Image Registration

Image registration: the process of aligning images

Purpose: essential for extraction of common spatial information

Typical applications:

- Integrating information from different sensors (eg. fusion)
- Finding changes in images taken at different times / under different conditions (eg. clinic studies)
- Inferring three-dimensional information from images when the sensor / objects have moved (eg. video)

Image registration consists of:

- **Feature space $\mathcal{F}$:** reduced dimensional representation of common information
- **Search space $\mathcal{T}$:** the class of spatial transformations
- **Dissimilarity metric:** a measure of difference between images

Given two images I_1 and I_2, and corresponding feature vectors F_1 and F_2, for $T \in \mathcal{T}$, define the dissimilarity metric

$$d_T(I_1, I_2) = \|T(F_1) - F_2\|$$

Registration problem: estimate the mapping T such that

$$T^* = \arg\min_{T \in \mathcal{T}} d_T(I_1, I_2)$$

Image registration steps:

1. Extract feature vectors $F_1, F_2 \in \mathcal{F}$ from both images
2. Apply candidate $T \in \mathcal{T}$, and compute the dissimilarity metric $d = \|T(F_1) - F_2\|$.
3. Refine T to reduce d.
4. Repeat 2 until $d = d_{min}$ achieved.

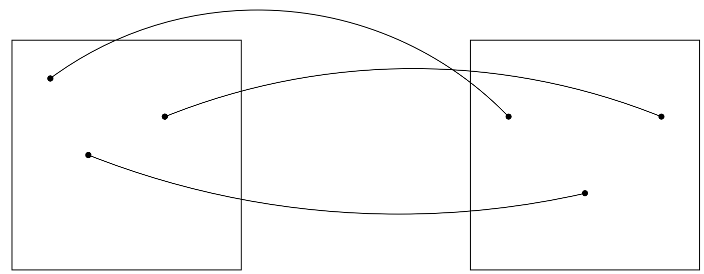
Image registration requirements:

- Robustness to small differences and outliers
- Computational feasibility
- At least semi-automatic feature extraction and selection

Previous registration methods:

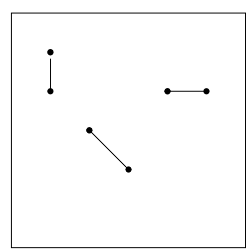
- Correlation and sequential methods
- Fourier methods
- Point mapping
- Model-based matching
- Mutual information method
  Viola and Wells ’96, Maes ’97, Thevenaz ’98

Registration Via Graph Matching

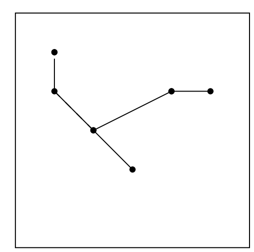


F_1 F_2

(a)



(b) Correspondence graph



(c) MST

MST

Let $\mathcal{X}_n = \{X_1, X_2, \ldots, X_n\}$ be a set of n feature vectors in $I\!R^d$.

- **Spanning Tree** $\mathcal{T}$ is a connected acyclic graph over $\mathcal{X}_n$.

  Power weighted length for Tree $\mathcal{T}$:

$$L(\mathcal{X}_n) = \sum_{e_{ij} \in \mathcal{T}} |e_{ij}|^\gamma$$

- **Minimal Spanning Tree (MST)** is the spanning tree which minimizes $L(\mathcal{X}_n)$.

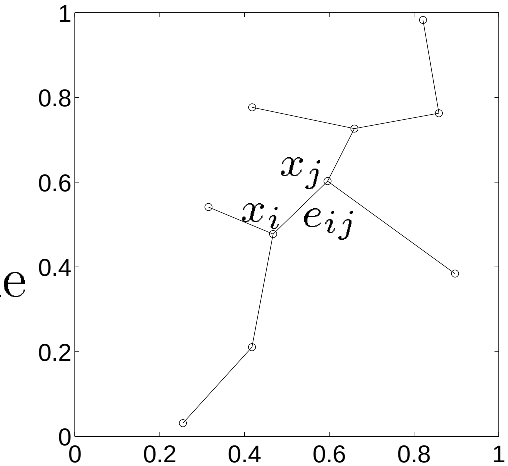


Figure 1: An MST example

Robustness via k-MST

Let $\mathcal{X}_{n,k} \subseteq \mathcal{X}_n$ contain k points.

- k-point MST is the MST spanning over $\mathcal{X}_{n,k}$.
- **The minimal k-point spanning tree (k-MST)** is the k-point MST of minimal length over all $\mathcal{X}_{n,k}$.

$$L(\mathcal{X}^*_{n,k}) = \min_{\mathcal{X}_{n,k}} L(\mathcal{X}_{n,k})$$

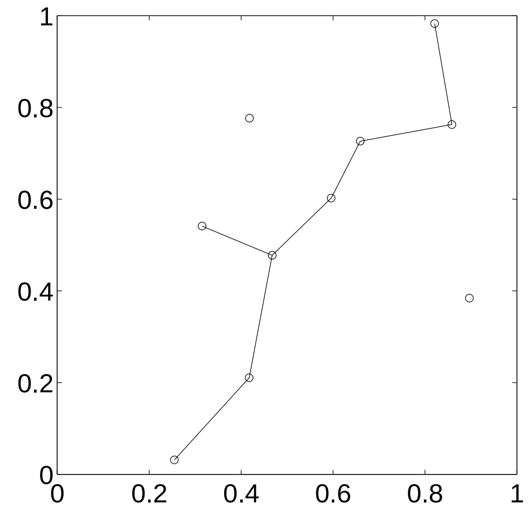


Figure 2: A k-MST example

Rényi Entropy and MST

Suppose $\mathcal{X}_n$ is a random sample from density f.
Rényi entropy of fractional order $\alpha = (d-\gamma)/d$:

$$R_\alpha(f) = \frac{1}{1-\alpha} \log \int_{I\!R^d} f^\alpha(x)dx$$

Let $L(\mathcal{X}_n)$ denote the γ-powered MST length function,

$$\lim_{n\to\infty} \frac{L(\mathcal{X}_n)}{n^\alpha} = \beta \int_{I\!R^d} f^\alpha(x)dx \qquad \text{(a.s.)}$$

Then

$$\hat{R}_\alpha(f) = \frac{1}{1-\alpha}\left[\log \frac{L(\mathcal{X}_n)}{n^\alpha} - \log \beta\right]$$

Register Image With MST

- Given two images I_1 and I_2
- Feature vectors F_1 and F_2
- Underlying densities f_1 and f_2

Image registration requires:

$$T^* = \arg\min_{T \in \mathcal{T}} R_\alpha(\epsilon f_1(T(\underline{x})) + (1-\epsilon) f_2(\underline{x}))$$

where

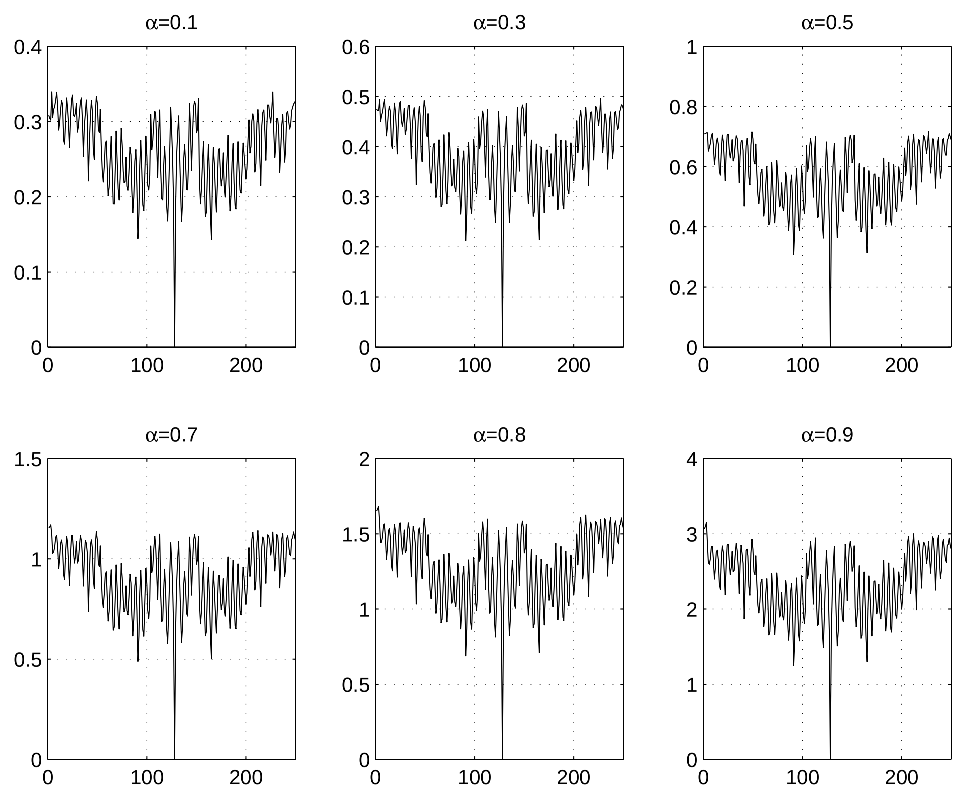
$$\epsilon = \frac{\mathrm{Card}(F_1)}{\mathrm{Card}(F_1) + \mathrm{Card}(F_2)}$$

Equally,

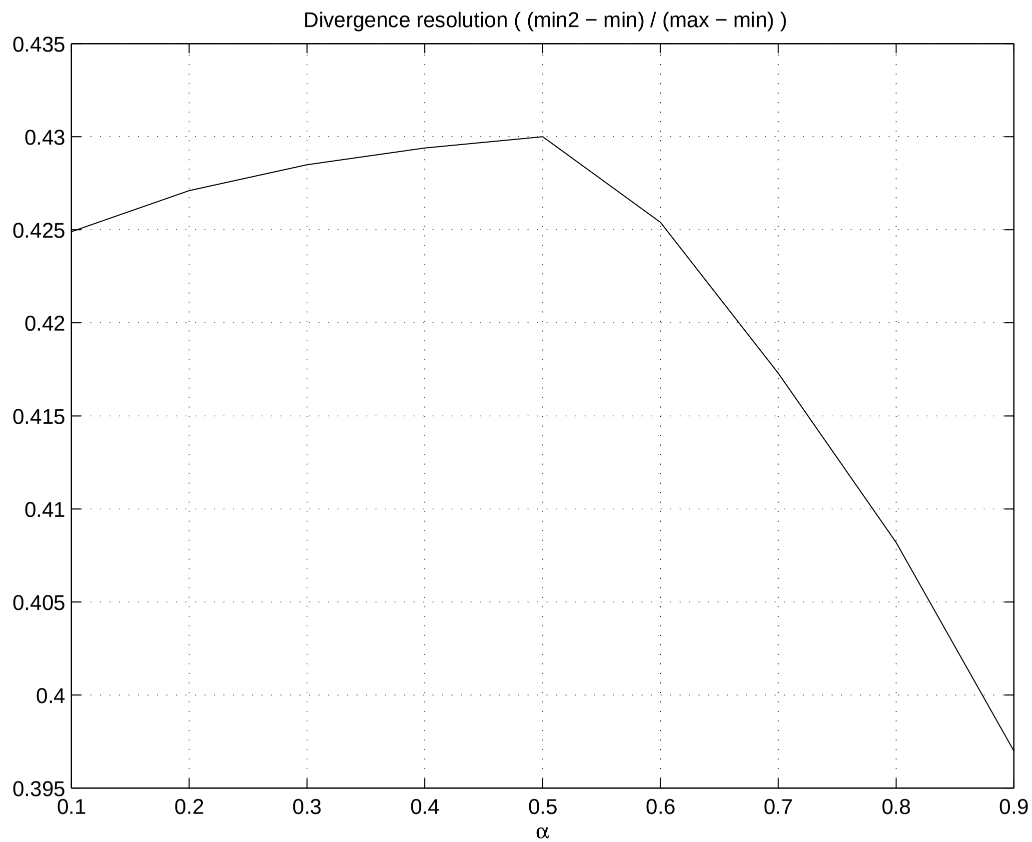
$$T^* = \arg\min_{T \in \mathcal{T}} L(T(F_1) + F_2)$$

Rényi information divergence:

$$I_\alpha(f, f_0) = \frac{1}{1-\alpha} \log \int \left(\frac{f(x)}{f_0(x)} \right)^\alpha f_0(x) dx$$



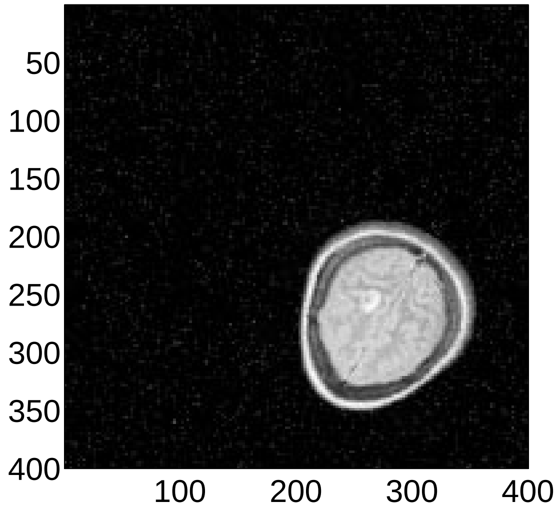
Divergence resolution:



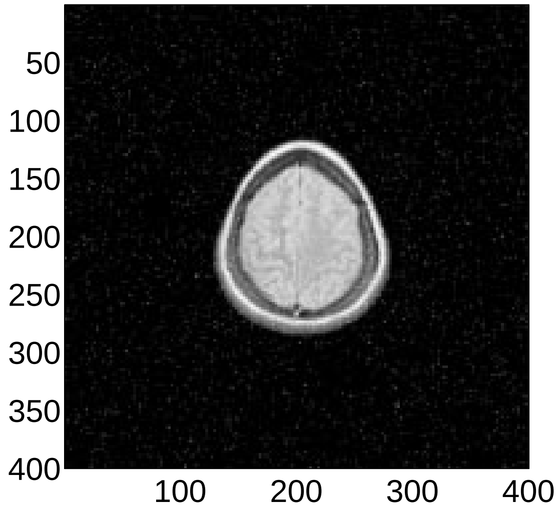
Experimental Results

Registering brain images taken at different times:

Noisy brain images for registration:

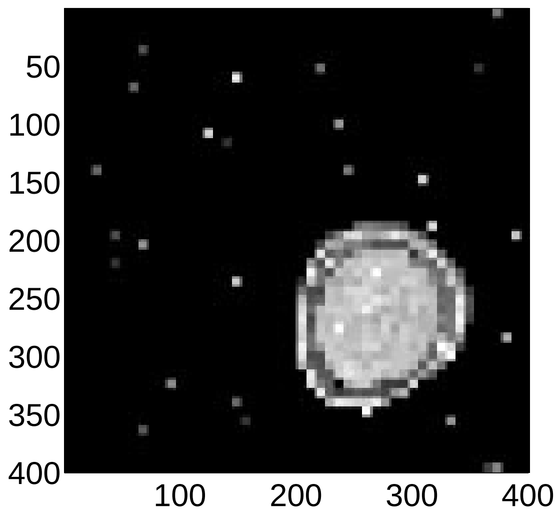


(a) Pre-operation

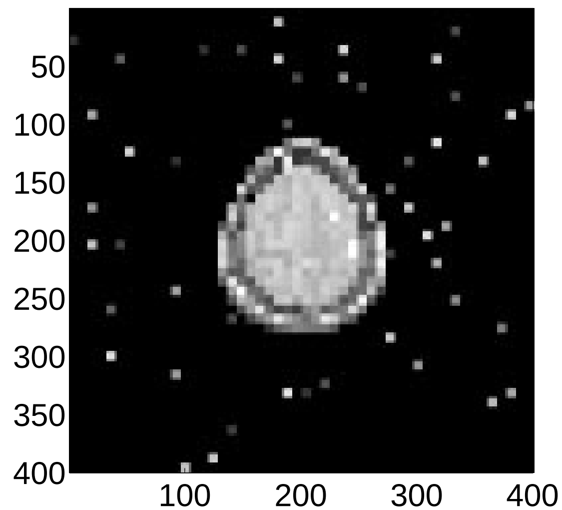


(b) Post-operation

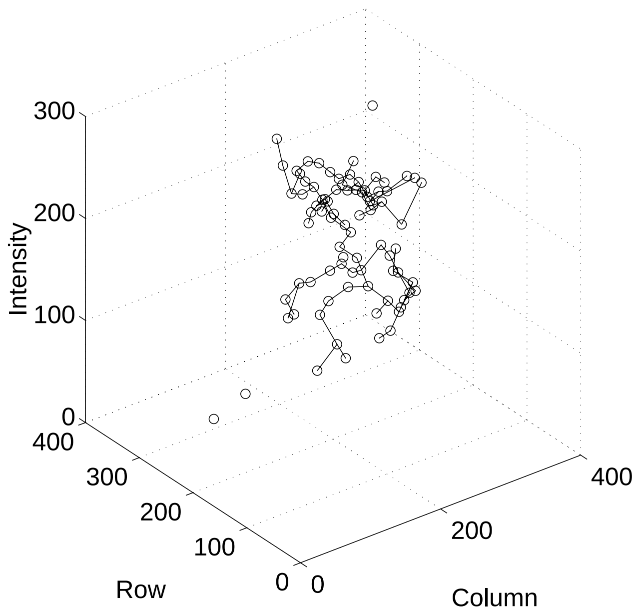
Subsampled images for registration:



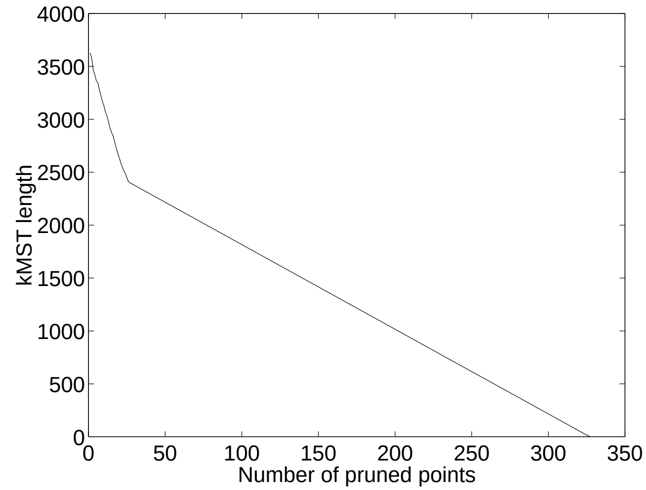
(a) Pre-operation



(b) Post-operation

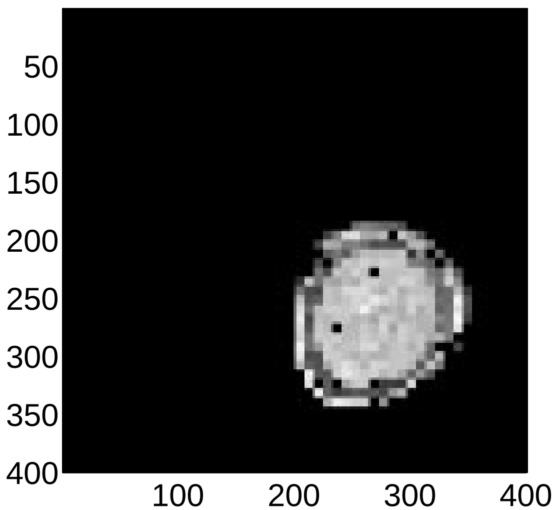


(a) k-MST for post-operation brain image

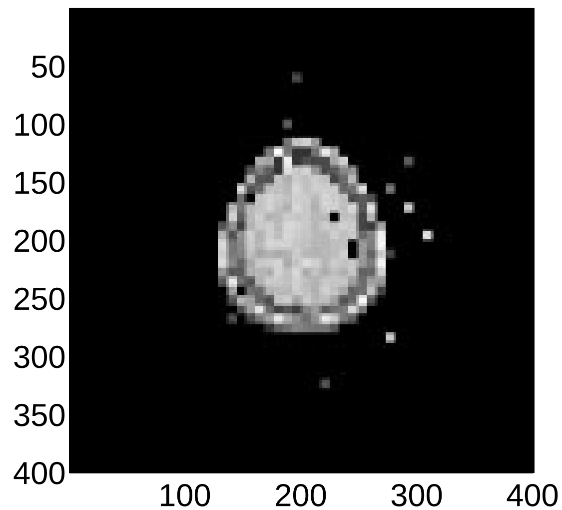


(b) k-MST length curve

Subsampled images after outlier removal:

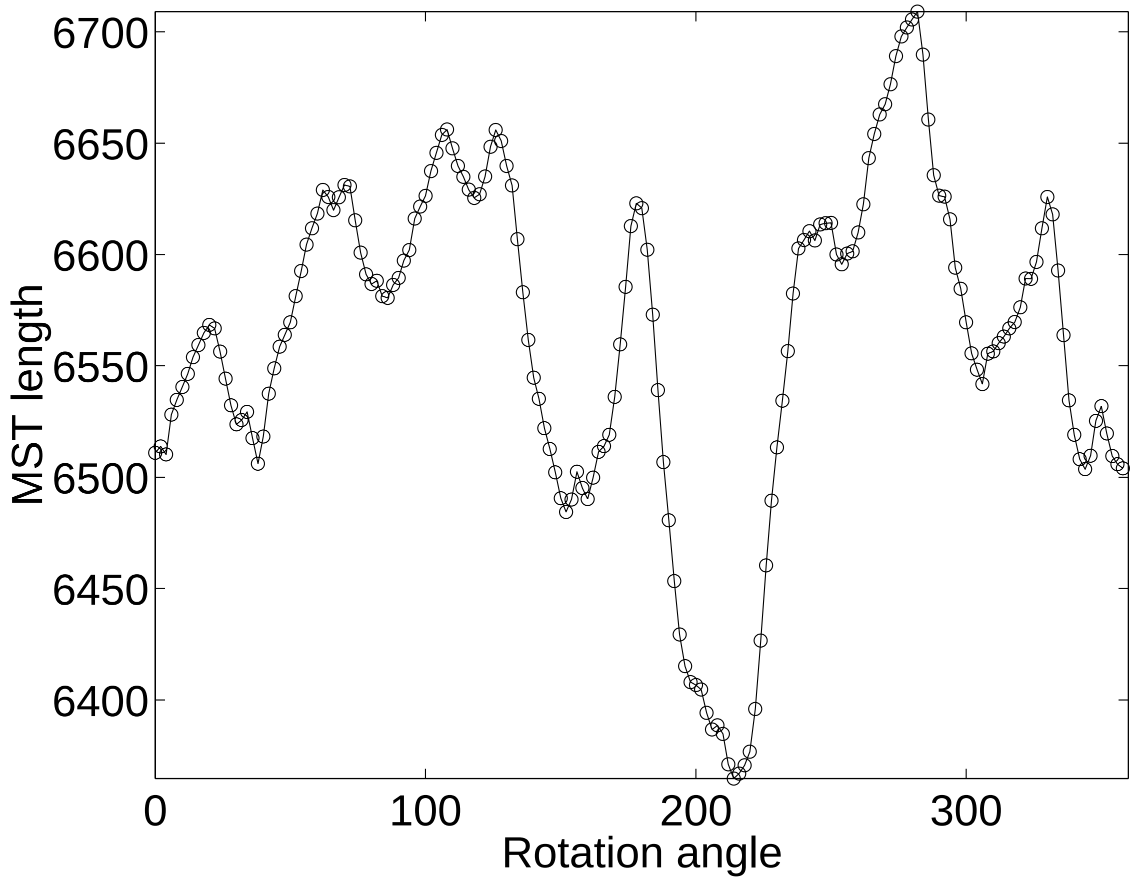


(a) Pre-operation

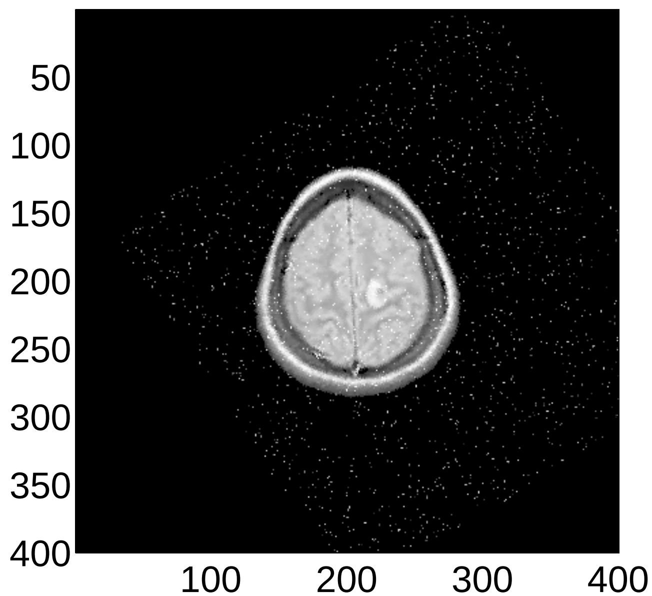


(b) Post-operation

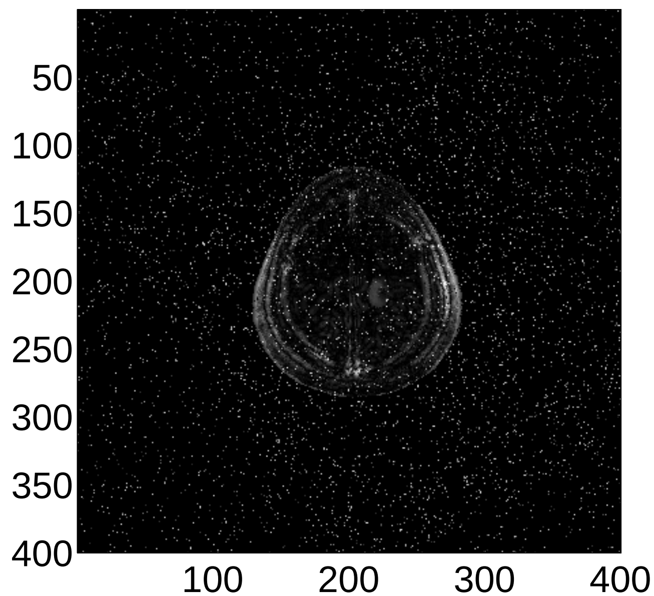
MST length as a function of rotation angle:



Registration result for brain images:



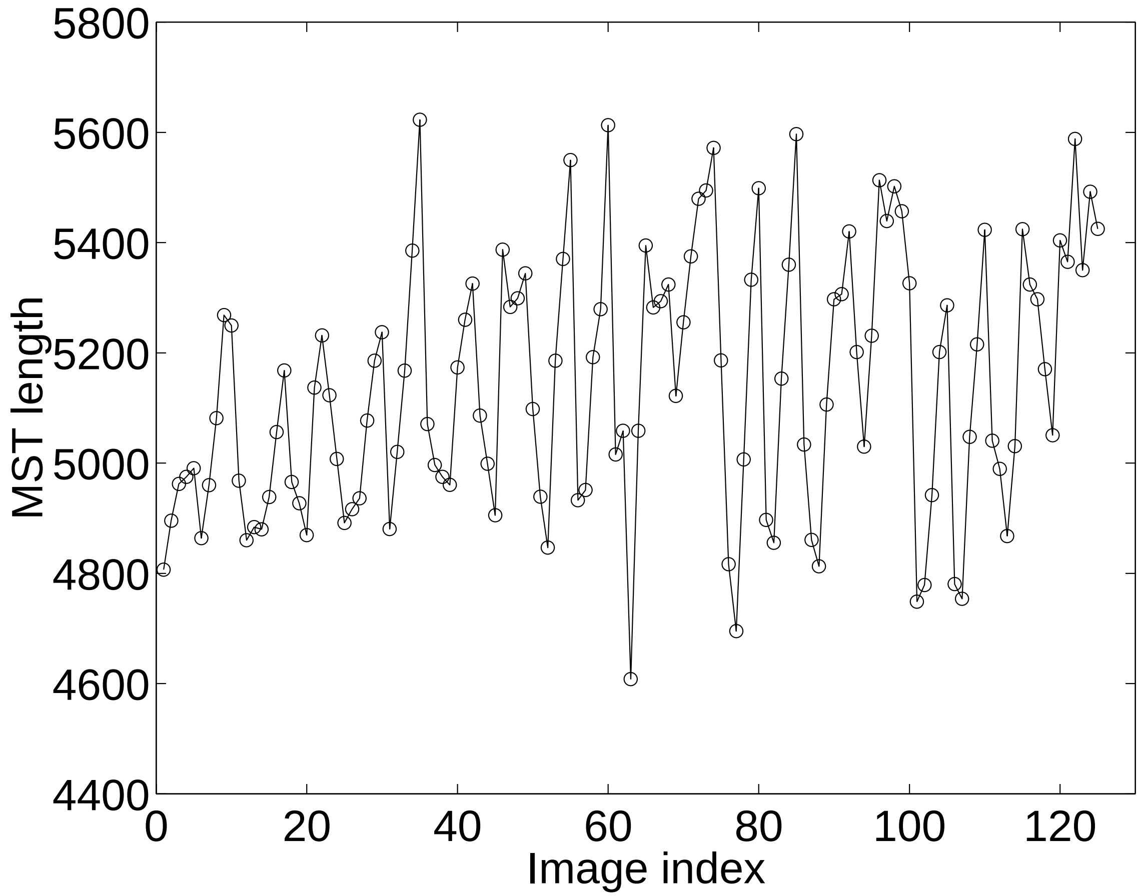
(a) Registered pre-operation brain image



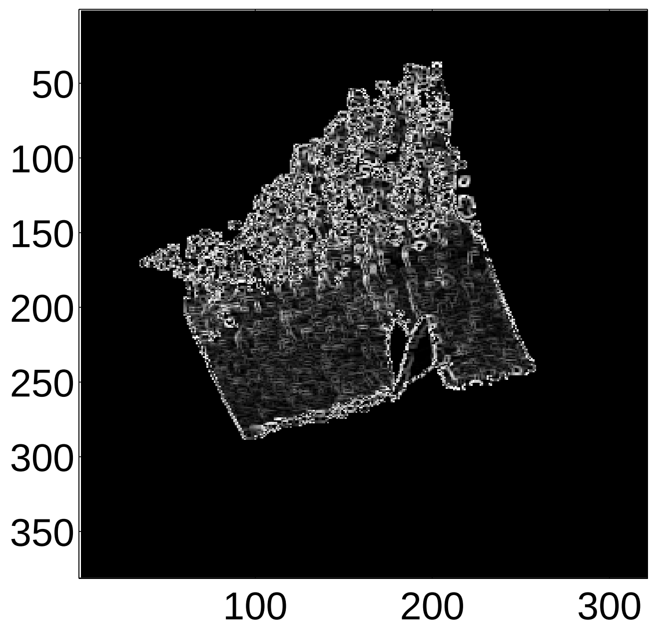
(b) Registration error

Geo-registration application:

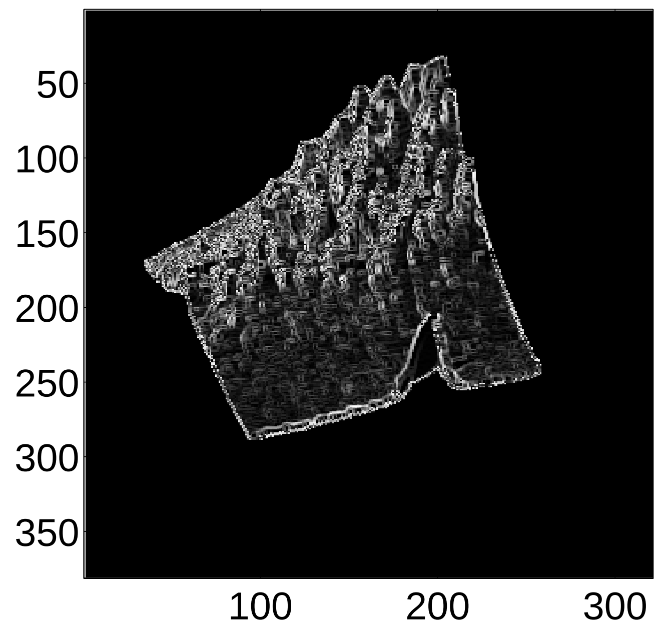
MST length for EO – terrain height map registration:



Registration result:



(c) EO image



(d) Projected terrain height map

Conclusions

- Proposed to register images by minimizing Rényi entropy.
- Implemented image registration by minimizing MST length.
- Employed k-MST to improve the registration robustness.
- Satisfactory algorithm performance was shown by experimental results.
- Will reduce computational complexity by extracting better features.