EECS 659
Adaptive Signal Processing

Fall 1998

Exam I

This is a two hour open book and open notes exam. Points for correctly answering each question are as indicated.

1. [30] Give true or false answers to the following questions providing justification of your answer for each case via example or counterexample.

   (a) [10] Newton's iterative method for function optimization always has better convergence properties than steepest descent methods.

   (b) [10] For adaptive channel equalization and deconvolution it is essential that any additive noise in the training signal be uncorrelated with the channel noise.

   (c) [10] The gain parameter μ in the SD algorithm can always be selected to ensure convergence as long as the reference has non-singular covariance matrix.

2. [30] Consider an adaptive FIR predictor (as in Fig. 2.1 of Kang and Solo text) for one step prediction of a sinusoid of known frequency, i.e. $x_k = cos(2\pi f_o k)$ and $y_k = cos(2\pi f_o (k+1))$. Let $\underline{w} = [w_1, \ldots, w_p]^T$ be the vector of p-th order optimal filter coefficients that minimizes the average of the squared errors $\frac{1}{M}\sum_{k=1}^{M} e_k^2$, where $M = 1/f_o$ is the sinusoidal period.

   (a) [10] Find the optimal filter $\underline{w}_o$ for the case $p = 1$. What is the resultant minimum average squared error?

   (b) [10] Find the optimal filter $\underline{w}_o$ for the case $p = 2$. What is the resultant minimum average squared error?

   (c) [10] What is the optimal filter $\underline{w}_o$ for $p > 2$ and what is its minimum average squared error?

3. [30] As an improvement on the momentum SD algorithm it is suggested to use a differentiation filter of the form

$$H(q^{-1}) = \frac{1 - \beta q^{-1}}{1 - \alpha q^{-1}}$$

   in the modified SD recursion:

$$\underline{w}_k = \underline{w}_{k-1} + H(q^{-1})\delta\underline{w}_{k-1}. \tag{1}$$

   where $\delta\underline{w}_k = \mu(R_{xy} - R_x\underline{w}_k)$.

   (a) [6] Rewrite the modified SD recursion (1) explicitly in terms of $\underline{w}_k$, $\underline{w}_{k-1}$, and $\underline{w}_{k-2}$ (without q^{-1} terms).

   (b) [6] Using the results of part (a) find a recursion for the weight error $\tilde{w}_k = \underline{w}_k - \underline{w}_{opt}$.

   (c) [6] Using the results of part (b) give a condition on μ involving α, β and the eigenvalues of R_x that ensures convergence.

   (d) [6] Now specialize to the scalar case $\underline{w} = w$ and derive the following explicit bound for $\mu\sigma_x^2$ when α and β are selected to satisfy the condition $\beta\mu\sigma_x^2 \approx \alpha$:

$$0 < \mu\sigma_x^2 < \frac{2}{1 - \beta}$$

   .

(e) [6] How does the result of part (d) compare to the conditions on μ for the standard SD and for the standard momentum filter?

4. [10] Briefly describe your project and state any progress made (be honest!).