Image Registration

The process of aligning images when objects have moved (e.g., video).

Purpose: Essential for extraction of common spatial information.

Typical applications:

- Integrating three-dimensional information from images taken at different times under different conditions (e.g., clinic studies).
- Finding changes in images taken at different times (e.g., fusion).
- Inferring three-dimensional information from images when objects have moved (e.g., video).
Image registration consists of:

- **Feature space** $F$: reduced dimensional representation of common information

- **Search space** $T$: the class of spatial transformations

- **Dissimilarity metric** $d$: a measure of difference between images $I_1$ and $I_2$

**Registration problem**: estimate the mapping $T$ such that

$$\min_{T \in T} d(I_1; I_2) = (I_1, I_2)_{L^p}$$

Given two images $I_1$ and $I_2$, and corresponding feature vectors $F_1$ and $F_2$, for $T_1 \in T$, for $T_2 \in T$, define the dissimilarity metric and $F_2$.
Imageregistration steps:

1. Extract feature vectors from both images.
2. Apply candidate $T$, and compute the dissimilarity metric $d = \| H - H(x) T \|$, and compute the dissimilarity.
3. Repeat to reduce $d$ until $d = d_{\min}$ achieved.
4. Repeat 2 until $p = p_{\text{desired}}$ achieved.

Imageregistration requirements:

- Robustness to small differences and outliers
- Computational feasibility
- At least semi-automatic feature extraction and selection
Previous registration methods:

Viola and Wells, '96, Maes, '97, Thevenaz, '98

Mutual information method •

Model-based matching •

Point mapping •

Fourier methods •

Correlation and sequential methods •
Registration via Graph Matching
Let $\mathcal{X}_n = \{X_1, X_2, \ldots, X_n\}$ be a set of $n$ feature vectors in $\mathbb{R}^d$.

- **Spanning Tree** $\mathcal{T}$ is a connected acyclic graph over $\mathcal{X}_n$.

Power weighted length for Tree $\mathcal{T}$:

$$L(\mathcal{X}_n) = \sum_{e_{ij} \in \mathcal{T}} |e_{ij}|^\gamma$$

- **Minimal Spanning Tree (MST)** is the spanning tree which minimizes $L(\mathcal{X}_n)$.

Figure 1: An MST example
Robustness via $k$-MST

Let $\mathcal{X}_{n,k} \subseteq \mathcal{X}_n$ contain $k$ points.

- $k$-point MST is the MST spanning over $\mathcal{X}_{n,k}$.

- The minimal $k$-point spanning tree ( $k$-MST ) is the $k$-point MST of minimal length over all $\mathcal{X}_{n,k}$.

$$L(\mathcal{X}^*_{n,k}) = \min_{\mathcal{X}_{n,k}} L(\mathcal{X}_{n,k})$$

Figure 2: A $k$-MST example
Suppose $X_n$ is a random sample from density $f$. Then

\[\left[ \log \frac{\nu u}{(u')^T} \right] \frac{\nu - 1}{1} = (f)^\nu H\]

Let \(L(X_n)\) denote the \(\nu\)-powered MST length function,

\[(\text{a.s.)}) \quad \lim_{n \to \infty} \frac{1}{n} \int \log \frac{\nu u}{(u')^T} \to \log \frac{\nu u}{(u')^T} \]

Let denote the \(\nu\)-powered MST length function,

\[xp(x) \nu \int \log \frac{\nu u}{(u')^T} \to \log \frac{\nu u}{(u')^T} \]

Then

\[xp(x) \nu \int \log \frac{\nu u}{(u')^T} = (f)^\nu H\]

\[p/(\nu - p) = \nu\]

Rényi entropy of fractional order \(\nu\) is a random sample from density $u'X$.
Given two images $I_1$ and $I_2$

Feature vectors $F_1$ and $F_2$

Underlying densities $f_1$ and $f_2$

Feature vectors $I_1$ and $I_2$

Given two images $I_1$ and $I_2$

Image registration requires:

Register Image With MST

Equivalently:

$$\begin{align*}
\left(\bar{H} + (\bar{H})L\right)_T^L & = \arg \min_T \mathcal{L} \\
\frac{\left(\bar{H}\right)}{\left(\bar{H}\right)} & = e
\end{align*}$$

where

$$(\bar{x})_{x, f}(e - 1) + (\bar{x})_L^L \arg \min_T \mathcal{L} = \arg \min_T \mathcal{L}$$
\[
x p(x) \log \left( \frac{f(x)}{f(x)} \right) \int_{\mathbb{R}} \frac{\nu - 1}{I} = (\nu f, f)^\nu \!
\]

Rényi information divergence:
Divergence resolution (\( \frac{\text{min} - \text{min}}{\text{max} - \text{min}} \))
Experimental Results

Registering brain images taken at different times:

Noisy brain images for registration:

(a) Pre-operation

(b) Post-operation
Subsampled images for registration:

(a) Pre-operation

(b) Post-operation
(a) k-MST for post-operation brain image

(b) k-MST length curve
Subsampled images after outlier removal:

(a) Pre-operation  (b) Post-operation
MST length as a function of rotation angle:
Registration result for brain images:

(a) Registered pre-operation brain image  (b) Registration error
Geo-registration application:

MST length for EO – terrain height map registration:
Registration result:

(c) EO image

(d) Projected terrain height map
Conclusions

- Proposed to register images by minimizing Rényi entropy.
- Implemented image registration by minimizing MST length.
- Employed k-MST to improve the registration robustness.
- Satisfactory algorithm performance was shown by experimental results.
- Will reduce computational complexity by extracting better features.