Optimal Detection of Single Spin MR

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Outline

- Cantilever-based single spin measurement system
- Signal detection framework
- Areas of research

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rugar2

rugar3

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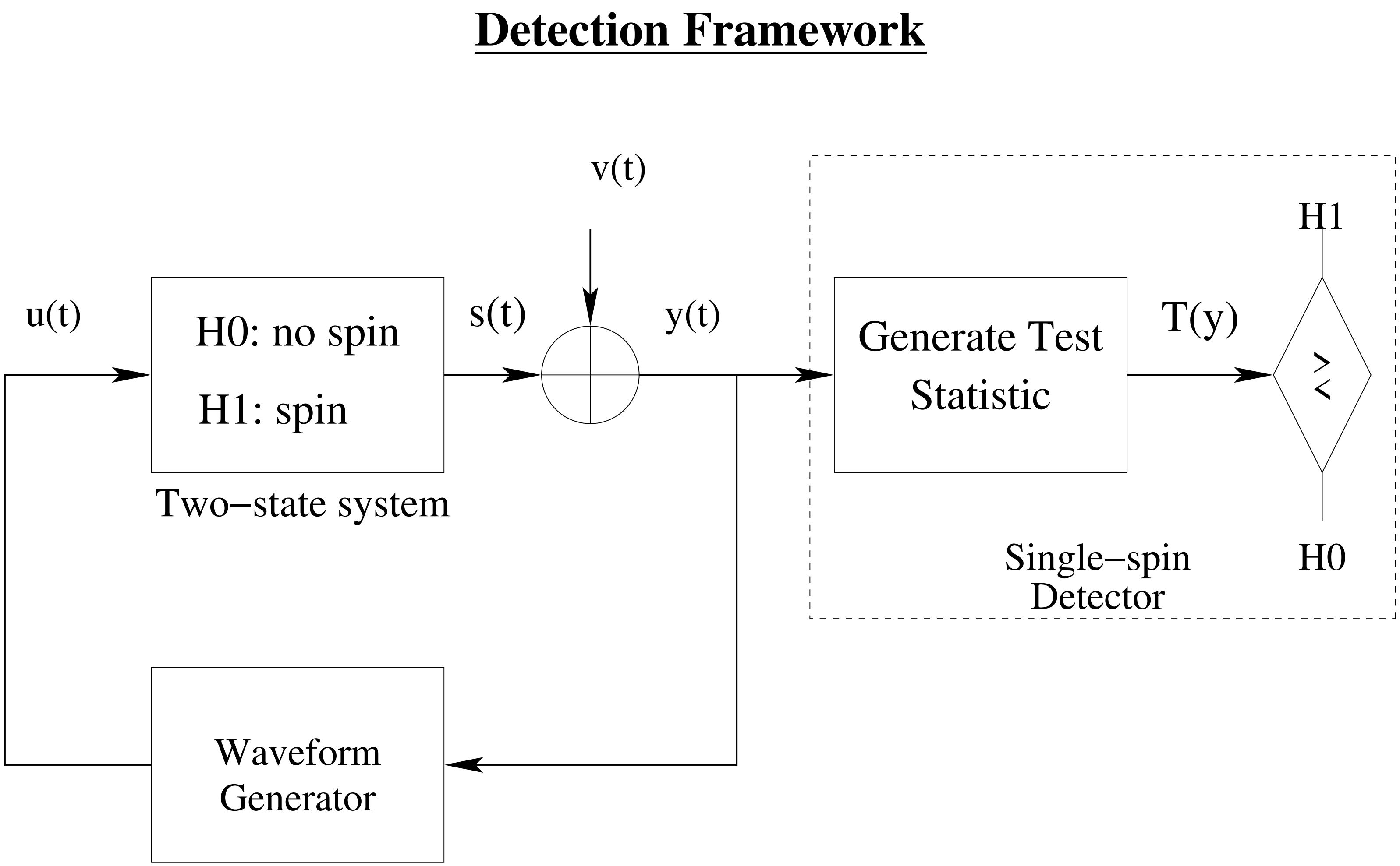

Figure 1: *Spin detection system model.*

Structure 1: Energy detector

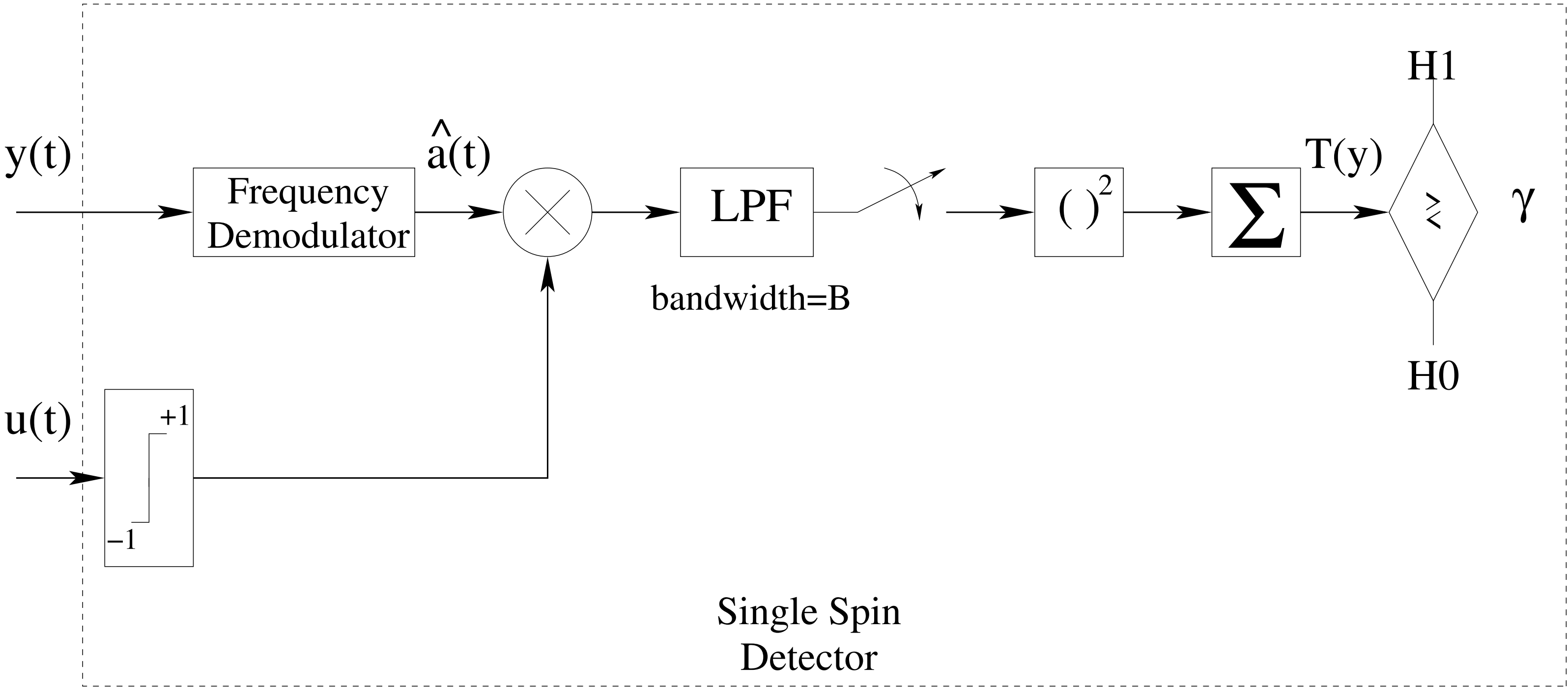

Figure 2: *Energy detector block diagram.*

- How to determine performance?

- How to determine required SNR for detection?

Energy Detector Performance

Assume:

- Random flip rate $\ll$ LPF bandwidth B

- $\hat{a}(t) = a(t) + w(t)$, $w(t)$ is zero mean Gaussian white noise of level $N_o/2$

- Detection performed over n periods of PM signal $u(t)$

Then ($\sigma^2 = BN_o$, $d =$SNR)

$$
\begin{aligned}
P_F &= P(T(y) > \gamma | H_0) \;=\; 1 - \chi_n(\gamma/\sigma^2) \;=:\; \alpha \\
P_D &= P(T(y) > \gamma | H_1) \;=\; 1 - \chi_{n,d}(\gamma/\sigma^2) \;=:\; \beta
\end{aligned}
$$

Receiver Operating Characteristic (ROC)

$$\beta = 1 - \chi_{n,d}(\chi_n^{-1}(1-\alpha))$$

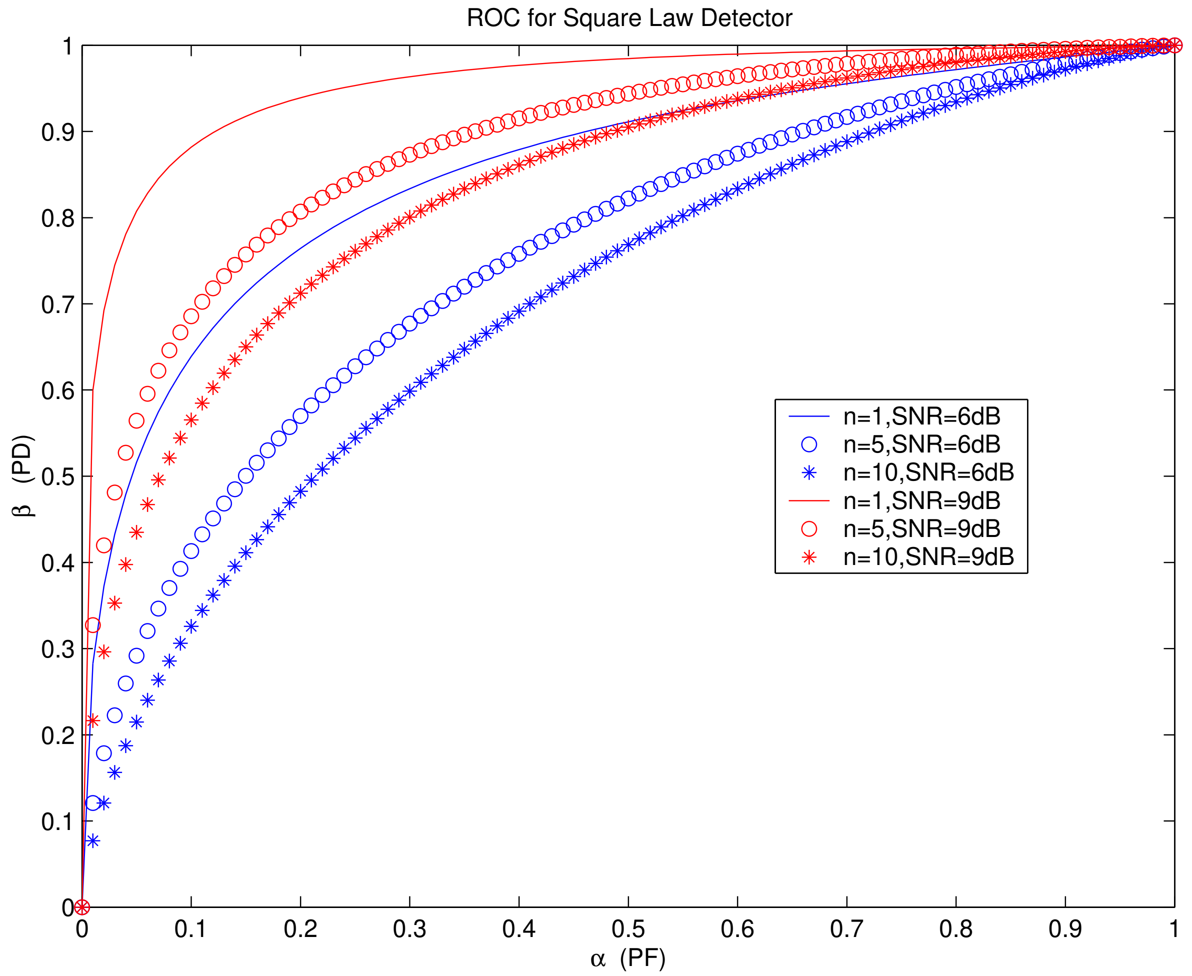

Figure 3: *ROC for two different SNR levels.*

Structure 2: GLRT

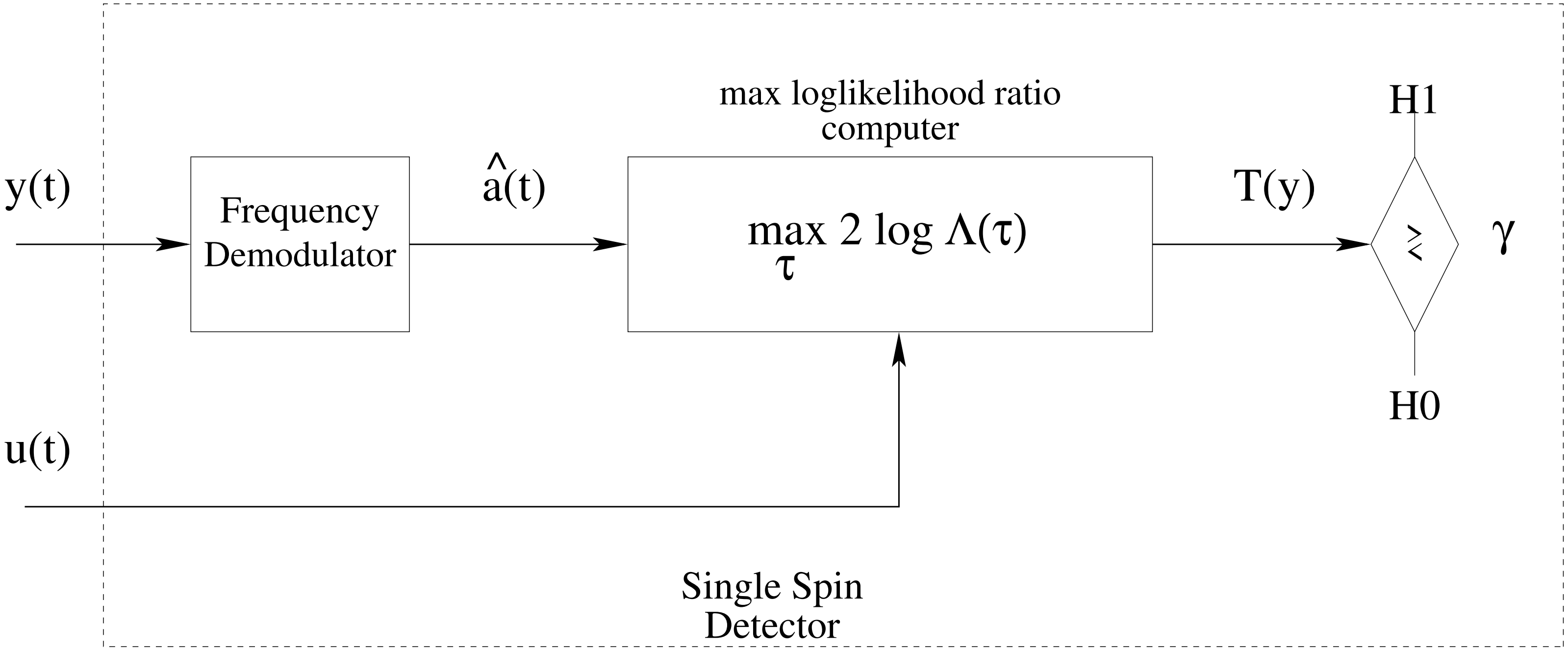

Figure 4: *Generalized likelihood ratio test under assumption of equiprobable initial spin polarity.*

$$2\log\Lambda(\tau) = 2\log\cosh\left(\sum_{i=1}^{n}(-1)^i z(\tau_i,\tau_{i-1})\right) - SNR$$

$$z(\tau_2,\tau_1) = c\int_{\tau_1}^{\tau_1} u(nT-t)\hat{a}(t)dt$$

Issues:

- ROC approximations and bounds?

- implementation complexity: multidimensional search?

- asymptotic approximations?

Structure 3: Bayes-optimal Detector

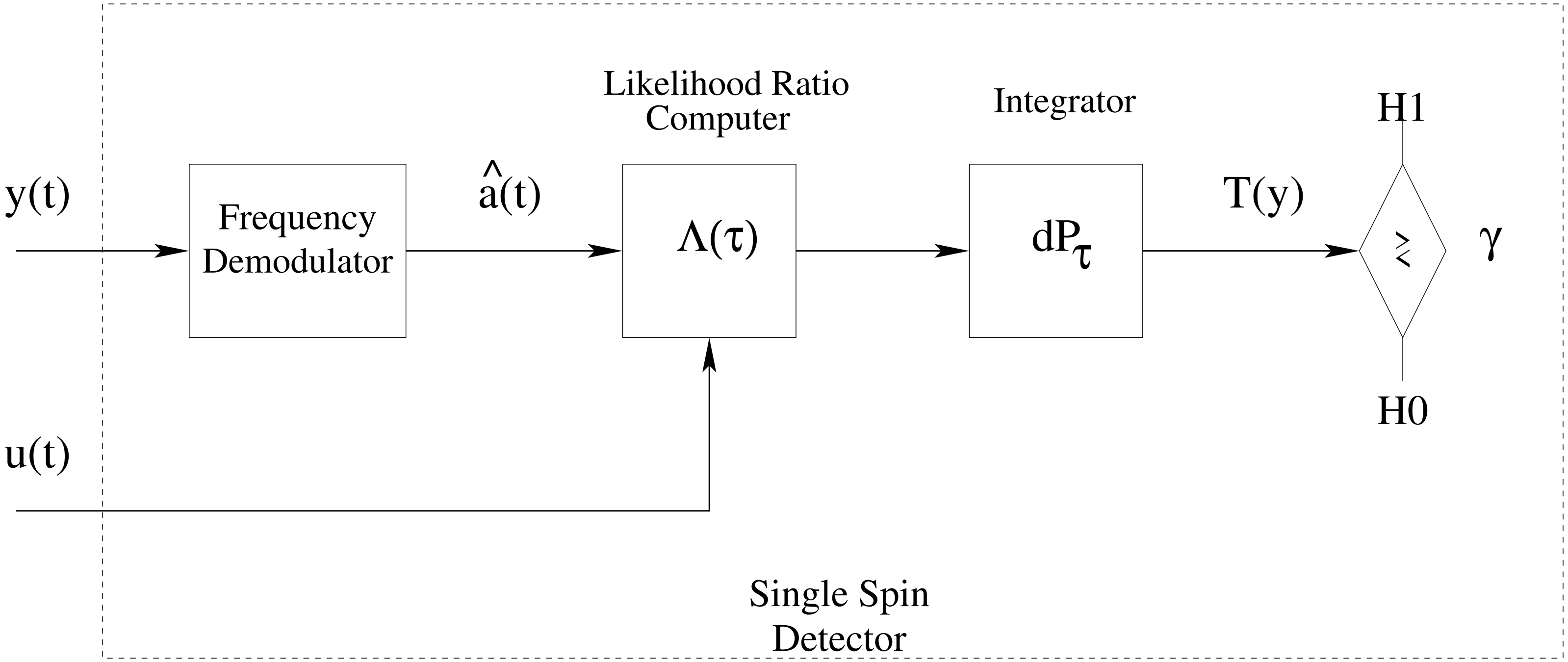

Figure 5: *Minimum P_e detector.*

Bayes detector minimizes average probability of error

$$P_e = \beta P(H_1) + \alpha P(H_0)$$

Issues:

- ROC bounds and approximations?

- implementation complexity: multiple dimensional integration?

- sensitivity to miss-specified priors?

Areas of Research

1. Determine performance and minimum SNR requirements for energy detector
2. Investigate implementation of GLR and optimal Bayes detectors
3. Find tight lower bounds and approximations on performance
4. Explore detector sensitivity/robustness to model mismatch
5. Consider more general setting of adaptive non-linear system identification
6. Integrated frequency demodulator/signal-detector
7. Signal waveform design as channel coding?
8. Single-spin tomography: reconstruction from multiple spin measurements.