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Robust Estimation of Point Process Intensity Features using k-minimal Spanning Trees[1]

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Abstract — **Minimal spanning trees (MST) have been applied to multi-dimensional random processes for pattern recognition and randomness testing (See [1] for references). In this paper we present a robust version of the MST to estimate complexity features of a point process intensity function under an epsilon contaminated model for the intensity. The principal feature considered is the Renyi entropy of the mixture and a strongly consistent entropy estimator is given which depends on the data only through the total length of the MST passing through the data points. Robustification of the MST estimator is achieved by applying the theory of k-minimum MST's [2].**

I. Renyi Features and Fractional Moments

Let $dN(x)$, $x \in \mathcal{X}$, be a multi-dimensional Poisson point process having normalized multivariate intensity $\lambda(x)$ which is the mixture: $\lambda(x) = (1-\epsilon)\lambda_s(x) + \epsilon\lambda_o(x)$, $0 \leq \epsilon < 1$. We assume that the "noise component" λ_o is a uniform intensity and ϵ is unknown. A spanning tree is a connected acyclic graph which passes through all coordinates associated with the point cloud generated by the process. It consists of an ordered list of normalized edge lengths along with a list of edge adjacency relations. The total length of the tree is defined as the sum of all edge lengths. The minimal spanning tree (MST) is the spanning tree which posesses minimal total length. The k-minimum spanning tree is the minimum length MST among those that pass through any k of the n points - thus the standard MST is equivalent to the n-minimum MST. The features are defined as the Renyi entropy $H_\alpha(dN)$ of fractional orders α, $0 < \alpha \leq 1$, associated with λ_s

$$H_\alpha(\lambda_s) = \frac{1}{1-\alpha} \ln \int_{\mathcal{X}} \lambda_s^\alpha(x) dx. \tag{1}$$

The Renyi entropy equals zero for $\alpha = 0$ and converges to the Shannon entropy $-\int_{\mathcal{X}} \lambda_s \ln \lambda_s$ as α approaches 1. For any α the Renyi entropy is maximized for a uniform intensity and minimized for an intensity concentrated at a single point.

Define the fractional moment of the edge lengths $\{l_i\}_{i=1}^{n-1}$ of an MST passing through n points of an observed p-dimensional point cloud

$$L_n^\alpha = \sum_{i=1}^{n-1} l_i^{(1-\alpha)p}. \tag{2}$$

When $\alpha = \frac{p-d}{p}$, d a positive integer, $(L_n^\alpha)^{\frac{1}{d}}$ is the Euclidean l_d norm of the lengths of the MST vertices. By convention, when $(1-\alpha)p = r/q$ is rational $l_i^{(1-\alpha)p} = (l_i^{\frac{1}{q}})^r$ denotes the r-th power of the real positive q-th root of l_i. It follows directly from the results of Steele [3] that the MST-based estimate $\hat{H}_\alpha$ defined below is a strongly consistent estimator of $H_\alpha(\lambda)$,

$$\hat{H}_\alpha(\lambda) = \frac{1}{1-\alpha} \ln \left(n^{-\alpha} L_n^\alpha\right) + \beta(\alpha, p), \tag{3}$$

where β is a constant independent of λ.

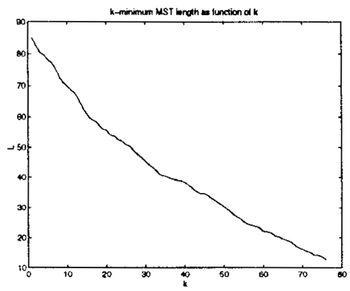

Figure 1: Length of the k-minimum MST plotted as a function of k for 60 points drawn from an annulus intensity + 40 points drawn from a uniform intensity.

II. Robust Estimation via k-MST Pruning

As compared to the length L_n^α of the original MST, the length $L_{n,k}^\alpha$ of the k-minimum MST is a robust Renyi entropy estimate. Robustness is attained since uniform additive noise will tend to produce points whose neighborhoods of "nearest neighbors" are larger than the corresponding neighborhoods of signal points. Thus exchanging a noise point with a signal point will typically reduce the total MST length and thus the k-minimal MST tends to eliminate only noise points in the early iterations $k = 1, 2, 3, \ldots$. By looking for the knee in the k-minimum MST total edge length curve, plotted as a function of k, it is possible to identify the best estimate of the number of offending noise points $N - k$; the corresponding k-minimum MST effectively eliminates these from the point cloud (see Figure 1).

References

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2. R. Ravi, R. Sundaram, M. Marathe, D. Rosenkrantz, and S. Ravi, "Spanning trees short and small,", pp. 546–555, 1994.
3. J. M. Steele, "Growth rates of euclidean minimal spanning trees with power weighted edges," *Ann. Probab.*, pp. 1767–1787, 1988.

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