ROBUST DETECTION, CLASSIFICATION AND CLUSTERING

Progress over period 11/95 - 8/98

Alfred Hero
Dept. of EECS
University of Michigan

Principal Objective: To develop weak-target detection/classification/clustering algorithms which are robust to target and clutter variability yet maintain highest possible discrimination capability.

Methods: CFAR target detectors, iterative estimation, graph theoretic clustering algorithms, subspace processed moment matrices.

RESEARCH ACCOMPLISHMENTS

- **Detection algorithms for inhomogeneous clutter**: Max invariant and GLR tests (Hero&Guillouet [1]).
- **Accelerated EM and SAGE ML/PML algorithms**: proximal-point with Kullback penalty (Chretien&Hero [2],[3], [4]).
- **Theory and application of k-shortest graphs**: entropy estimation, robust clustering, image registration, resource allocation (Hero&Michel [5],[6],[7],[8],)
- PCA for higher order moment matrices: non-parametric target detection and classification (Hero, O'Neill&Williams [9],[10]).

Detection of Deep Hide Targets

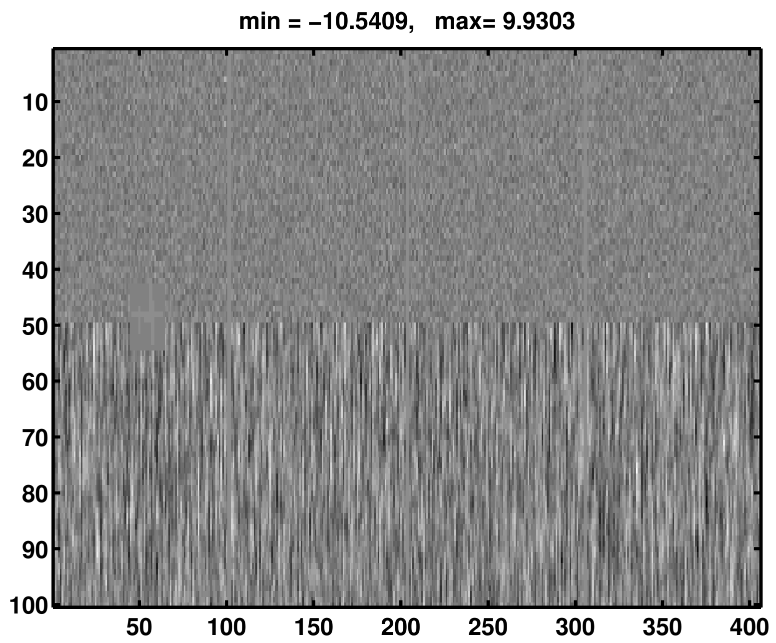


Figure 1: Known deep hide target on clutter boundary

Objective: Adaptive target detection in structured unknown clutter.

Methods: GLR and maximal invariant tests for structured MANOVA

Challenges:

- previous GLR work applies only to non-structured homogeneous clutter (Kelly:AES86, Kelly&Forsythe:TR848, Chen&Reed:AES87, ...)
- extension of group invariance approaches to SAR/IR imaging

Accomplishments:

- GLR extended to case of block structured clutter covariance.
- Maximal invariant tests extended to multiple targets (Hero&Guillouet:ICIP97 [1])
- quantitative improvement observed in P_D for $P_F < 0.1$.

Hypothesis testing problem:

$$H_0 \;:\; \mathbf{Y} = \mathbf{N}$$

$$H_1 \;:\; \mathbf{Y} = \mathbf{S}\,\mathbf{a}\,\mathbf{B} + \mathbf{N}$$

- $\mathbf{S} = [\underline{s}_1, \ldots, \underline{s}_p]$ = known $m \times p$ matrix of spatial target signatures
- $\mathbf{a}$ = unknown $p \times p$ matrix
- $\mathbf{B}$ known $p \times n$ indicator matrix of target-containing chips
- $\text{vec}\{\mathbf{N}\} \sim \mathcal{N}_{mn}\left(\underline{0}, \mathbf{R} \otimes \mathbf{I}_n\right)$
- $\mathbf{R}$ unknown **block diagonal** clutter covariance

GLR Test:

$$\Lambda(\mathbf{Y}) = \frac{\max_{\mathbf{a} \neq 0, \mathbf{R} > 0} f(\mathbf{Y};\; \mathbf{a},\; \mathbf{R})}{\max_{\mathbf{a} = 0, \mathbf{R} > 0} f(\mathbf{Y};\; \mathbf{a},\; \mathbf{R})} \quad \overset{H_1}{\underset{H_0}{\gtrless}} \quad \eta$$

$\mathbf{R}_{AB}$	$\mathbf{R}_A$	$\mathbf{R}_B$	Log GLR
?	?	?	$2/n\ \ln(\Lambda) = \ln\left(\frac{1+q(0,\hat{R})}{1+q(\hat{a}_0,\hat{R})}\right)$
0	?	?	$2/n\ \ln(\Lambda) = \ln\left(\frac{1+q_A(0,\hat{R}_A)}{1+q_A(\hat{a}_1,\hat{R}_A)}\ \frac{1+q_B(0,\hat{R}_B)}{1+q_B(\hat{a}_1,\hat{R}_B)}\right)$
0	$\sigma^2\mathbf{C}_A$	?	$2/n\ \ln(\Lambda) = \ln\left(\frac{1+q_B(0,\hat{R}_B)}{1+q_B(\hat{a}_2,\hat{R}_B)}\frac{q_A(0,C_A)}{q_A(\hat{a}_2,C_A)}\right)$
0	$\mathbf{C}_A$	?	$2/n\ \ln(\Lambda) = \ln\left(\frac{1+q_B(0,\hat{R}_B)}{1+q_B(\hat{a}_3,\hat{R}_B)}\right) + q_A(0,C_A) - q_A(\hat{a}_3,C_A)$

Table 1: GLR Test statistics for various block structured clutter covariance

Special case: Single known target $\mathbf{S} = \underline{s}\ (m \times 1)$

- $q_A(a, R)$ is a quadratic form:

$$q_A(a, R_A) = (\underline{\bar{x}}_i - a\underline{s}_A)^H \mathbf{R}_A^{-1} (\underline{\bar{x}}_i - a\underline{s}_A)$$

- $\hat{a}_0, \hat{a}_1, \hat{a}_2, \hat{a}_3$ are solutions to cubic equation

Group Invariance approach:

- Find group of transformations $\mathcal{G}$ leaving model invariant
- Find maximal invariant $\mathbf{Z}$ under group actions
- Compute MPI detector using $\mathbf{Z}$

$$\Lambda(\mathbf{Z}) = \frac{f(\mathbf{Z};\ \mathrm{a}, \mathbf{R})}{f(\mathbf{Z};\ 0, \mathbf{R})} \quad \overset{H_1}{\underset{H_0}{\gtrless}} \quad \eta$$

OR

- Implement deflection criterion constructed on $\mathbf{Z}$

$$Q(\mathbf{Z}) \quad \overset{H_1}{\underset{H_0}{\gtrless}} \quad \eta$$

Numerical Example

Assumptions

- presegmented clutter regions
- 51 subimages acquired from 20×40 sliding data windows
- crosshair targets

Target detection algorithms implemented:

- Maximal invariant test
- Mismatched GLR using global clutter covariance matrix R
- Block covariance constrained GLR's implemented with cube rooting

Legends for graphs

- Kelly(uns) is GLRT which ignores the boundary information and uses the full covariance matrix estimate.
- Kelly(str) is the Kelly(uns) test implemented with a block diagonal covariance estimator.
- B-S (Bose-Steinhardt) is maximal invariant test for known target which accounts for block diagonal clutter covariance.
- GLRT1 is GLR test which assumes unknown clutter covariance on each side of boundary
- GLRT2 is GLR test which assumes clutter on one side of boundary is known white ($\mathbf{R}_A$ = identity matrix).
- GLRT3 is GLR test which assumes clutter on one side of boundary is white with unknown power ($\mathbf{R}_A = \sigma^2\mathbf{I}$)

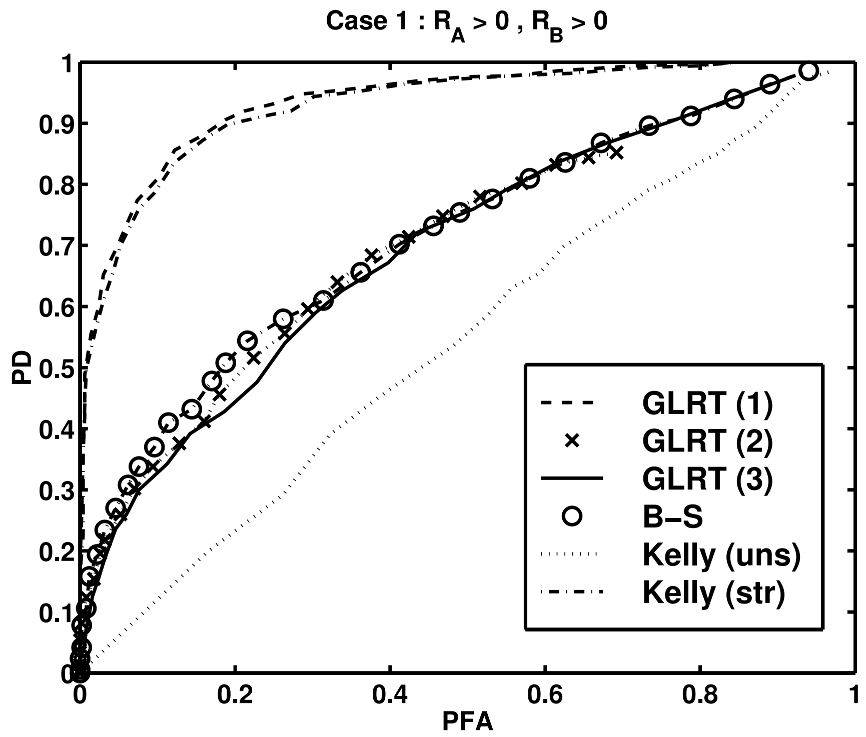


Figure 2: ROC comparisons for the case of a known target at known location on boundary between lowpass and highpass clutter regions. Note GLRT1 performs slightly better than Kelly(uns).

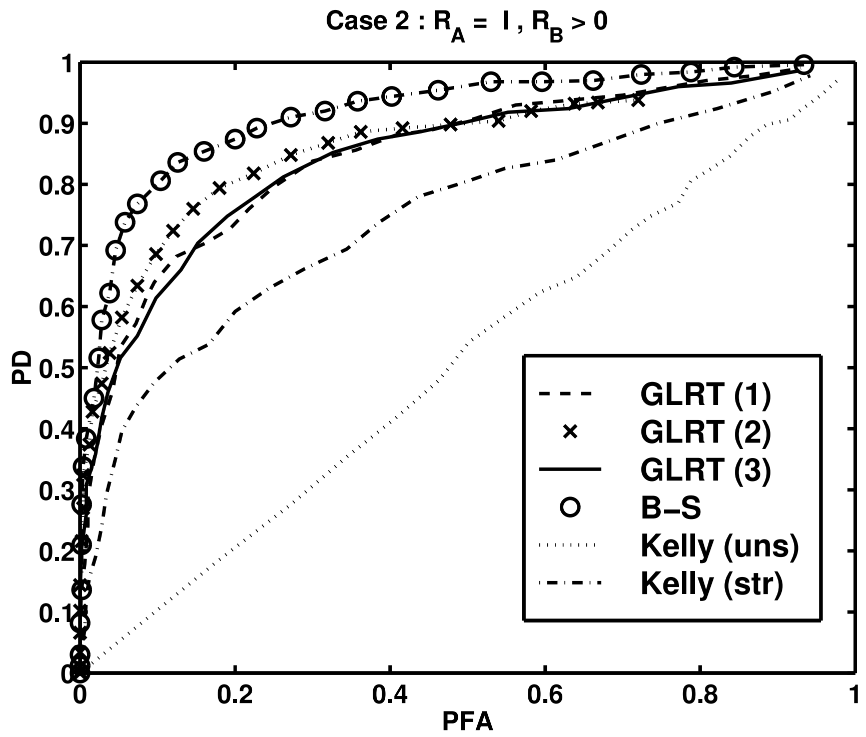


Figure 3: ROC comparisons for the case of a known target at known location on boundary between lowpass clutter region and a white clutter regionwith known power. Note B-S performs better than GLRT2.

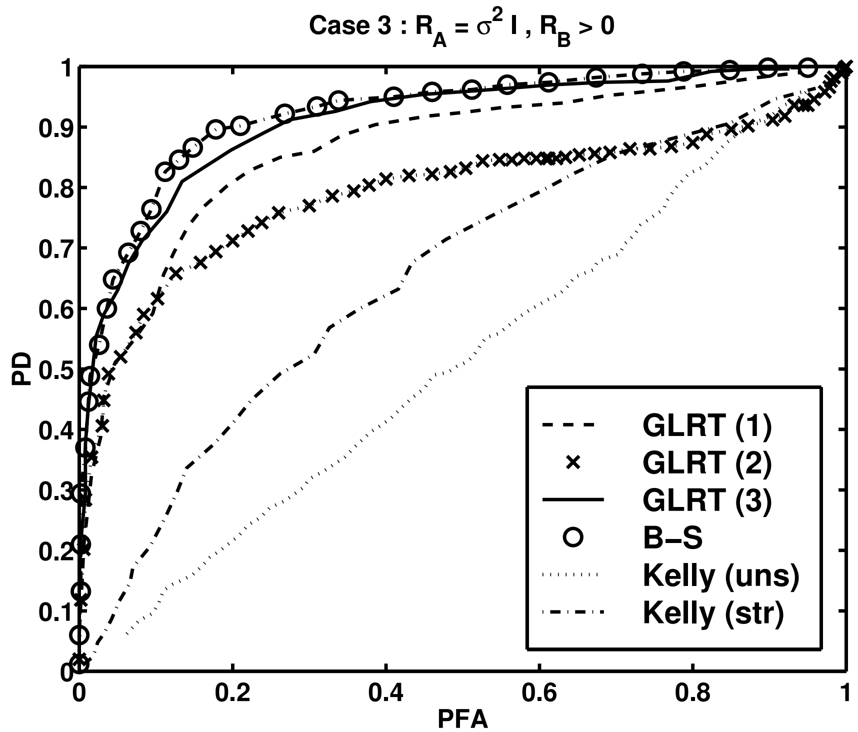
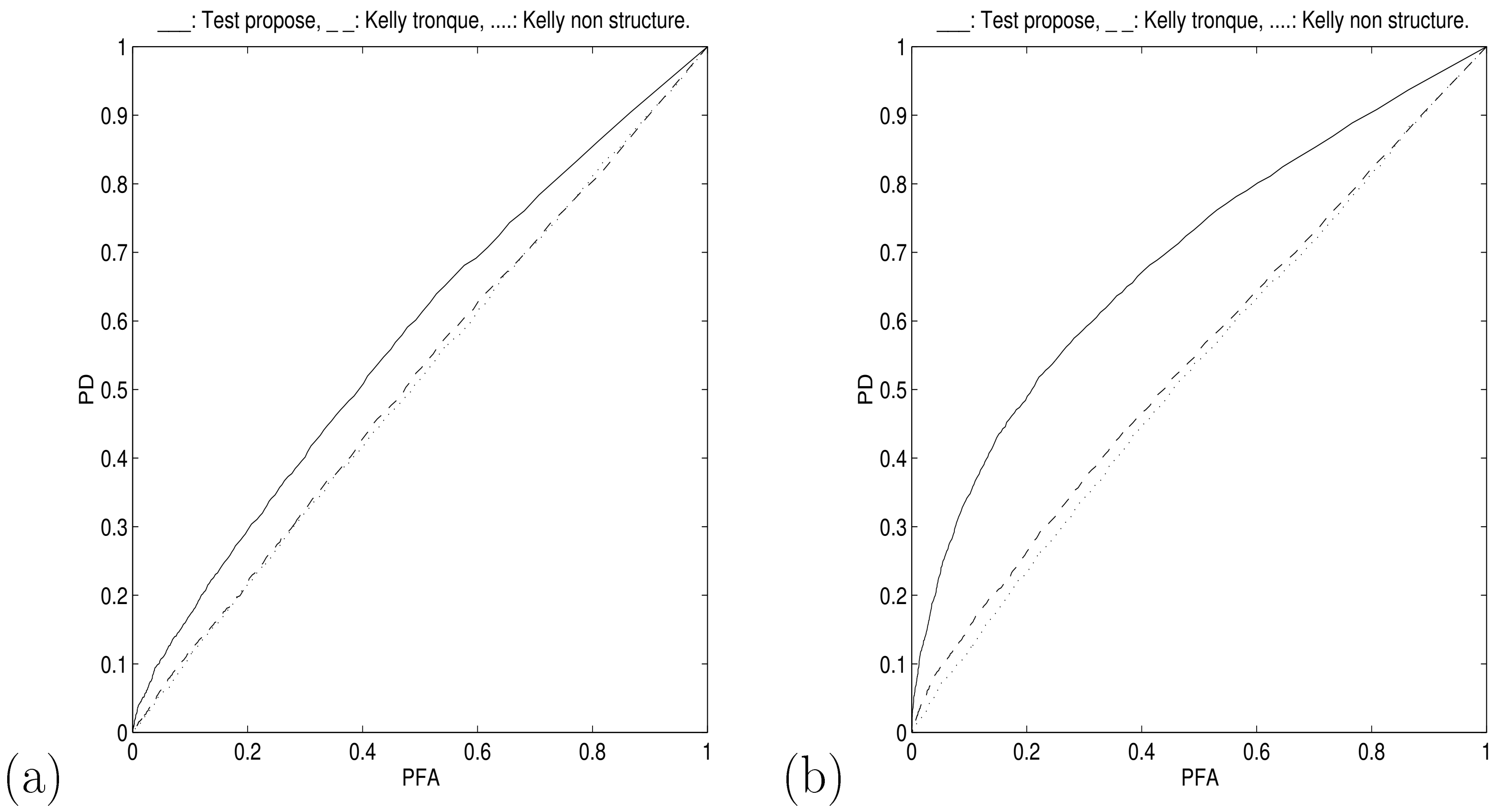


Figure 4: ROC comparisons for the case of a known target at known location on boundary between lowpass clutter region and a white clutter region with unknown power. Note B-S performs slightly better than GLRT3.



Single target at **unknown** location. ROC curves for Kelly's GLRT (structured and unstructured) and new max invariant test: (a) SNR = 0dB, (b) SNR = 20dB (From Hero&Guillouet:ICIP97).

Extensions

- Sensitivity to boundary segmentation errors
- Pose uncertainty: target subspace detection
- Polarimetric SAR
- Multiple regions: b-block structured maximal invariant
- Non-Gaussian clutter: elliptical family of densities, heavy-tailed mixture models
- **GLR via iterative optimization**: Proximal point-EM

Proximal-Point/EM Algorithms

Objective: maximize the convex (penalized) log likelihood

$$\ln f(Y;\ \underline{\theta}) - \beta P(\underline{\theta})$$

Challenges: ensuring fast convergence rate, low computation per iteration, stability, monotonicity.

<u>Our Solution</u>: (Chretien&Hero:ISIT98, Chretien&Hero:SIAM98)

Develop new class of stable, monotonic and rapidly convergent hybrid EM algorithms

Expectation Maximization Algorithm (Dempster, Laird&Rubin:JRSS78)

$$\underline{\theta}^{k+1} = \mathrm{argmax}_{\underline{\theta}} Q(\underline{\theta}|\underline{\theta}^k), \qquad k = 1, 2, \ldots$$

- $Q(\underline{\theta}|\underline{\theta}^k) = E[\ln f(X;\ \underline{\theta})|Y,\ \underline{\theta}^k]$
- X is more informative "complete data"

Advantages of EM:

- Stability: likelihood is guaranteed to increase in k
- Globally convergent for convex likelihood:
- Likelihood typically increases to $> 95\%$ of maximum within few iterations

Disadvantage of EM: Slow asymptotic convergence $\Rightarrow$ SAGE

Proximal Point Algorithm (PPA) (Rockafellar:SIAM76)

$$\underline{\theta}^{k+1} = \text{argmax}_{\underline{\theta}}\{\ln f(Y;\underline{\theta}) - \lambda_k \|\underline{\theta} - \underline{\theta}^k\|^2\}, \qquad k = 1, 2, \ldots$$

PPA with Kullback Penalty (Chretien&Hero:SIAM98)

$$\underline{\theta}^{k+1} = \text{argmax}_{\underline{\theta}}\left\{\ln f(Y;\underline{\theta}) - \lambda_k K(\underline{\theta},\ \underline{\theta}^k|Y)\right\}$$

- $K(\underline{\theta},\ \underline{\theta}^k|Y) = E\left[\ln \frac{f(X|Y;\underline{\theta})}{f(X|Y;\underline{\theta}^k)}\right]$
- $\{\lambda_k\} > 0$ sequence of relaxation parameters

$$\lambda_k > 0, \qquad \text{and} \qquad \lambda_k \to 0$$

Computational form of Kullback-PPA:

$$\underline{\theta}^{k+1} = \text{argmax}_{\underline{\theta}} \left\{ (1-\lambda_k) \ln f(Y;\underline{\theta}) + \lambda_k Q(\underline{\theta} \mid \underline{\theta}^k) \right\}, \qquad k = 1, 2, \ldots$$

Properties: (Chretien&Hero:SIAM98)

- Kullback-PPA has monotone likelihood property for any $\lambda_k > 0$
- Bundle mechanism (conjugate subgradient) (LeMarechal:75) can be applied for non-differentiable $f(Y;\underline{\theta})$, $Q(\underline{\theta};\underline{\theta}^k)$
- Obtain superlinear convergence rate for differentiable case
- Under local quadratic approximation to $\ln f(Y;\underline{\theta})$ Kullback-PPA becomes hybrid EM/Newton algorithm
- Kullback-PPA generalizes to coordinatewise optimization: hybrid SAGE/Newton

Example: Maximum likelihood deblurring

$$\underline{Y} = \mathbf{A}\underline{\theta} + \underline{n}_y \quad (n \text{ data points})$$

* $\mathbf{A}$: known blurring matrix

* $\underline{n}_y$: Poisson noise with covariance

$$\mathrm{cov}_{\underline{\theta}}\,(\mathrm{diag}[\mathbf{A}\underline{\theta} + \underline{r}])$$

* $\underline{\theta}$: unknown vector (image)

$\Rightarrow$ Compare EM, SAGE, NCA, Prox-SAGE

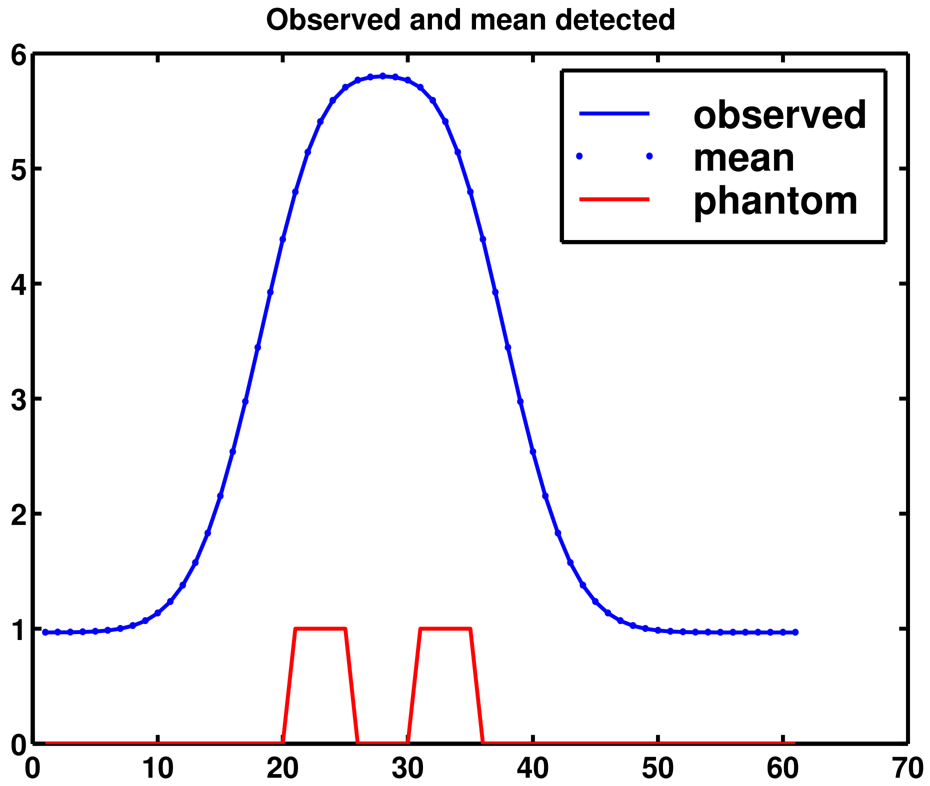


Figure 6: True bilevel signal and blurred signal

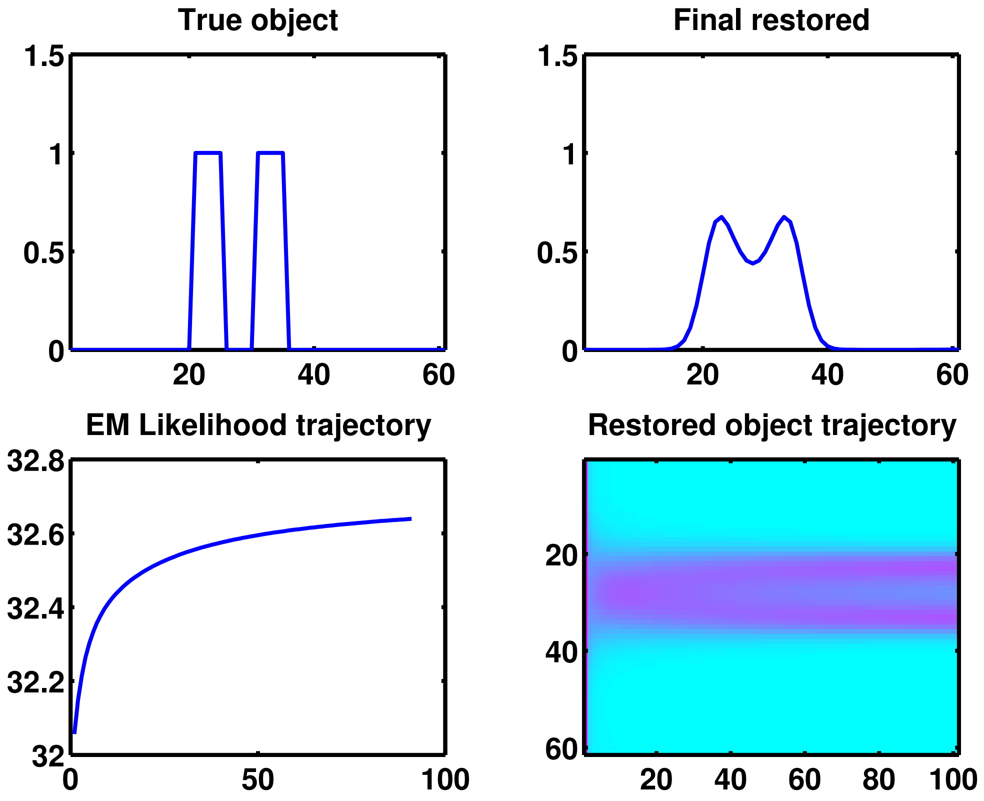


Figure 7: EM restoration

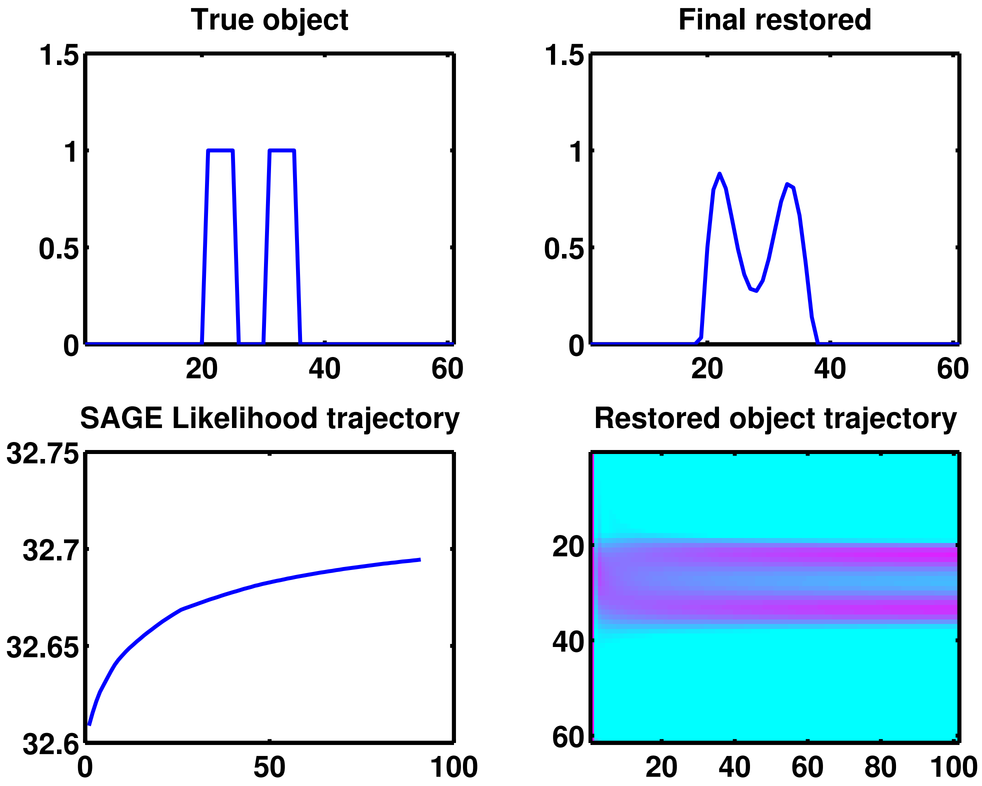


Figure 8: SAGE restoration

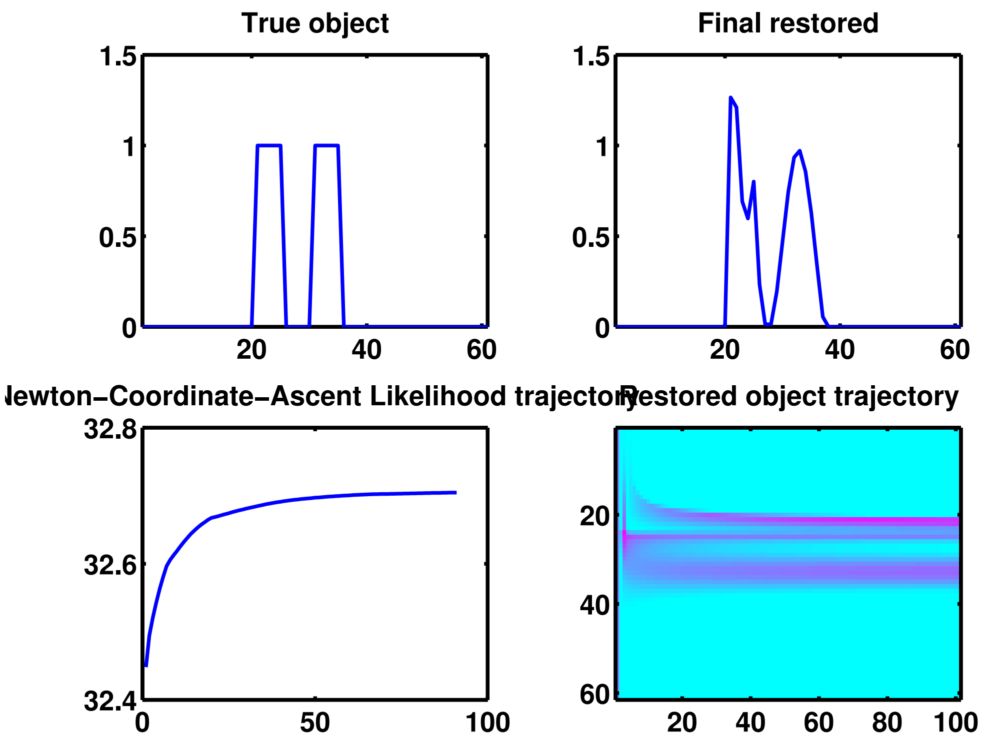


Figure 9: Netwon Coordinate Ascent reconstruction

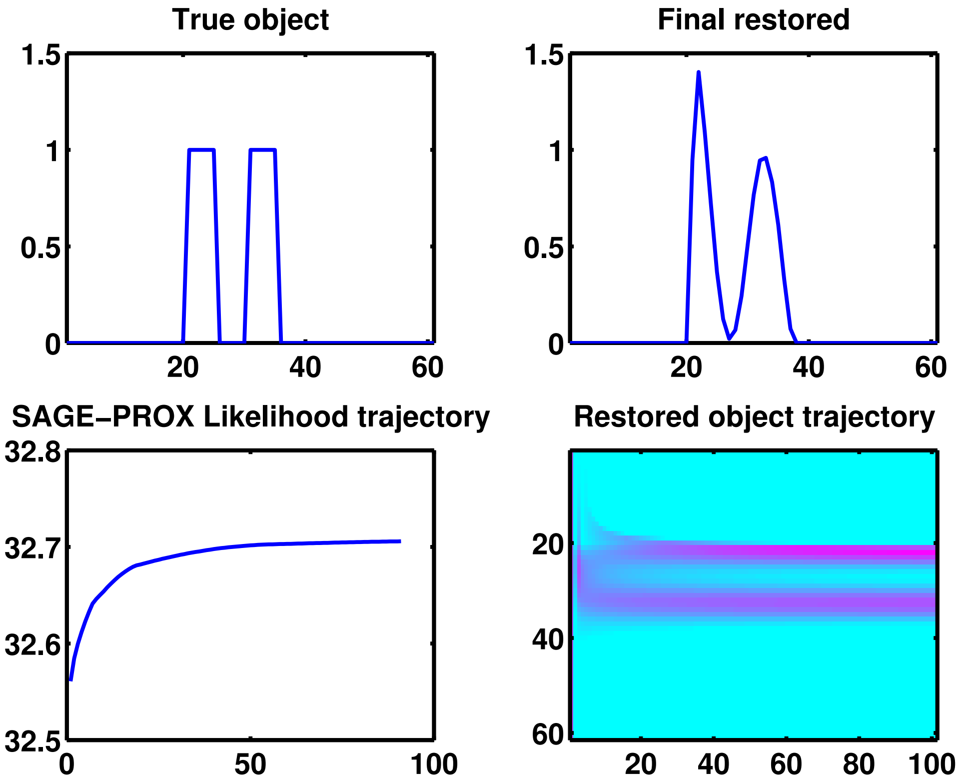


Figure 10: Prox-SAGE reconstruction

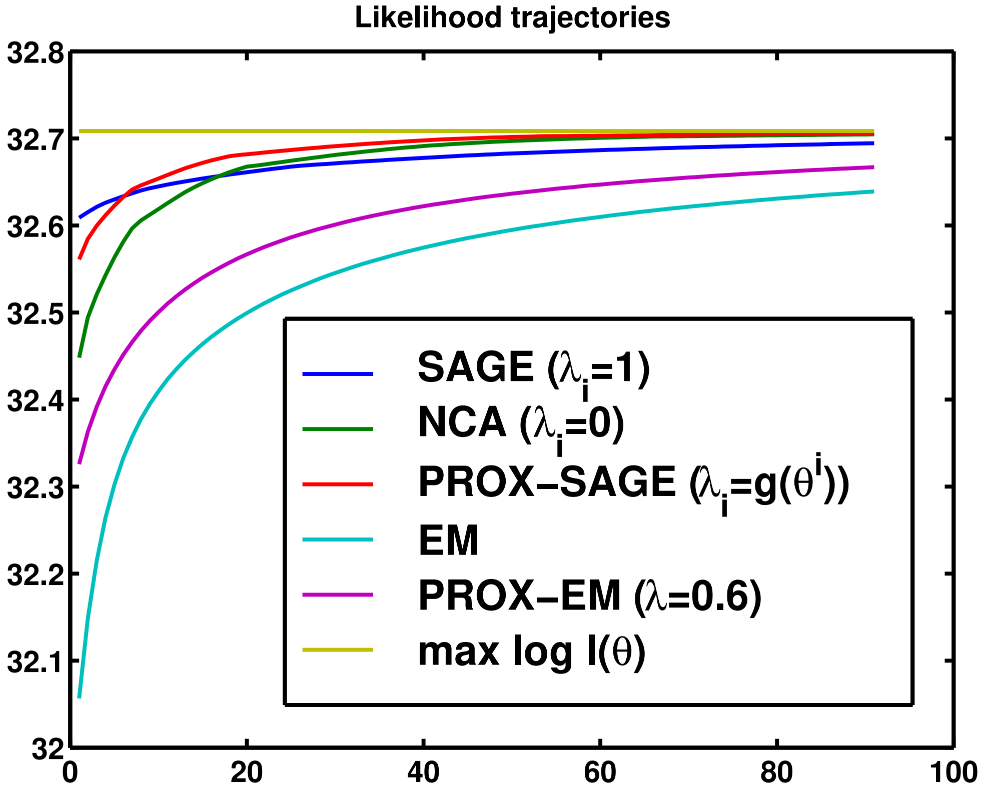
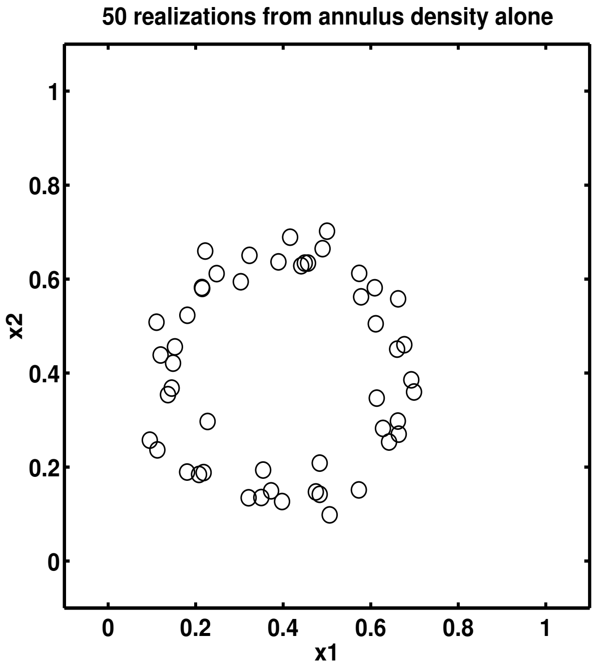
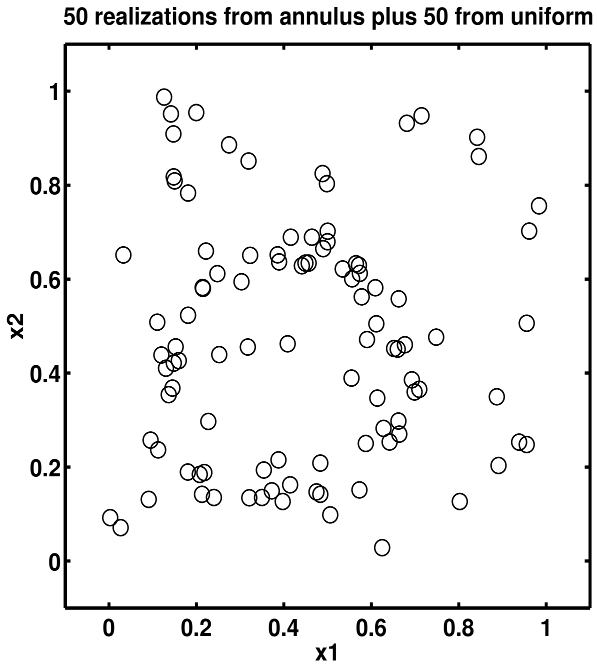


Figure 11: Likelihood trajectory comparisons

Theory and Application of k-shortest Graphs

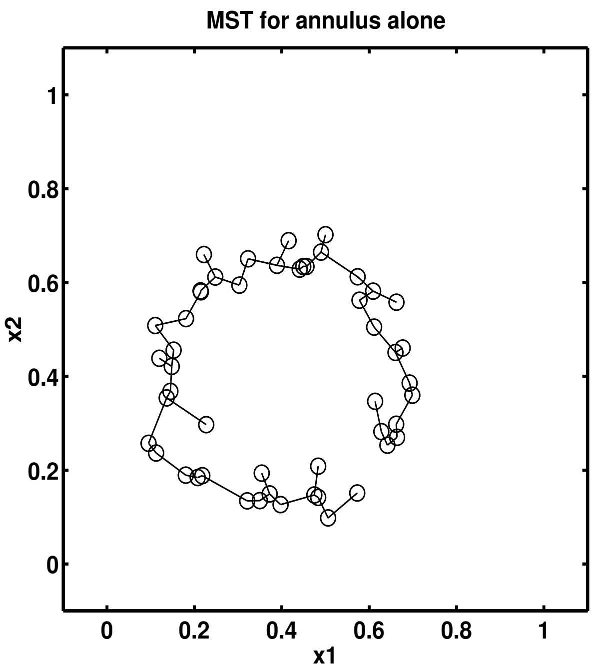


(a)

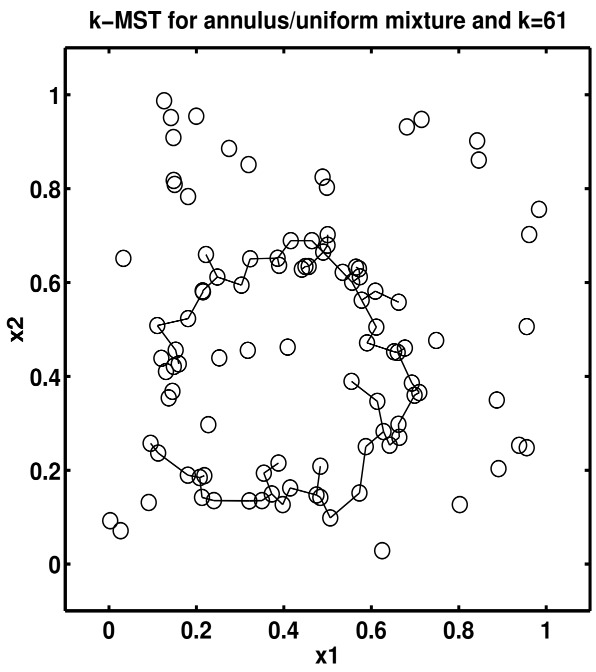


(b)

Definition: MST over $\{x_i\}_{i=1}^n$ in $\mathbb{R}^n$ is **shortest tree spanning the** n **points**



(a)



(b)

Definition: k-shortest MST over $\{x_i\}_{i=1}^n$ in $\mathbb{R}^n$ is **shortest MST spanning any k points**

Accomplishments (Hero&Michel:IT98, Hero&Michel:SPIE98):

- Developed polynomial-time greedy approximations to k-MST.
- Established asymptotic properties of k-MST length function (Renyi entropy).
- Established quantitative robustness of k-MST via influence curve analysis (multi-dimensional rank order statistics).

Applications:

- Robust entropy estimation - extension of Ahmad&Lin:IT76, Vasicek:JRSS76, Hall&Morton:AnnInstStatMath93
- Clustering/classification with false alarms - extension of Toussaint:PR80, Zahn:TC71, Banks:Interface92
- Robust image registration and two-sample matching - extension of Viola&Wells:MIT95, Friedman&Rafsky:AnnStat79)

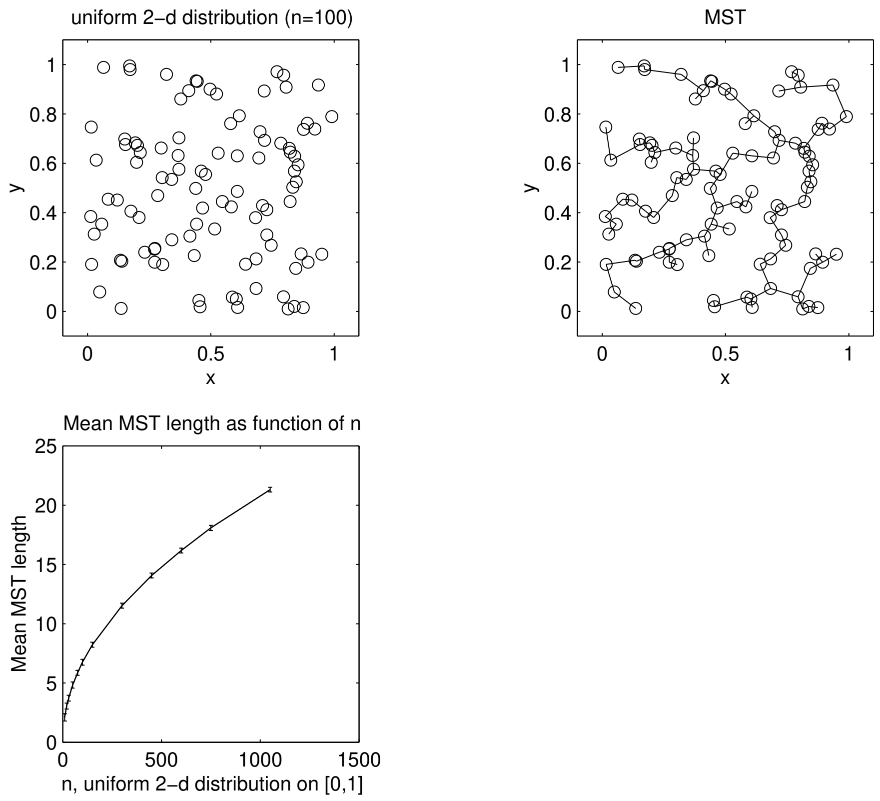


Figure 14: 2D Uniform sample study.

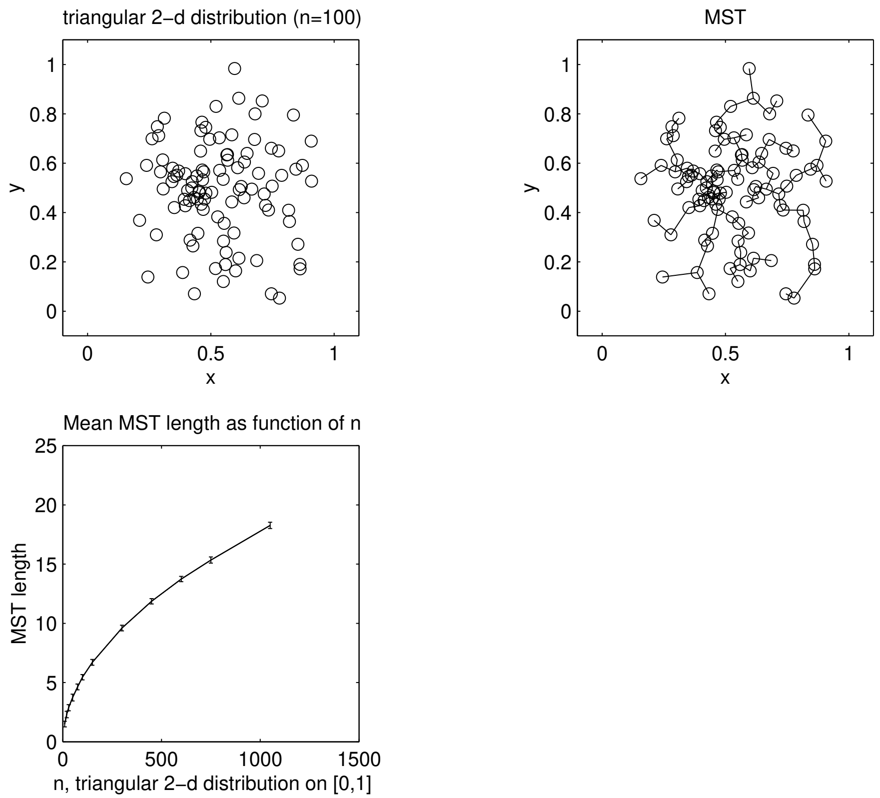


Figure 15: 2D Triangular sample study.

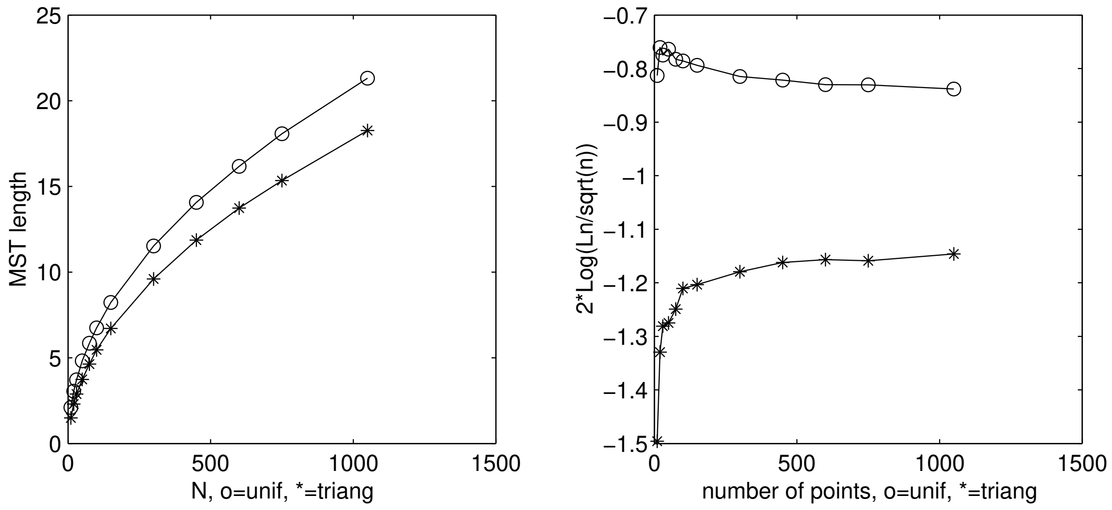


Figure 16: MST Length and log length comparisons.

Principal asymptotic result: extension of Steele:AnnAppProb1988, Redmond&Yukich:AnnAppProb94:

Theorem (Hero&Michel:IT98): For any $\alpha \in (0,1]$ the sum $L_{n,k} = \Sigma\,|e|^\gamma$ of the power-weighted edges of the k-MST satisfies

$$\frac{1}{1-\nu} \ln\left(L_{n,\lfloor \alpha n \rfloor}/(\lfloor \alpha n \rfloor)^\nu\right) \;\rightarrow\; \min_{A:P(A)\geq\alpha} R_\nu(X|A) + \ln \beta_{d,\gamma} \quad (a.s.)$$

where $R_\nu(X|A)$ is the Renyi entropy of the conditional distribution $f(x|X \in A)$ of order $\nu = (d-\gamma)/d$

$$R_\nu(X|A) = \frac{1}{1-\nu}\ \ln \int f^\nu(x|X \in A)dx.$$

Application to Image Registration

Minimum entropy property:

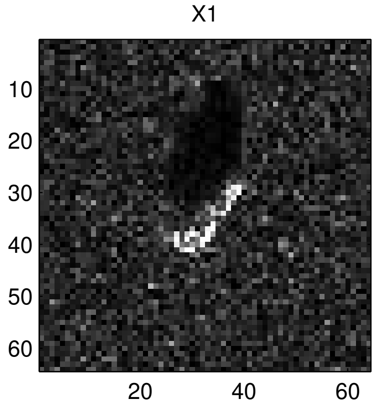
$$R_\nu([X,Y]) \geq R_\nu(X)$$

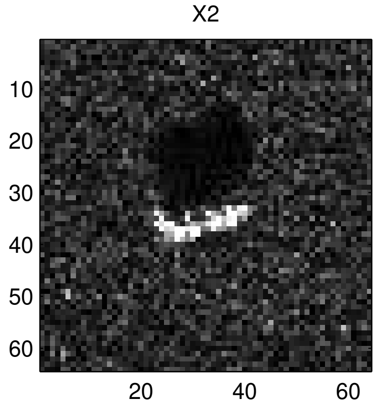
Two step registration process:

1. Select α, $\nu \in [0,1]$
2. Implement outlier rejection on X, Y via k-MST
3. Entropy minimization via MST over coordinate transformations on Y

Applications:

1. **MSTAR targets in clutter**
2. MRI brain image registration



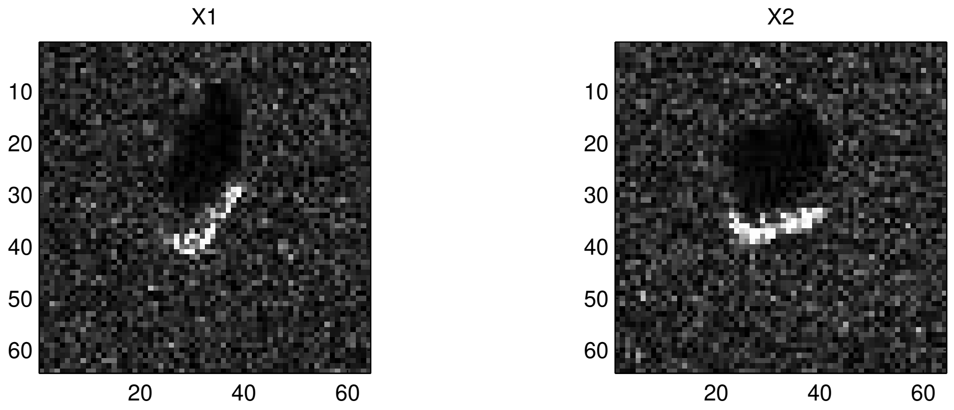


BTR-70 Personnel Carrier - Quadrant 3 ($180° - 270°$)

Targets at 214° and 255° (spotlight mode)

Depression angle 14.95° and 15.12°

Images from MBVLab's MSTAR Radar Image Gallery
(http://www.mbvlab.wpafb.af.mil/public/MBVDATA/img_intro.htm)


X1
X2
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20
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50
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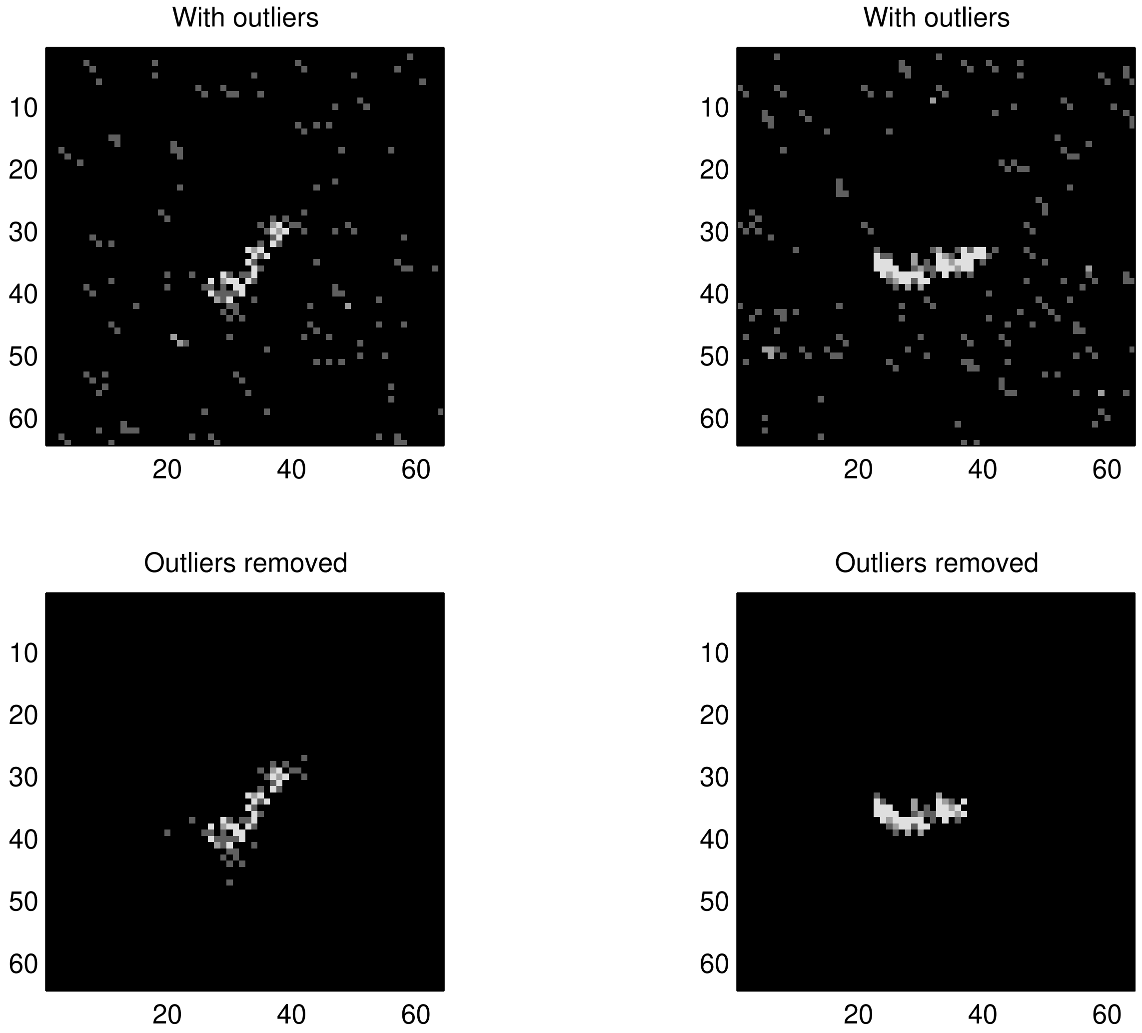
60


With outliers
With outliers
Outliers removed
Outliers removed
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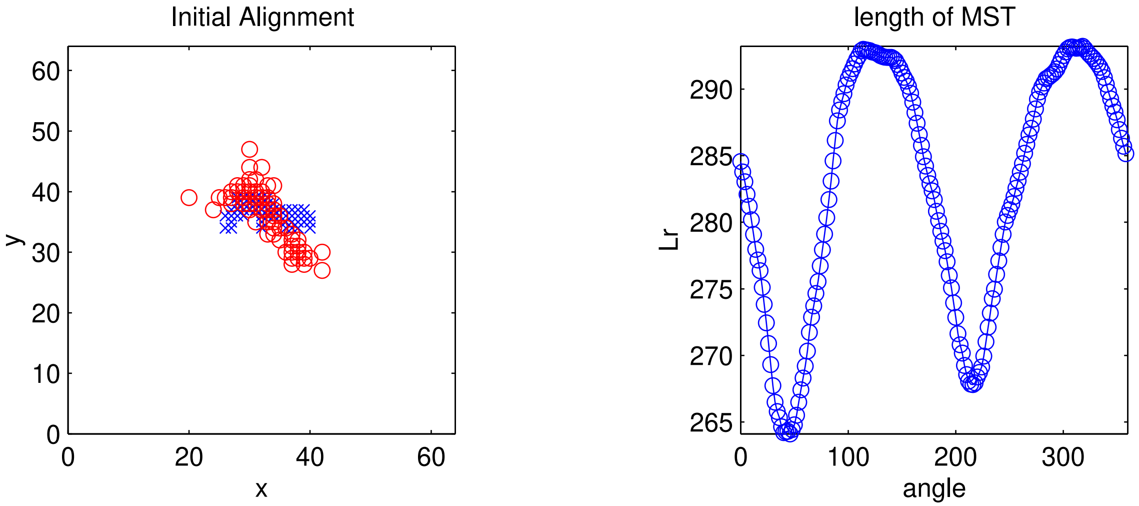

Initial Alignment
60
50
40
30
20
10
0
y
0
20
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x
length of MST
290
285
280
275
270
265
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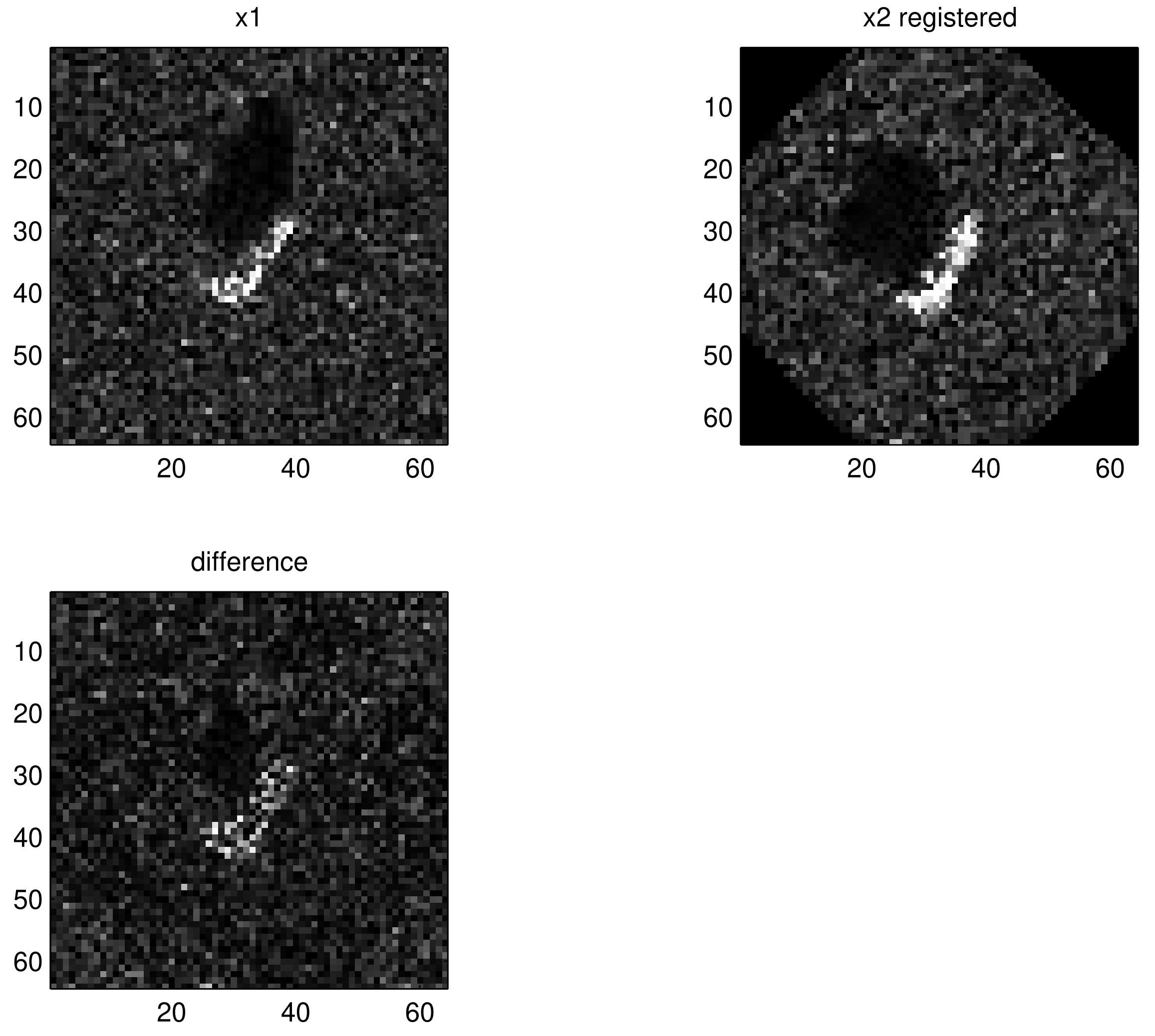
0
100
200
300
angle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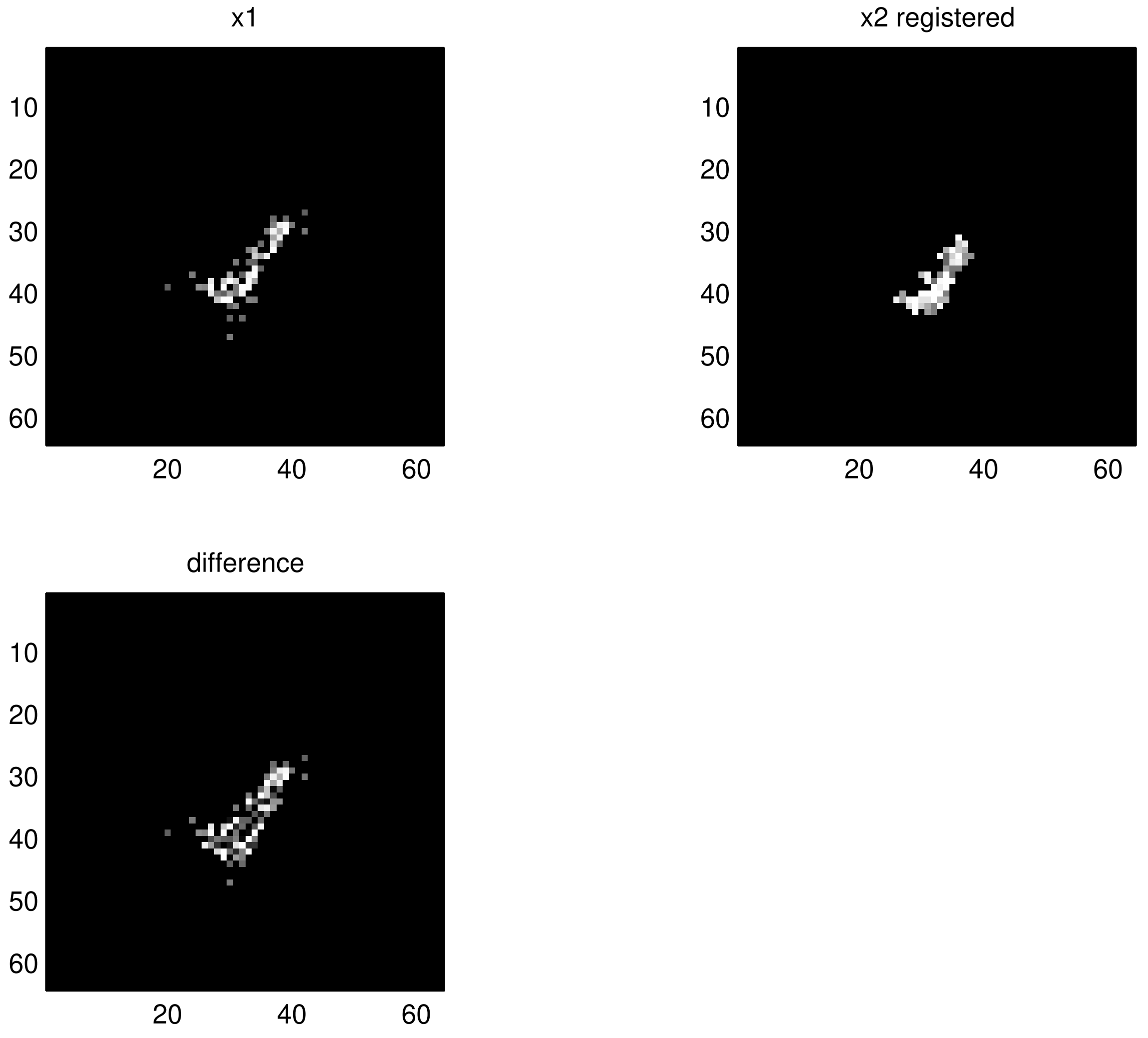
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Future Work:

- Selection of α for outlier rejection, e.g. linear prediction criteria.
- Quantitative comparison to histogram, RBF, kernel estimators of entropy.
- Convergence analysis of k-MST for spatially correlated feature vectors.

*

References

[1] A. Hero and C. Guillouet, "Robust detection of SAR/IR targets via invariance," in *IEEE Int. Conf. on Image Processing*, pp. 472–475, 1997.

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