Definition of Query Procedure
Given any \( n \in \mathbb{N}, \) and \( \varepsilon > 0, \) we search for a moving target \( (n, d, \varepsilon) \)-non-adaptive query procedure consisting of

- \( n \) queries \( A^n \), where for each \( i \in [n] \), \( A_i \) is a yes/no question with \( \Pr_{(x, y) \sim (S, V) \sim \mathcal{D}}[A_i] = \frac{1}{2} \).
- A decoder \( D \) that outputs \( \hat{x} \).

such that the excess-resolution probability satisfies

\[
\Pr[(n, d, \varepsilon)] = \sup_{A^n, D} \max_{V \in \mathcal{D}} \left[ \Pr[[D(x), (S, V), (x, y)] > \varepsilon] \right] \leq \varepsilon.
\]

Accurate estimation of the trajectory implies accurate estimate of the initial location and velocity, and vice versa.

- \( |S_i - S_j| < \frac{\varepsilon}{n} \) and \( |V_i - V_j| < \frac{\varepsilon}{n} \) implies accurate estimation of the trajectory, i.e., \( \text{NLL}_{(S, V)} = \mathbb{O}(\varepsilon) \) implies \( |S_i - S_j| < \frac{\varepsilon}{n} \) or \( |V_i - V_j| < \frac{\varepsilon}{n} \).

- \( |S_i - S_j| > \frac{\varepsilon}{n} \) or \( |V_i - V_j| < \frac{\varepsilon}{n} \) implies poor estimate

Fundamental Limit

- Given any number of queries \( n \in \mathbb{N} \) and \( \varepsilon \in [0, 3], \)

\[
\delta(n, d, \varepsilon) = \inf \{k \in \mathbb{N} : \exists \text{ an \( (n, d, \varepsilon) \)-non-adaptive query procedure}\}.
\]

- Best non-asymptotic resolution achievable by any non-adaptive query procedure with \( n \) queries and excess-resolution probability \( \varepsilon \).

Dual quantity (sample complexity):

\[
n^*\delta(d, \varepsilon) = \inf \{n \in \mathbb{N} : \delta(n, d, \varepsilon) \leq \delta\}
\]

Preliminaries

- \( \mathcal{P}_X = \mathbb{B}(p_X) \) denotes the Bernoulli distribution

\[
p_X^\mathbb{B}(y) = \mathbb{B}(p_X(y)) \quad \text{when } t(\{A(y)\} = q)
\]

- \( \mathbb{F}_{Y|X}^p \) denotes the distribution on \( Y \) induced by \( \mathcal{P}_X \) and \( \mathbb{F}_{Y|X}^p \)

- For any \( (x, y) \in X \times Y \), define the mutual information density

\[
I(x, y) = \log \mathbb{F}_{Y|X}^p(y | x) - \log \mathbb{F}_Y^p(y)
\]

- "Capacity" of measurement dependent channels \( (\mathcal{P}_{Y|X}^p)_{x \sim [p]}

\[
\mathcal{C} = \max_{x \sim [p]} I(x, y) \quad \text{where } \mathcal{P}_Y = \mathbb{B}(p_Y(y))
\]

- "Dispersion" of measurement dependent channels

\[
\mathcal{V}_d = \inf_{x \sim [p]} \mathbb{E}_{Y|X}[\mathcal{V}_d(X, Y)] \quad \text{if } \varepsilon < 0.5
\]

- "Dispersion" of measurement dependent channels

\[
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\]

Main Result

**Theorem 1** For any \( \varepsilon \in (0, 1) \) and finite \( d \in \mathbb{N} \), the minimal achievable resolution \( \delta(n, d, \varepsilon) \) satisfies the following properties

- If \( \omega_{\varepsilon} = O(d^4) \) for \( t \in [0.5, 1], \)

\[
-2d \log \delta(n, d, \varepsilon) = -nC + O(\omega_{\varepsilon})
\]

- If \( \omega_{\varepsilon} = O(d^2) \) for \( t \in [0.5, 1], \)

\[
-2d \log \delta(n, d, \varepsilon) = -nC + O(\omega_{\varepsilon})
\]

Discussions

- Theorem 1 is tight under maximal speed constraint \( \omega_{\varepsilon} \)

- Refines the result by Kaspi et al., ITT 2018 (Theorem 3):

- Non-asymptotic, non-vanishing vs asymptotic, vanishing

- Any measurement dependent channel vs a measurement dependent BSC

- Multidimensional vs one-dimensional

- Strong converse holds

\[
\lim_{n \to \infty} \frac{1}{n} \sum_{i=1}^{n} I(X_i, Y_i) = \mathcal{C}
\]

- Proof ideas: finite blocklength channel coding + analysis of the number of quantized trajectories