Figure 3. Optimal bit allocation and resultant MSE for LMS channel equalization example.**

## **Partial Update LMS Algorithm**

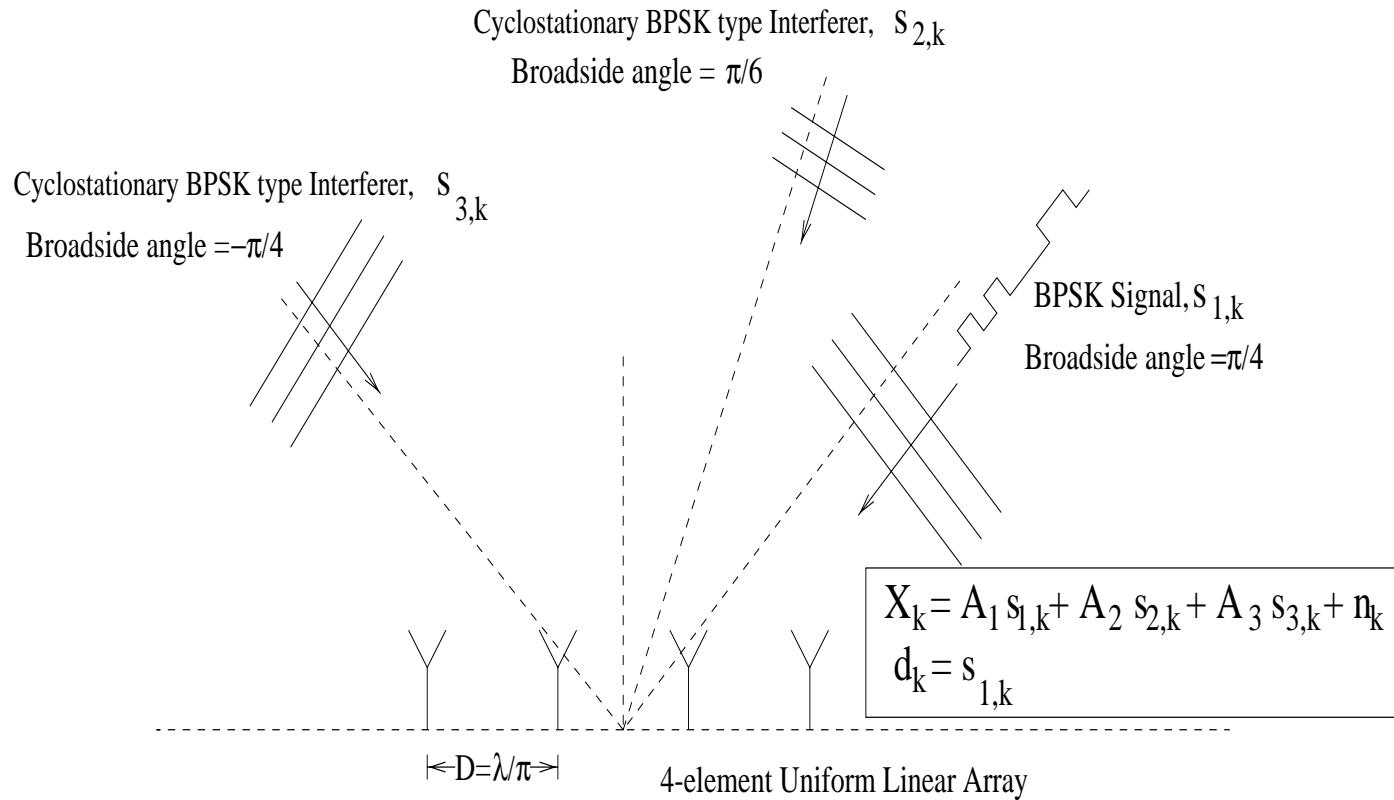
- Partial Update LMS Algorithm: Only a subset of LMS coefficients updated per iteration (Douglas, Godavarti, Messerschmitt)
- Advantages:
  - Computational and memory savings
  - Power savings
- Existing Partial Update LMS (PU-LMS) Algorithms
  - Periodic (**P-LMS**) and Sequential (**S-LMS**) Partial Update LMS
- Contributions
  - Tight convergence conditions on **P-LMS** and **S-LMS** (Godavarti&Hero ICASSP99, ICASSP01)
  - New Stochastic P-LMS Algorithm (**SPU-LMS**) with better convergence properties (Godavarti&Hero SAM00)

## Sequential Partial Update LMS Algorithm



**Figure 4. Block diagram of the sequential LMS algorithm.**

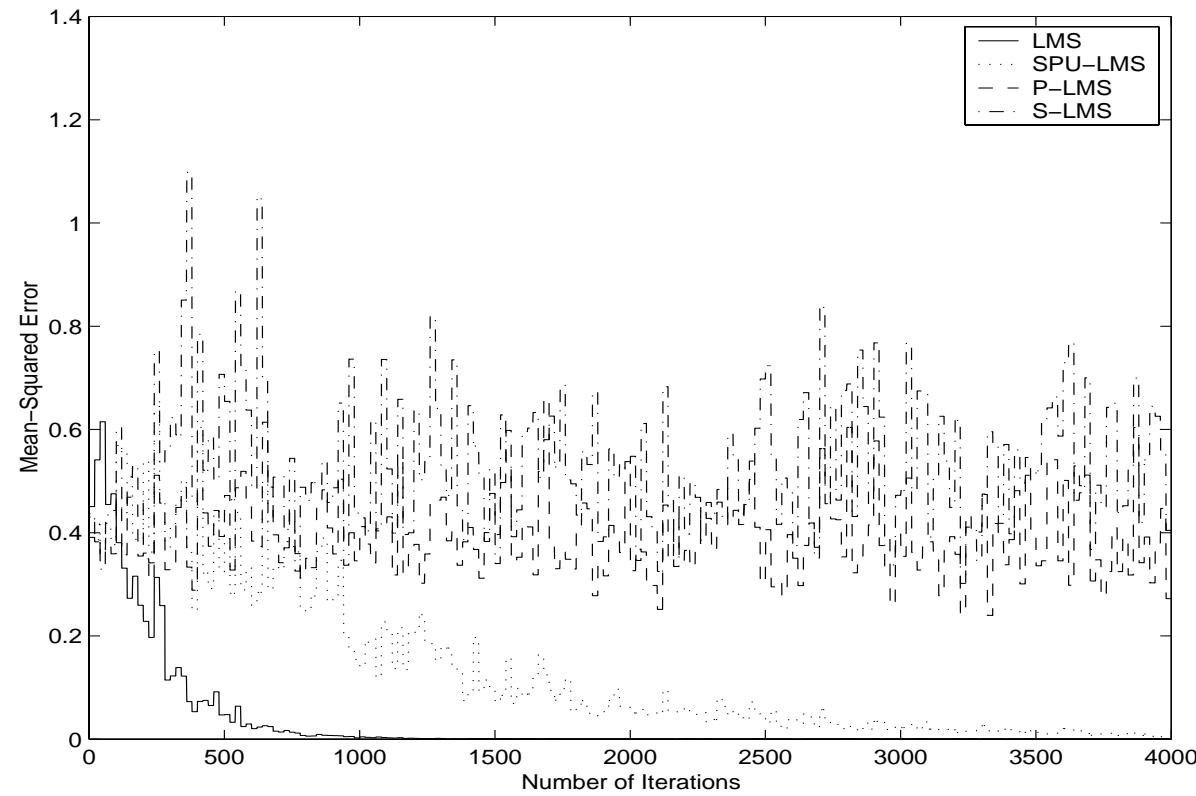
## Example: Non-Stationary Signal Scenario



**Figure 5. Signal scenario for the example**

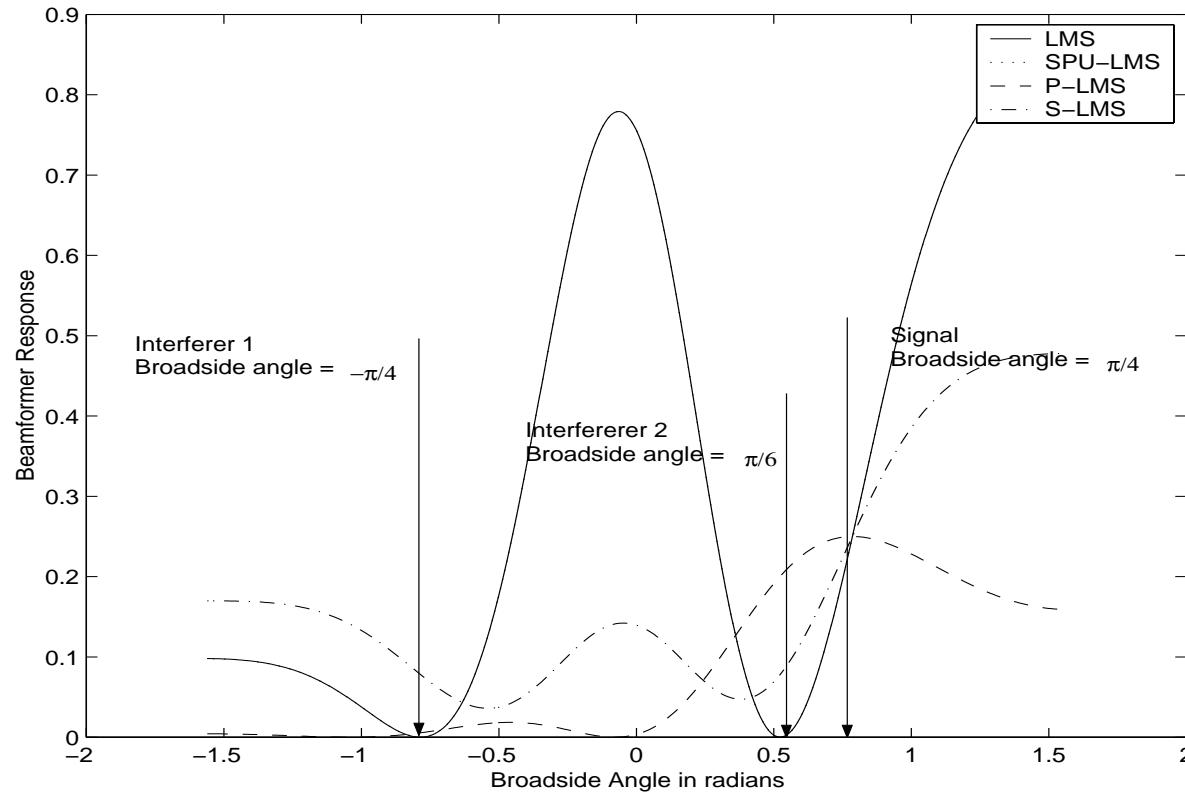
- **SPU-LMS** randomly updates one coefficient per iteration
- **P-LMS** updates every fourth instant
- **S-LMS** updates one coefficient per iteration in the order {1, 2, 3, 4}.

## Example cont'd



**Figure 6. Envelopes of trajectories of all algorithms for Example 2. S-LMS and P-LMS failed to converge for all observed values of  $\mu > 0$**

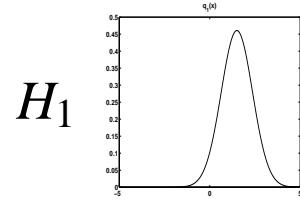
## Example cont'd



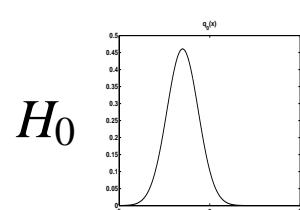
**Figure 7. Best possible beampatterns generated by all algorithms. LMS and SPU-LMS have indentical beamformer response characteristics.**

## HYPOTHESIS TESTING FROM Q/VQ DATA

Objective: Observe  $\hat{x} = Q(x)$  and decide between  $H_0$  and  $H_1$

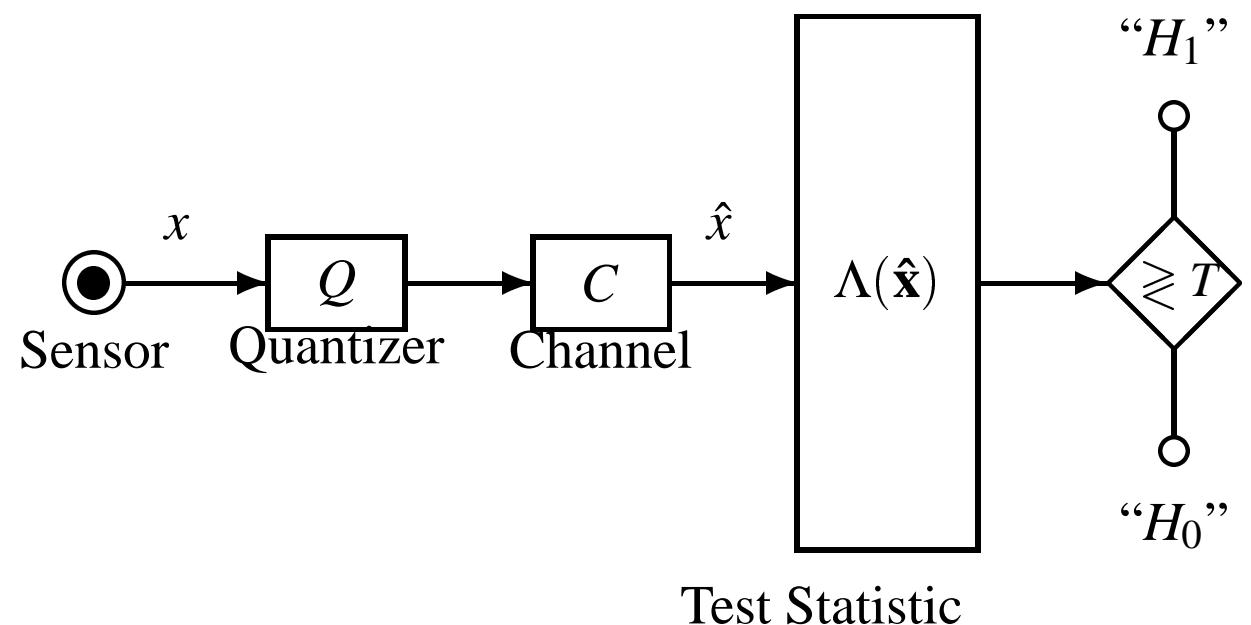


$H_1$



$H_0$

Environment



Test Statistic

**Objective:** What is best way to compress images sequence to minimize loss in detection performance?

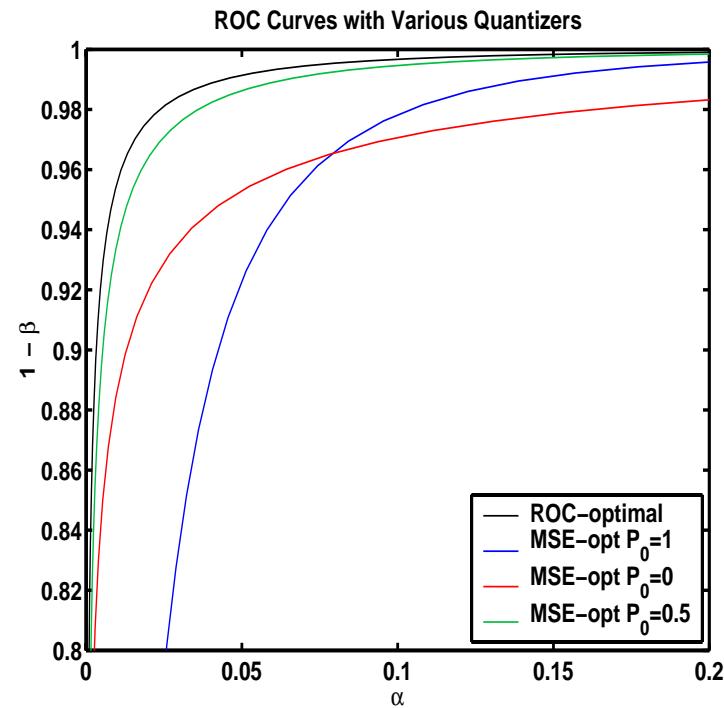
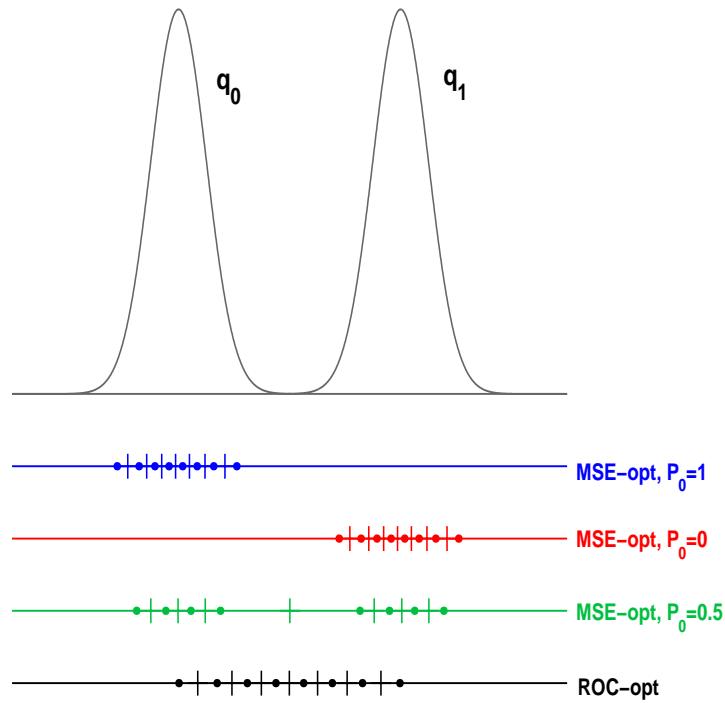
**Other applications:**

- Target recognition
- Classification
- Indexing databases
- Medical imaging

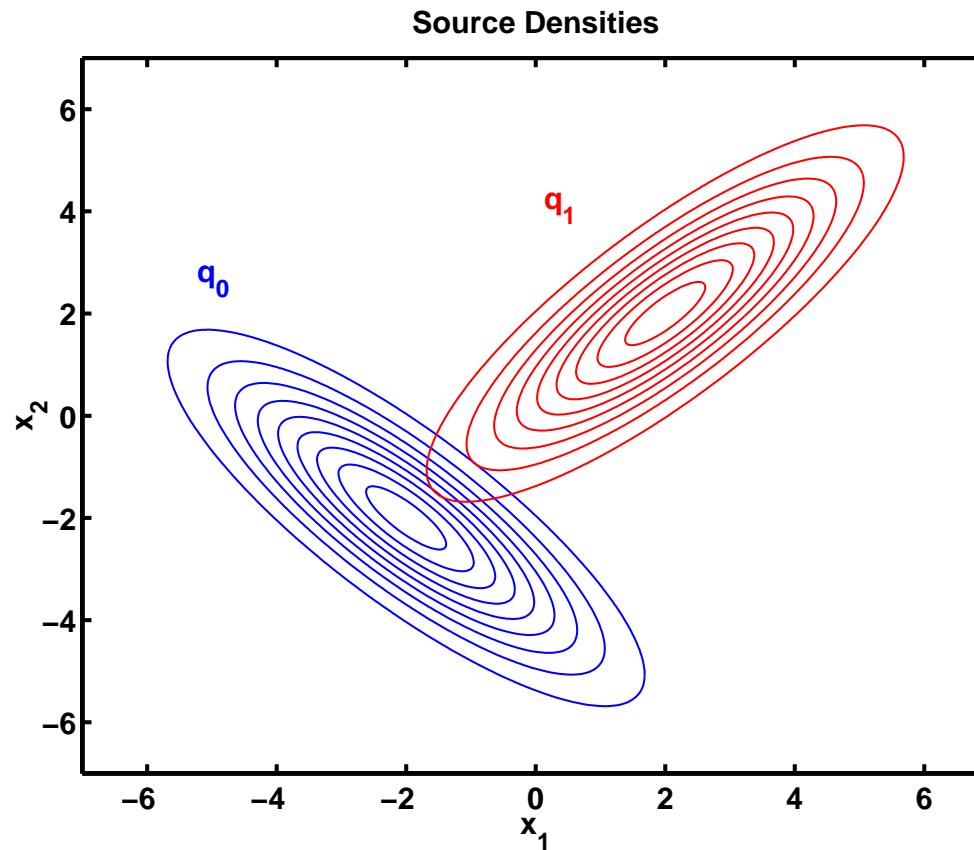
**Innovation on previous work:** High-rate, detection-optimal vector quantization.

- VQ: Oehler, Gray 96; Perlmutter et. al. 96.
- Scalar Quantization: Poor, Thomas 77, 83; Benitz, Bucklew 89; Picinbono, Duvaut 88.
- MSE-opt VQ: Gersho 79; Zador 82; Na, Neuhoff 95.

## ROC Performance of Various Quantizers

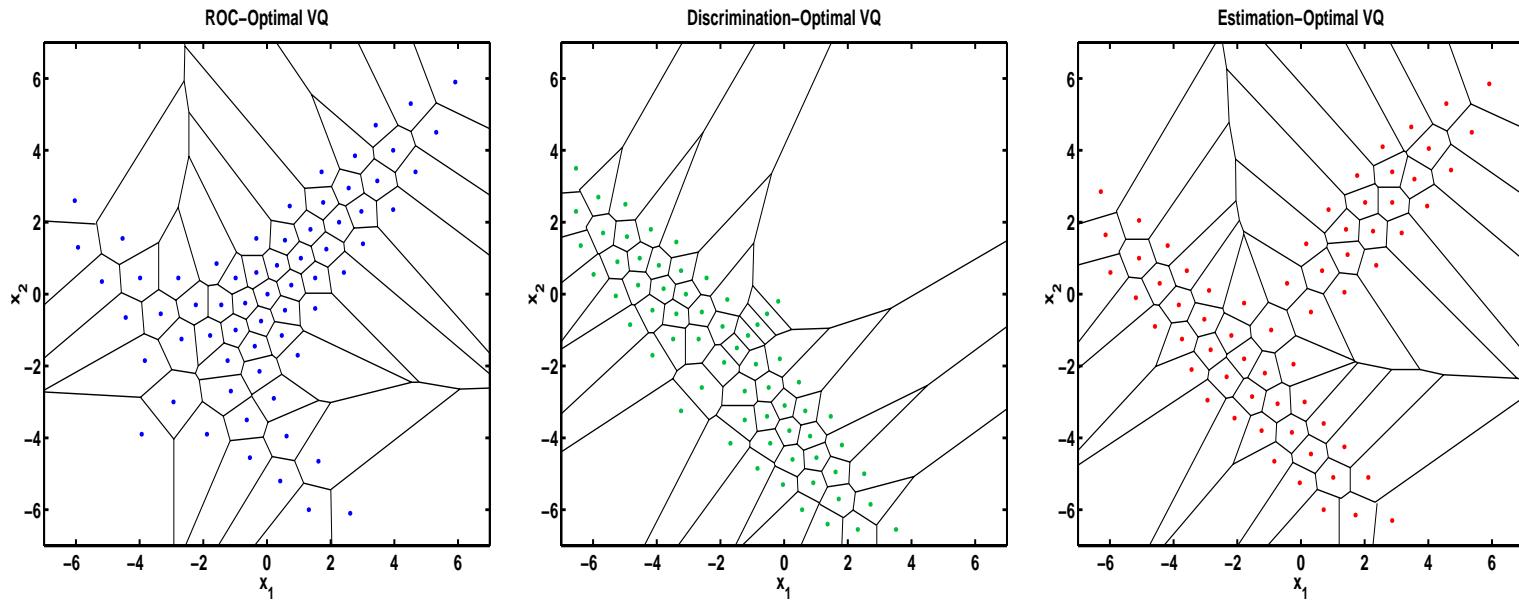


## **2-D Anisotropic Gaussian Example**



**Figure 8.** Source densities for 2-D anisotropic Gaussian example.

## 2-D Anisotropic Gaussian Example



**Figure 9. ROC-optimal congruent-cell VQ (left), Discrimination-optimal congruent-cell VQ (middle), and Estimation-optimal VQ (right) with  $N = 64$ .**

## LOW POWER STRATEGIES FOR MIMO

### **Objectives:**

- Combat co-channel interference and fading
- Achieve higher data rates for fixed BER
- Assure information security

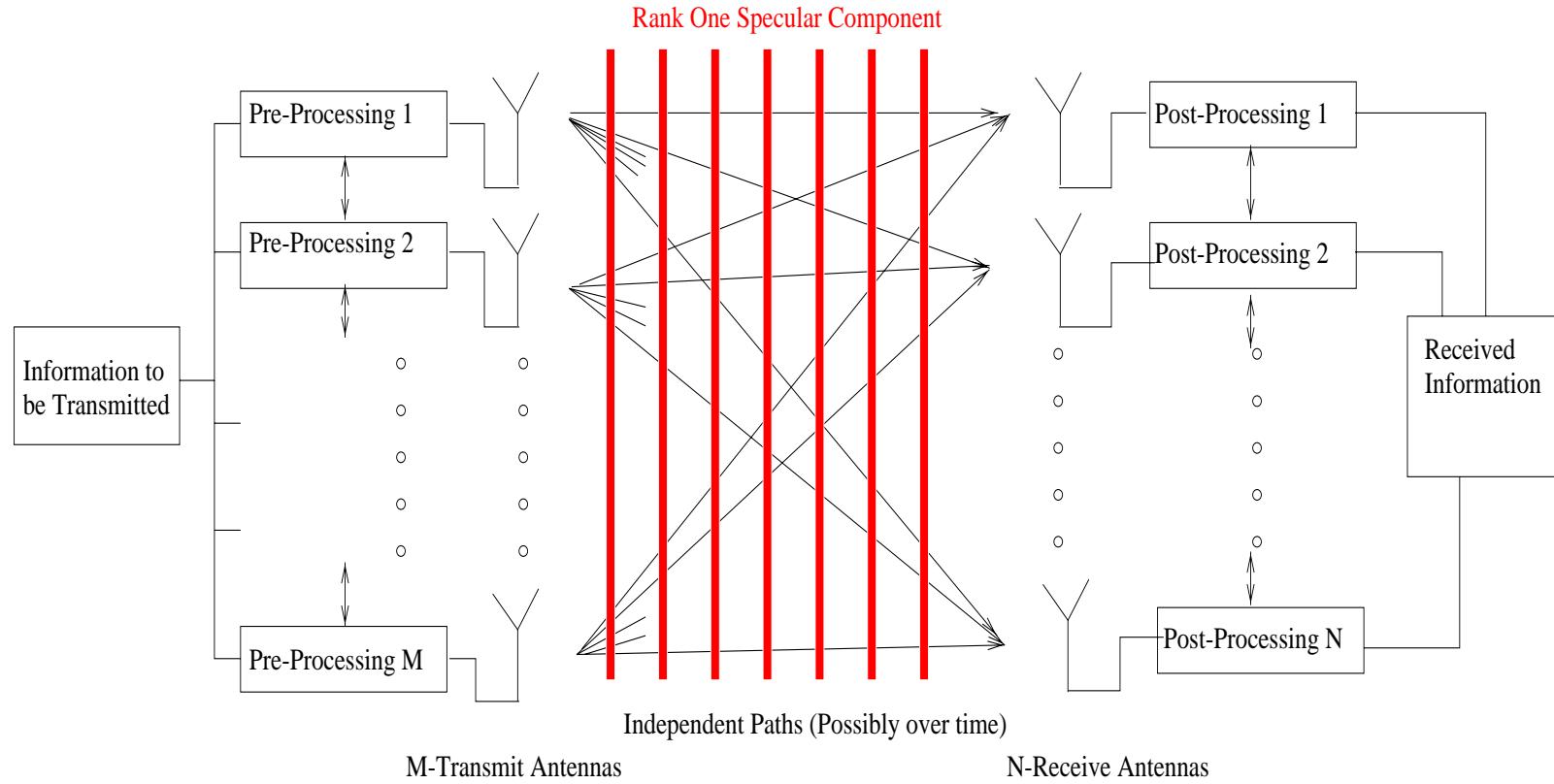
### **Previous Work:**

- BLAST (Foschini BLTJ96, WPC98)
- Channel capacity Rayleigh fading (Telatar ET98; Marzetta&Hoc
- Sub-optimal Space-Time codes (Seshadri IT98 Hochwald&Marz

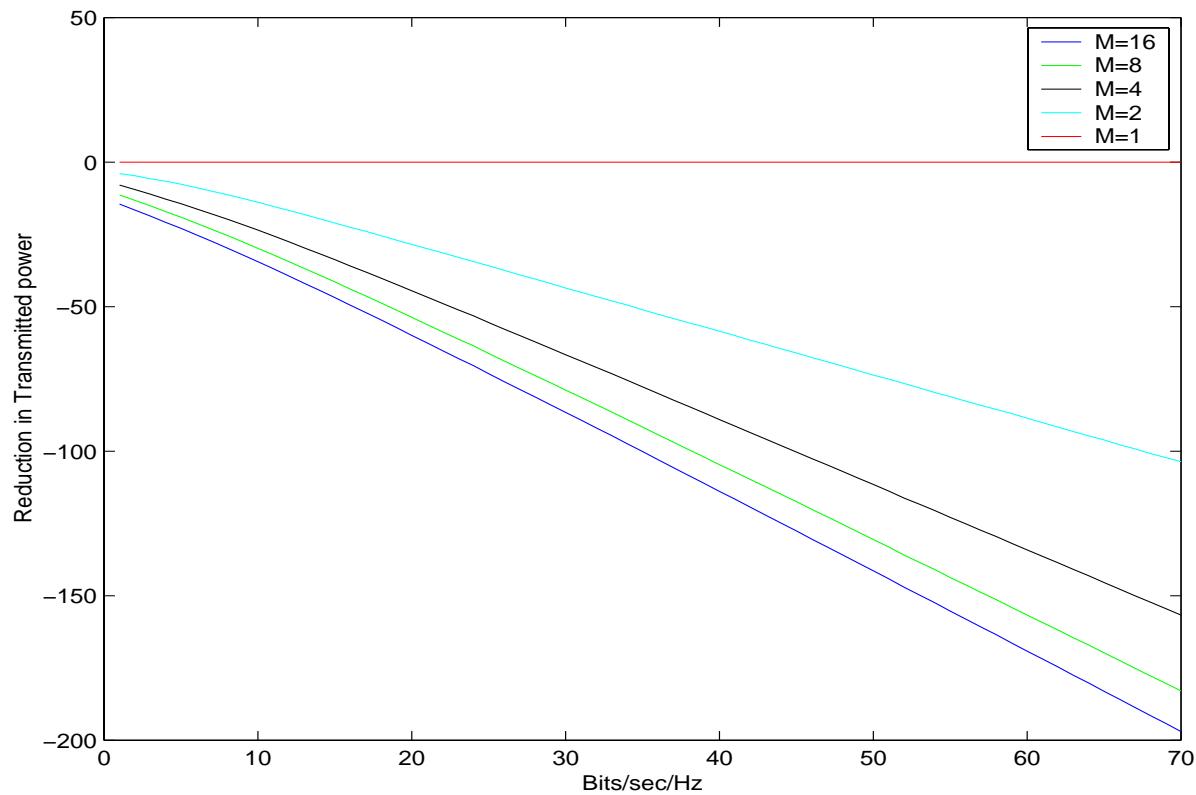
### **Contributions:**

- optimal signal design for Rayleigh fading
- Rician fading extensions
- Secure space-time modulation

$$X = \eta SH + N, \quad H = (\sqrt{1-r})G + (\sqrt{rMN})\alpha\beta^\dagger$$

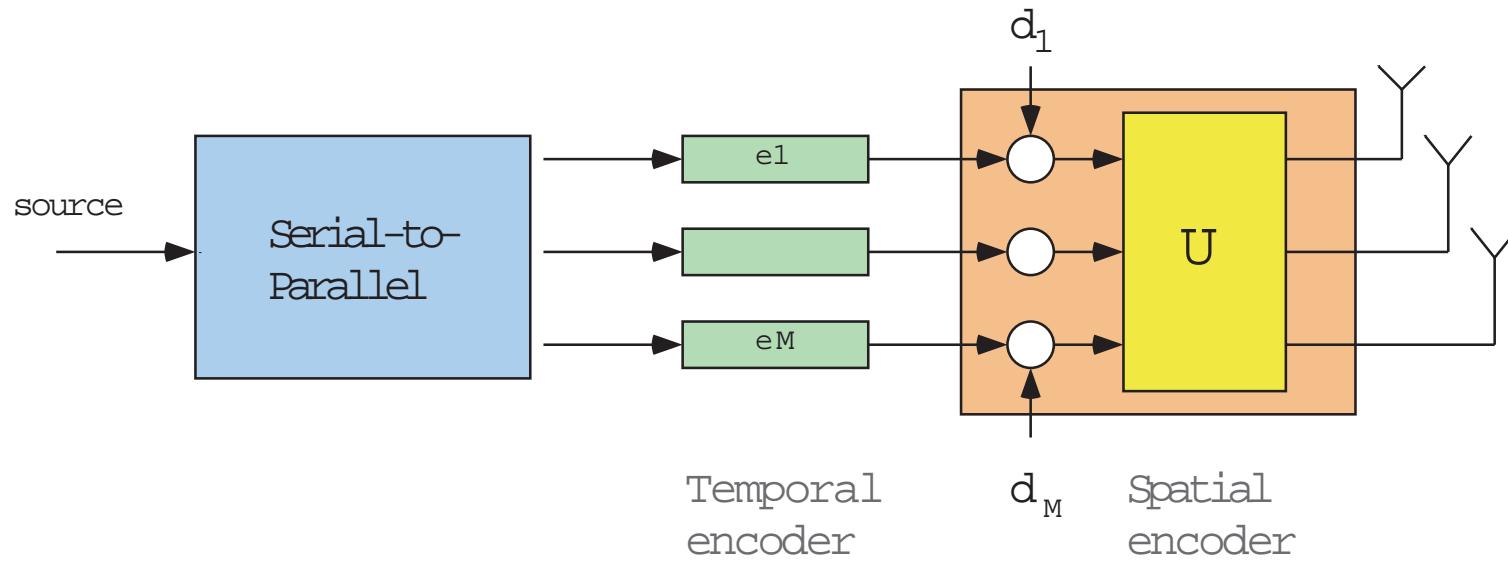


**Figure 10. Multiple antenna transmitter and receiver channel.  $M$  transmitters,  $N$  receivers,  $T$  time samples.**

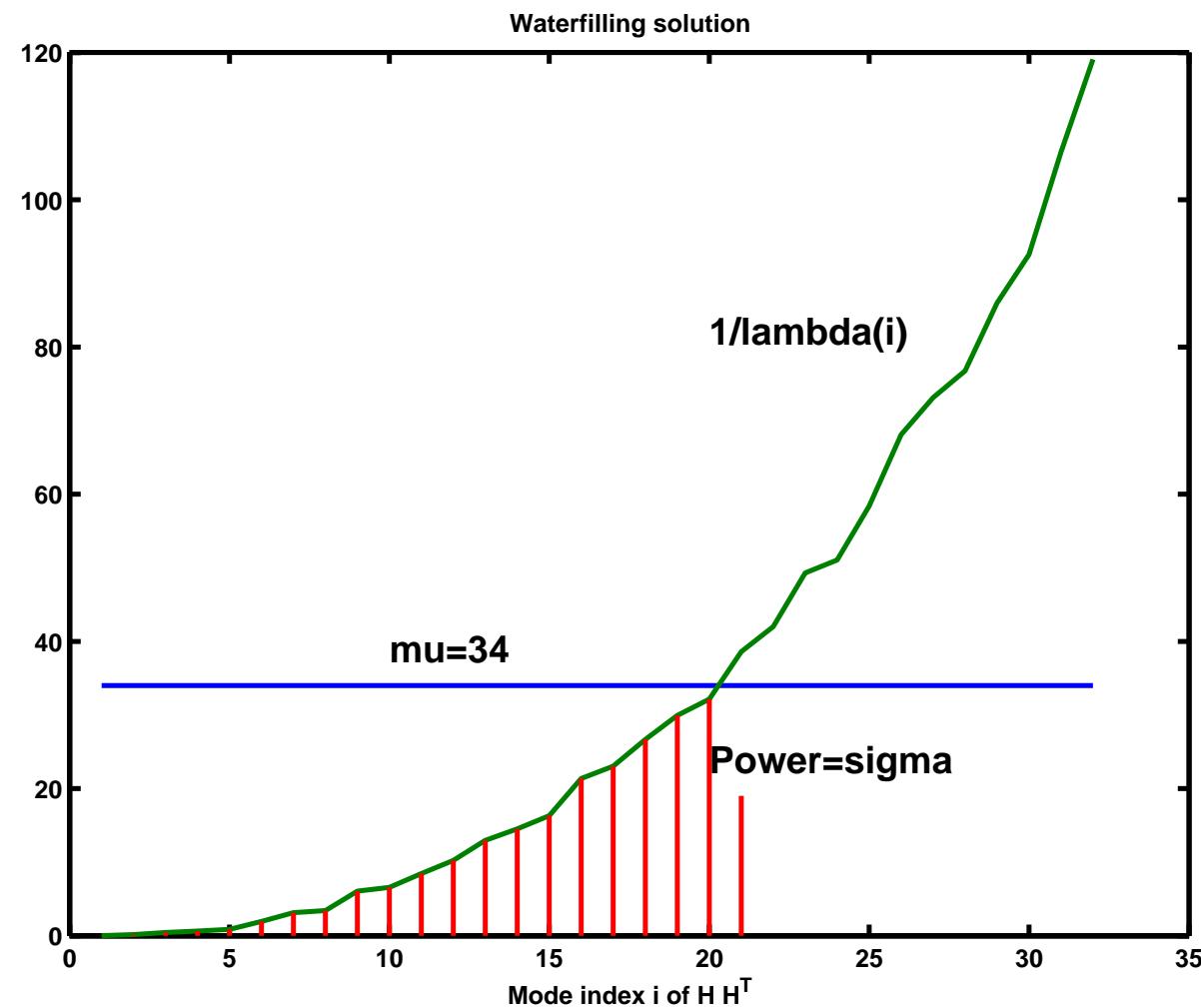


**Figure 11. Reduction in Transmitted power for different number of antennas as a function of capacity (bps/Hz). Rayleigh fading channel is assumed known by receiver but not by transmitter.**

## IT-IR Link

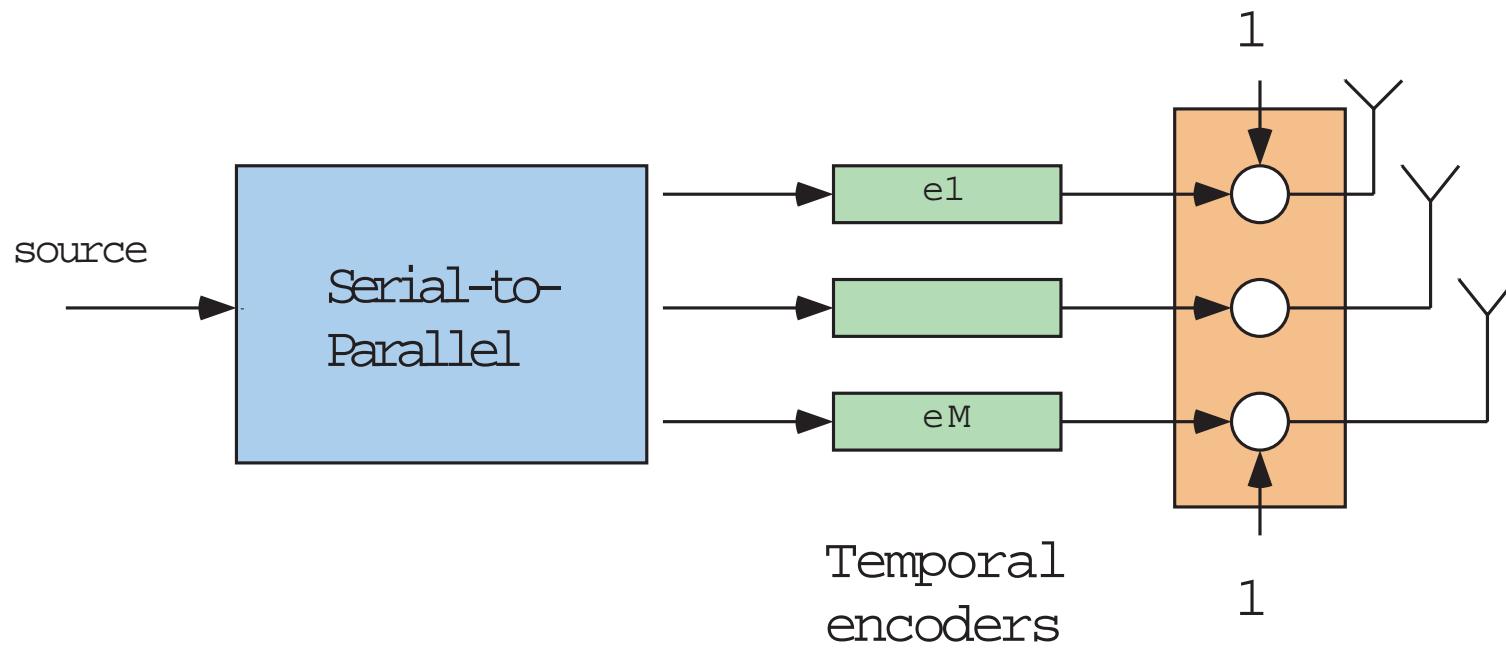


**Figure 12.** Optimal STC for informed-transmitter informed-receiver



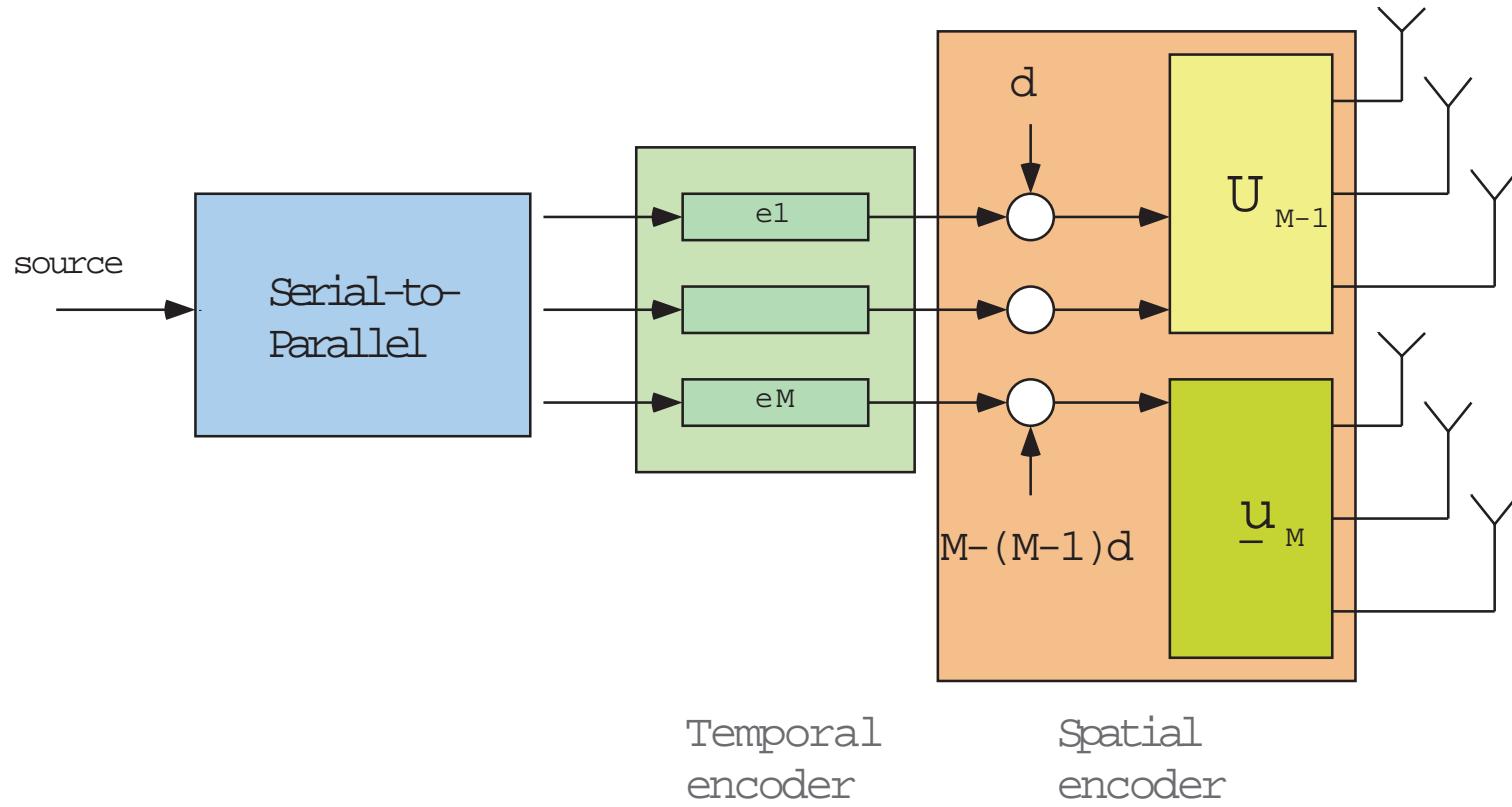
**Figure 13.** Waterpouring solution for power-capacity achieving mode allocation( $N = M = 32$ )

### UT-IR Link

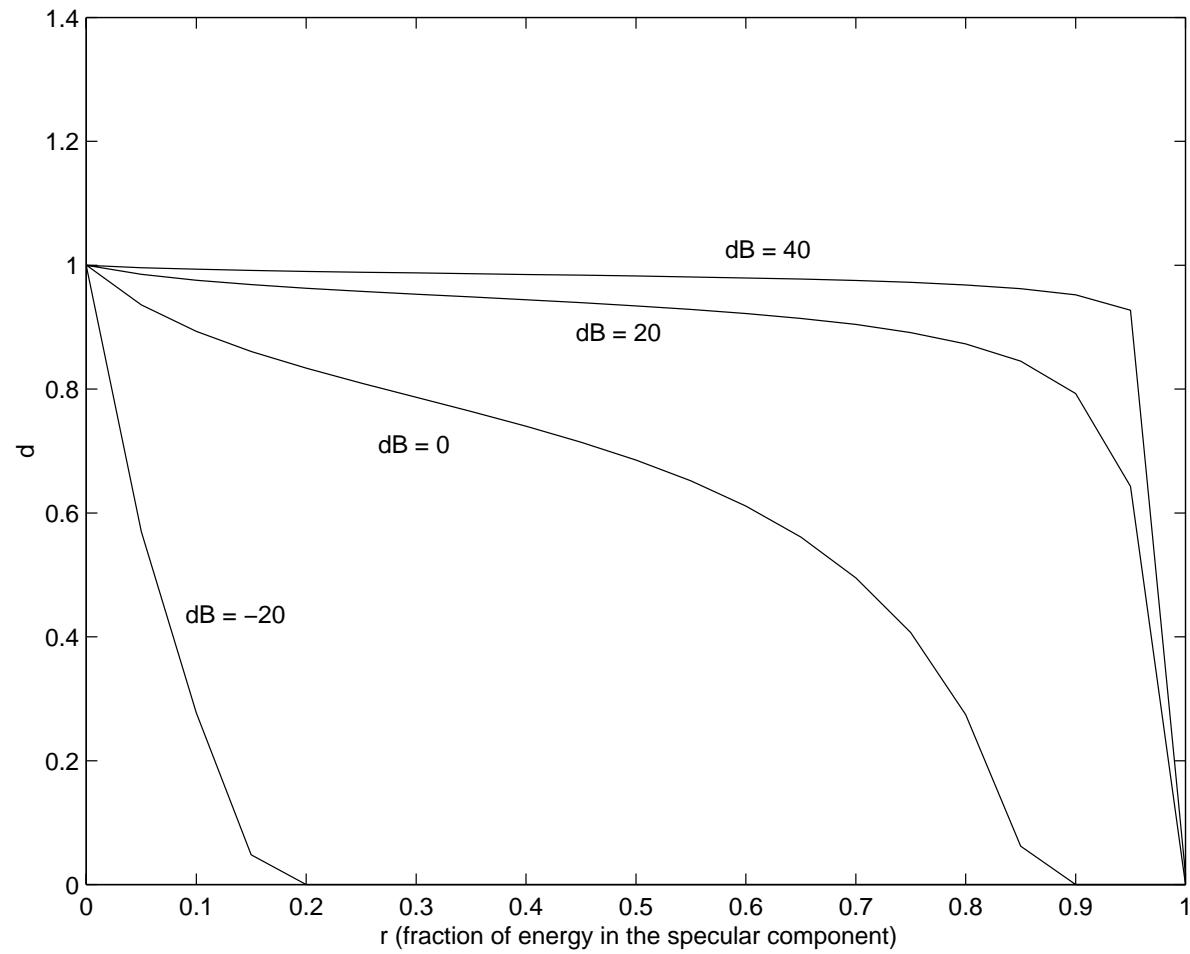


**Figure 14.** Optimal STC for UT-IR: Rayleigh link

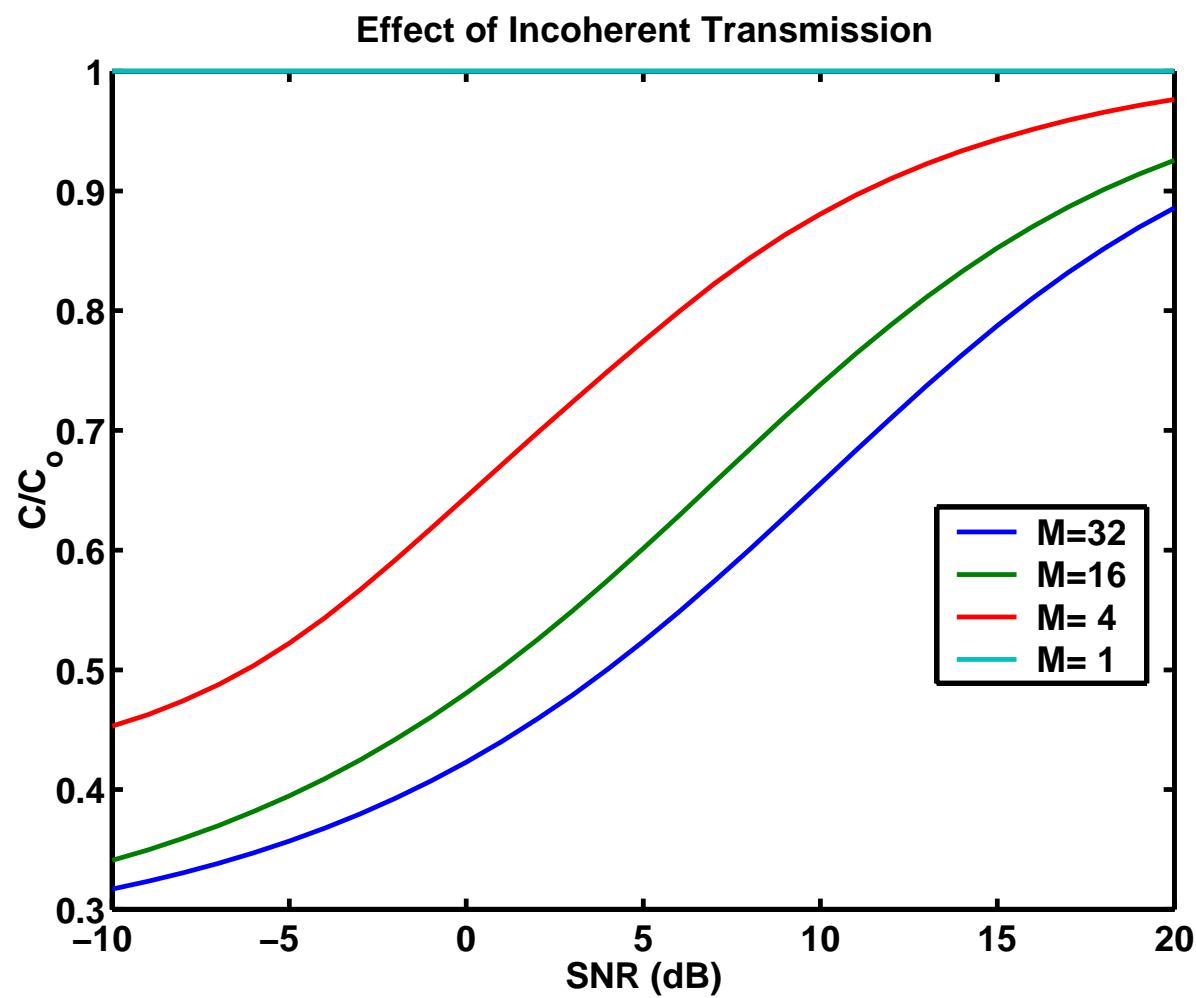
## Optimal STC for UT-IR: Rician Link



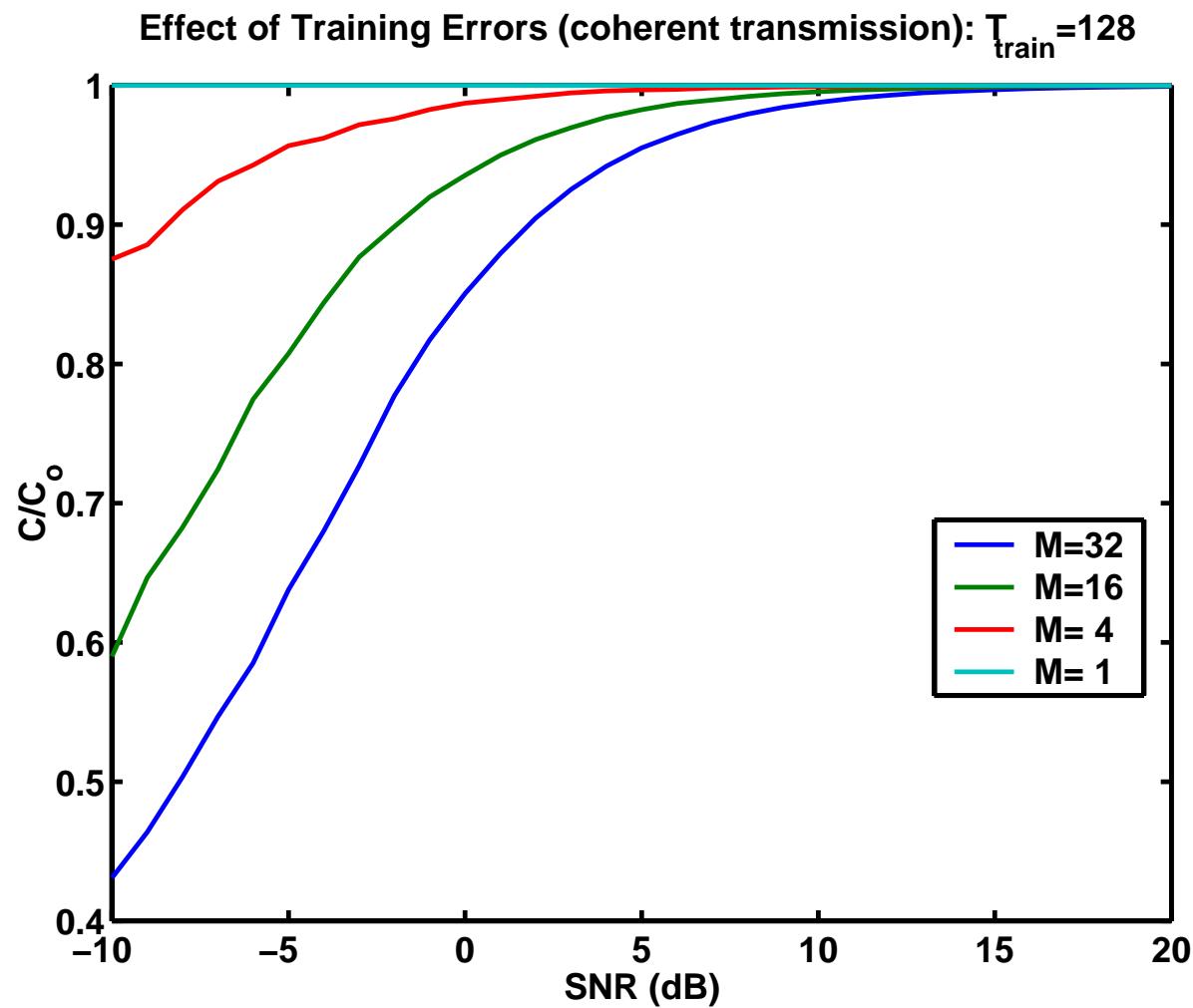
**Figure 15.** *Optimal STC for Rician uninformed-transmitter informed-receiver*



**Figure 16.** Numerical optimization yields  $l = 0$  and values of  $d$  shown as a function of  $r$  for different values of  $\eta$  (SNR).

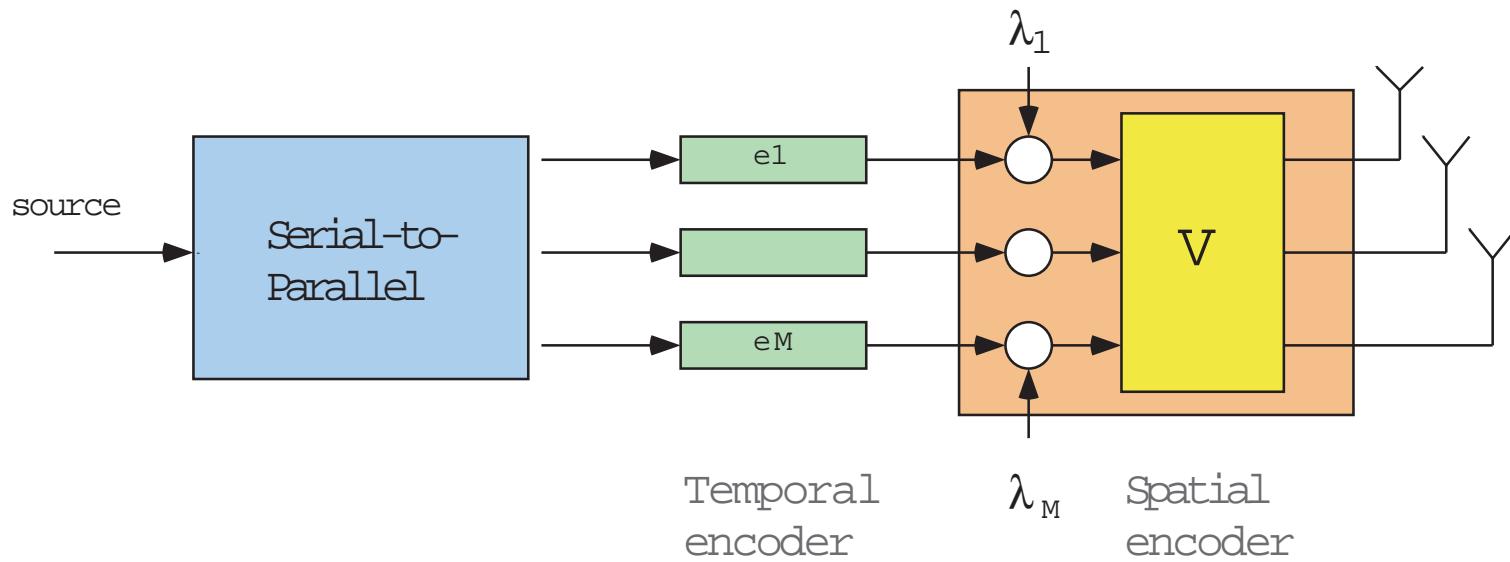


**Figure 17.** Capacity loss due to uninformed transmission (UT-IR).



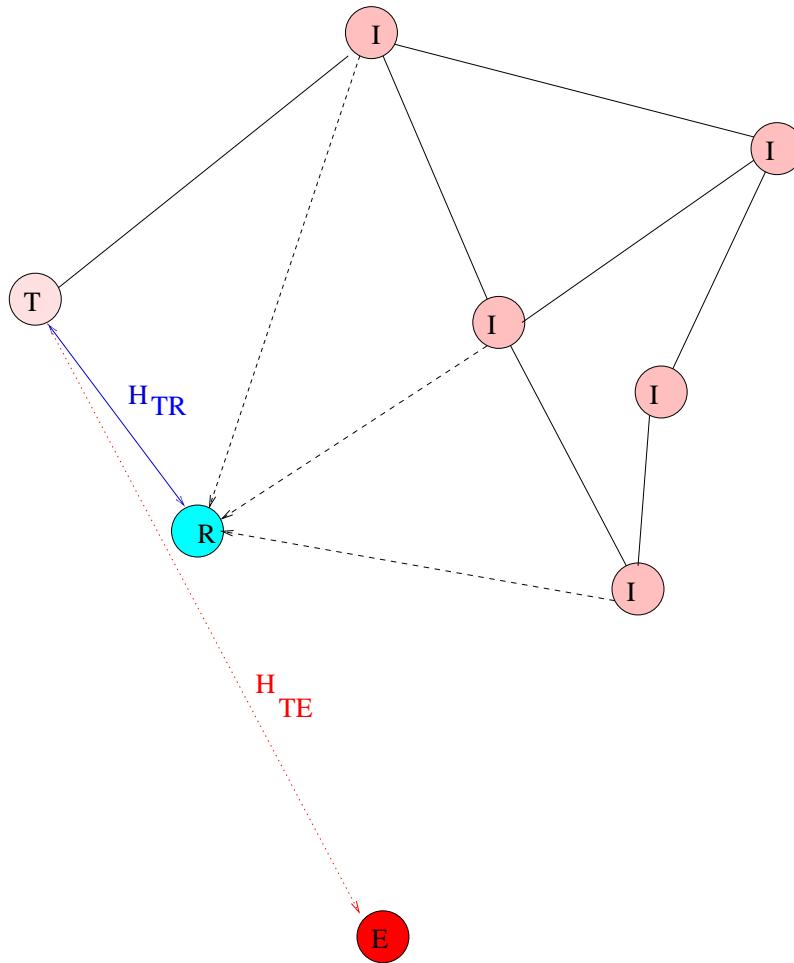
**Figure 18.** Capacity loss due to T/R channel estimation errors.

### UT-UR Link



**Figure 19.** Optimal STC for uninformed-transmitter uninformed-receiver

## MIMO INFORMATION SECURITY



**Figure 20.** Wireless network with eavesdropper

## **Information Security: Eavesdropper Resistance**

Hypotheses:

1. Subscriber links have *informed* transmitters/receivers (IT-IR):
  - $H_{TR}$  is known to both parties over a hop
  - Training generally required to learn channel
  - Feedback required to inform transmitter of channel
2. Eavesdropper link has *uninformed* transmitter (UT)
  - $H_{TE}$  unknown to transmitter
  - $S, H_{TE}$  may be known or unknown to eavesdropper
  - Modulation type, signal constellations, source density, may be known to eavesdropper

## LPD constraints

The eavesdropper must make a decision between

$$H_0 : \quad X_i = W_i, \quad i = 1, \dots, L$$

$$H_1 : \quad X_i = S_i H_i + W_i, \quad i = 1, \dots, L$$

His minimum attainable detection error probability has exponential rate

$$\liminf_{L \rightarrow \infty} \frac{1}{L} \ln P_e = \rho$$

$$\rho = \inf_{\alpha \in [0,1]} \lim_{L \rightarrow \infty} \frac{1}{L} \ln \int f_{H_1}^{1-\alpha}(X) f_{H_0}^{\alpha}(X) dX$$

- $\rho$  is Chernoff error exponent ( $\rho \leq 0$ )
- $\rho$  is minimal  $\alpha$ -divergence between densities  $f_{H_1}$  and  $f_{H_0}$
- Chernoff exponent is achieved for Bayes test

## Uninformed Eavesdropper: Low SNR

$$\begin{aligned}\rho &= \min_{\alpha \in [0,1]} \left( -\frac{\alpha(1-\alpha)\eta_e^2}{2} \operatorname{tr}\{\overline{SS^\dagger} \overline{SS^\dagger}\} + o(\eta_e^2) \right) \\ &= -\frac{\eta_e^2}{8} \operatorname{tr}\{\overline{SS^\dagger} \overline{SS^\dagger}\} + o(\eta_e^2)\end{aligned}$$

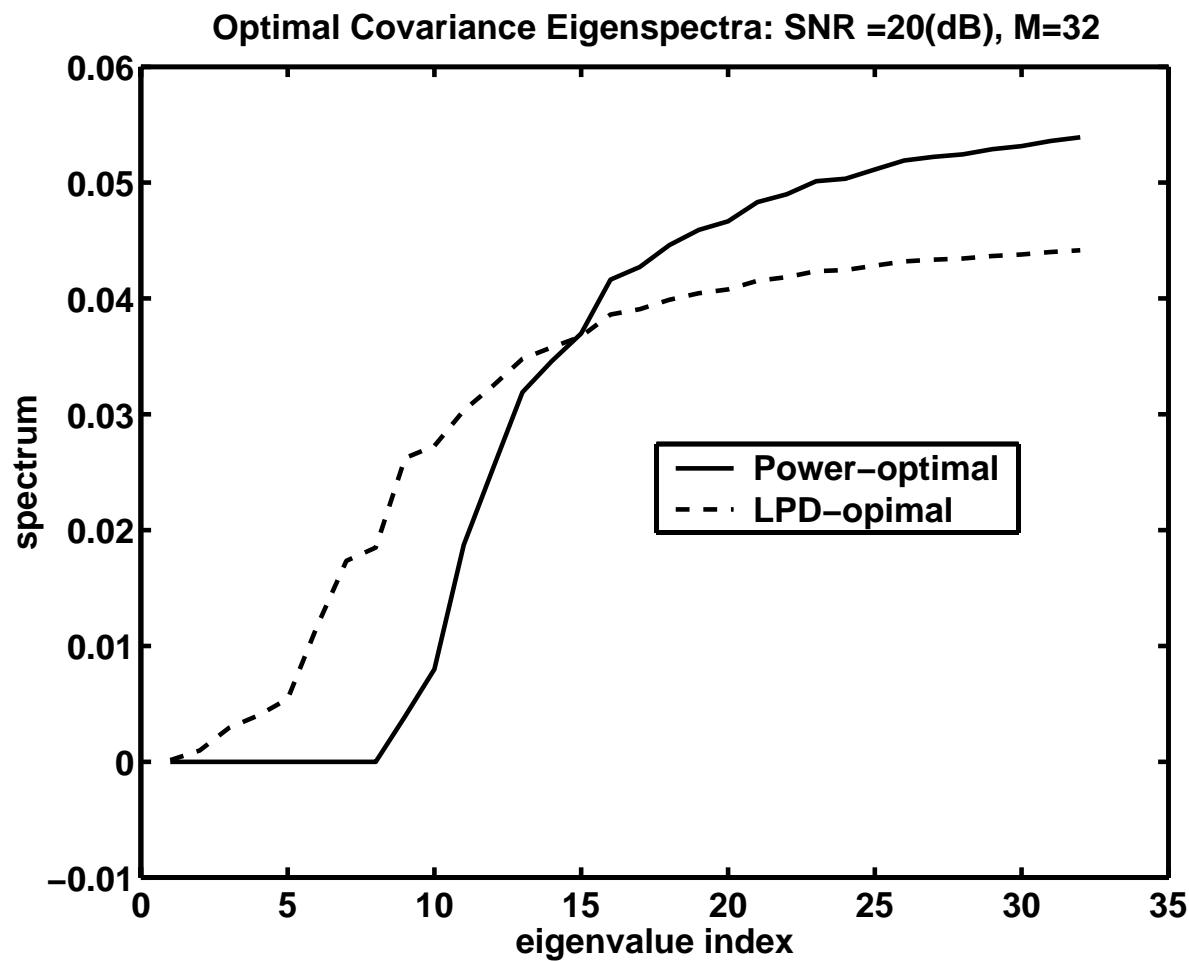
**Transmitter strategy:**

Attain  $E[\max_{P(S)} \ln P(X|H_{TR}, S)/P(X|H_{TR})]$  subject to

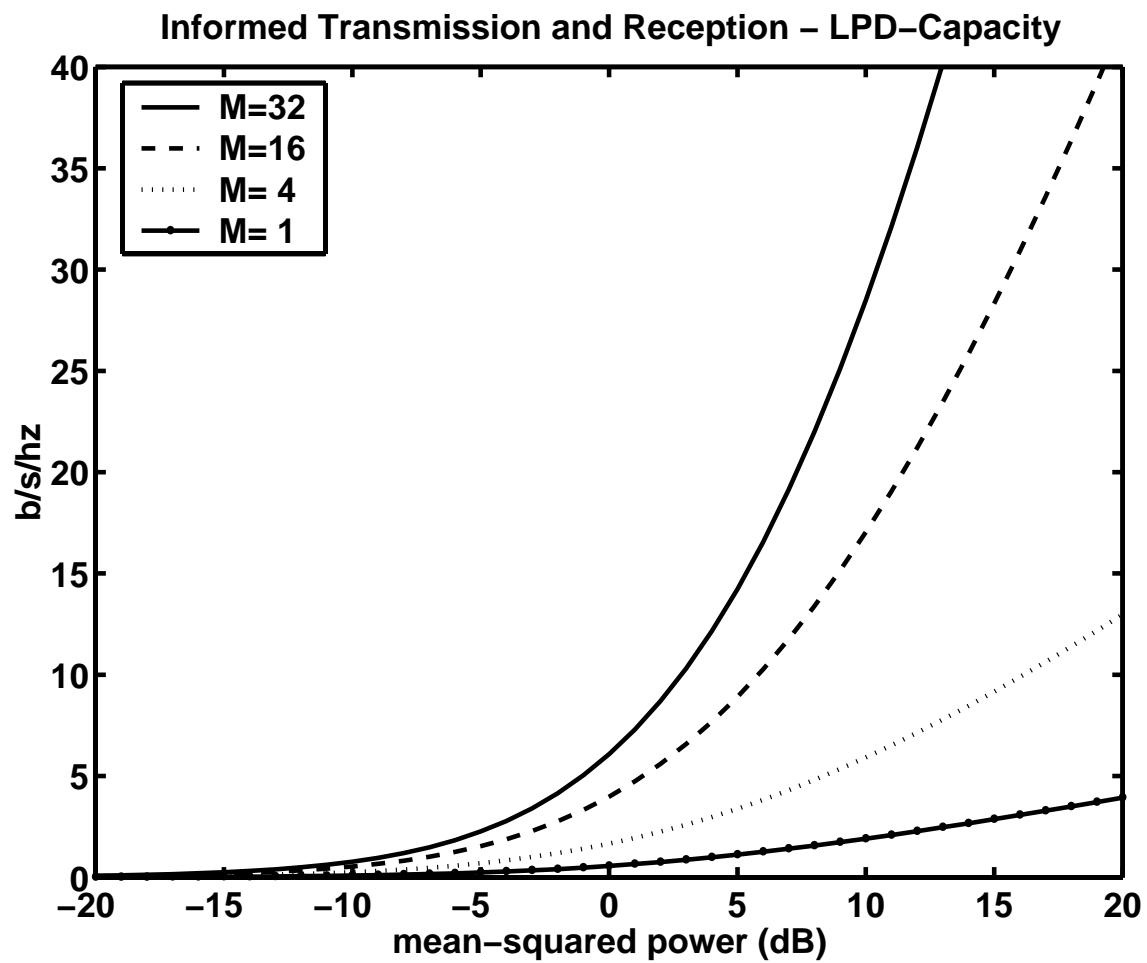
$$\operatorname{tr}\{\overline{SS^\dagger} \overline{SS^\dagger}\} \leq P_{4avg}$$

- Equivalent to constraining  $S$  to Gaussian source with

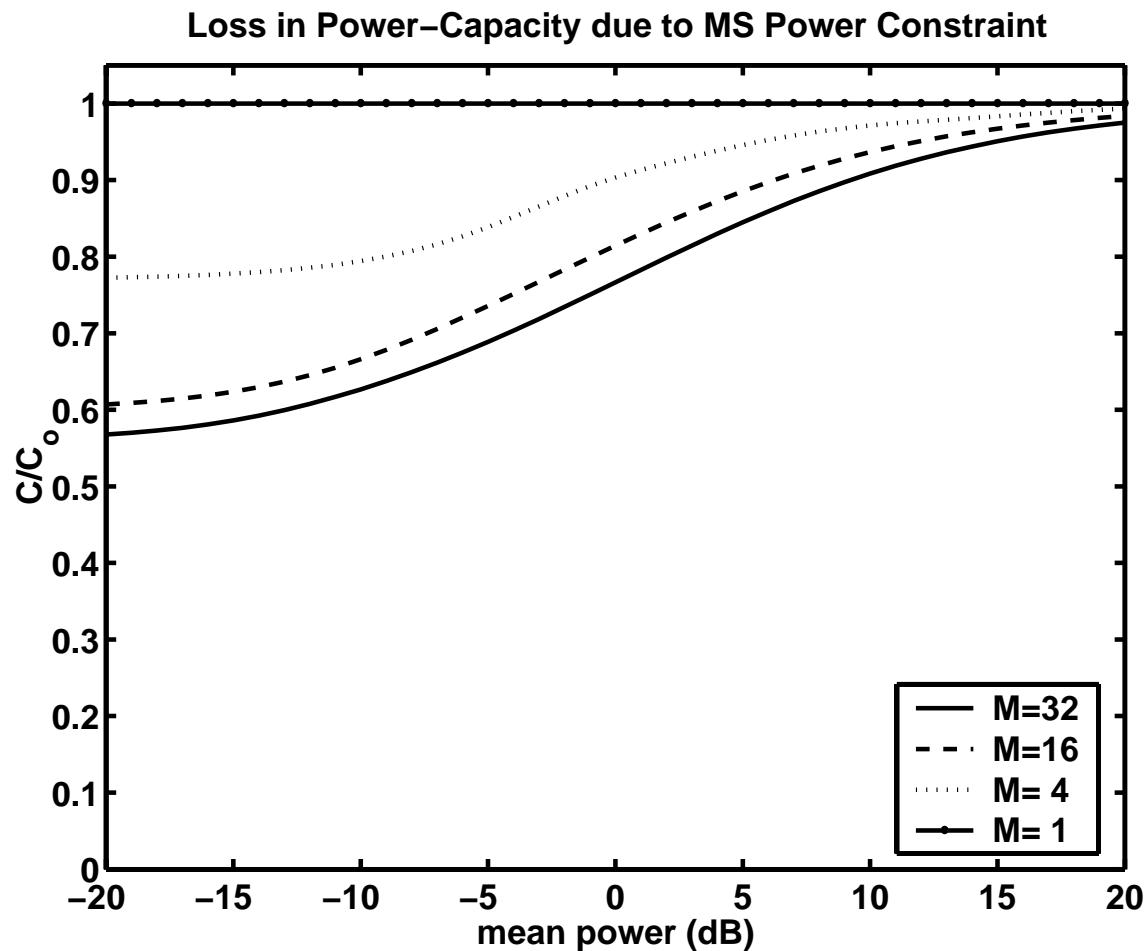
$$\operatorname{tr}\{\overline{SS^\dagger SS^\dagger}\} \leq P_{4avg}/3$$



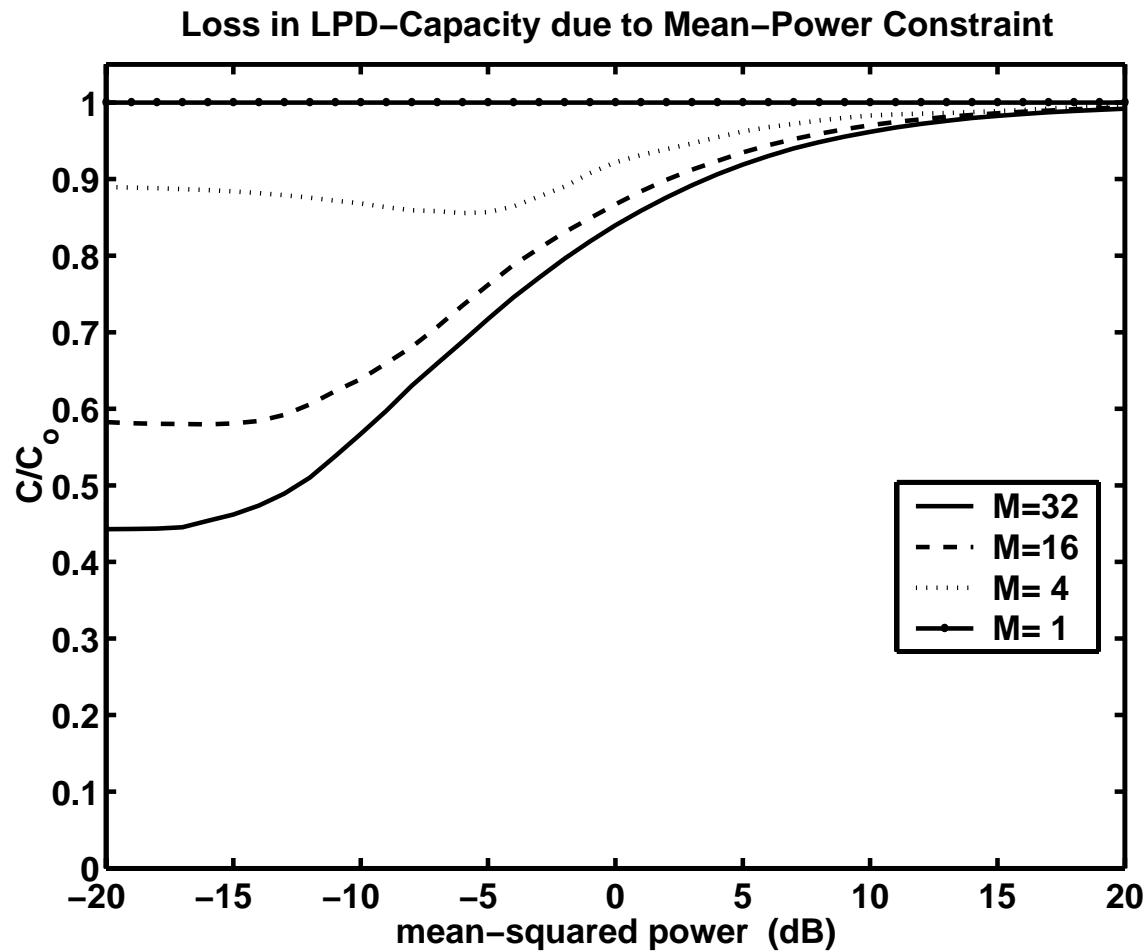
**Figure 21.** Optimal source spectra:  $SNR = 20dB, M = N = 32$



**Figure 22.** *IT-IR LPD-constrained capacity ( $N = M$ )*



**Figure 23.** Loss in power-capacity due to LPD constraint ( $N = M$ )



**Figure 24.** Loss in LPD-capacity due to  $P_{avg}$  constraint ( $N = M$ )

## Conclusions

1. Signal processing can greatly impact terminal power consumption
2. Strategies to tradeoff power vs algorithm performance have been developed to optimize
  - Adaptive filter resolution vs convergence rate and steady state error
  - Data compression vs. detection performance
  - MIMO information rates vs. information security
3. Signal processing performance must be considered for maximum power reduction